2D Eigenmodes and space quantisation (I)

- Theories of **self-focusing** based on the **paraxial** NSE lead to the run-away process of optical collapse in which light focuses down to an **unphysical singularity**. In contrast to this, it has been demonstrated experimentally that beams can propagate beyond the predicted focal points as self-trapped filaments [7,8]. **Nonparaxiality** is **fundamental** to light itself and it has been shown both numerically [9] and analytically [10] that, by the inclusion of this feature alone, the **collapse** singularity can be **avoided**.

- We focus on the fundamental properties of **localised solutions** possessing **soliton-like** uniform phase profiles. In particular, modal patterns which involve **optical vortex** structures are considered. While these were originally proposed for propagation in self-defocusing media [11], there is experimental evidence that optical vortices also appear spontaneously in systems which have a **self-focusing Kerr** nonlinearity.

- For the **2D NNSE**, we adopt a normalisation that is directly applicable to a paraxial Gaussian beam in linear propagation

$$i\frac{\partial u}{\partial \zeta} = \frac{\tan^2\Theta}{4}\frac{\partial^2 u}{\partial \zeta^2} + \frac{1}{4}\nabla_t^2 u + \eta\,|u|^2 u \quad (1)$$

where $\Theta = \tan^{-1}(2/kw_0)$ can be identified as the **far-field** beam **angle** from the theory of linear propagation and the **nonlinear parameter**, η, is the normalised power of the input beam.

- We seek **localised solutions** for the field propagating in a homogeneous nonlinear Kerr medium of the form

$$u(R,\theta,\zeta) = U(R)P(\theta)\exp(-i\beta\zeta) \quad (2)$$

where $R=\eta\rho$, $U(R)$ is a radial profile of uniform phase which we take to be real and $P(\theta)$ is the azimutal variation which is allowed to be complex. We examine the case where the transverse intensity profile of the solution is given by $U^2(R)$ and set $P(\theta)=\exp(im\theta)$ where, for continuity purposes, m is restricted to integer values.

- Substituting (2) in (3), we obtain

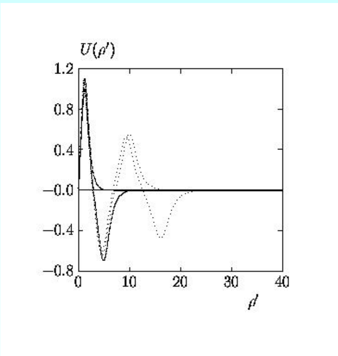
$$\frac{1}{4}\left(\frac{d^2U}{dR^2} + \frac{1}{R}\frac{dU}{dR} - \frac{m^2u}{R^2}\right) - \beta' U + |U|^2 U = 0 \quad (3)$$

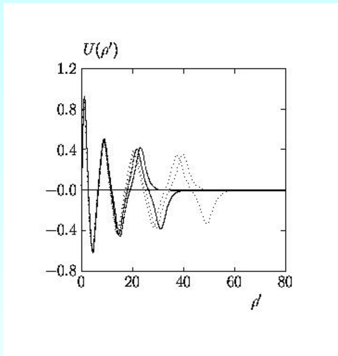
where

$$\beta' = \frac{1}{\eta}\left(\frac{\tan^2\Theta}{4}\beta^2 + \beta\right) \quad (4)$$

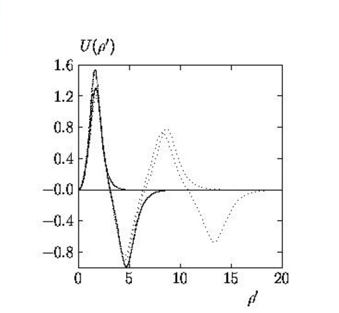
2D Eigenmodes and space quantisation (II)

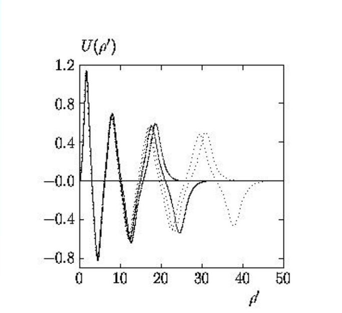
- The following figures show the, numerically computed, profiles of the **vortex modes** with *m*=1 and, respectively, *n*=0,1,2,3 and *n*=4,5,6,7.



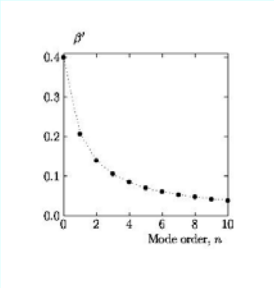


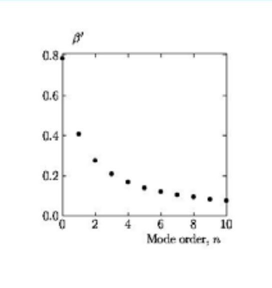
- Vortex modes with *m*=2 and *n*=0,1,2,3 and *n*=4,5,6,7, respectively:





- The **eigenvalues**, β', corresponding to the previously shown eigenmodes are, for *m*=1 and *m*=2:





- We have discovered that **the product of the integrated intensity**, η, **and the eigenvalue**, β', **scales linearly with the order of the eigenmode**. This implies the **quantisation** of the propagation wavenumber for the paraxial 2D solitons and a modified (but related) form of quantisation for the nonparaxial 2D solitons. The figures below show these relations for *m*=1 and *m*=2 modes.