Relativistic Effects in Non-Relativistic Systems

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We propose a new model for describing the evolution of scalar optical pulses in Kerr-type planar waveguides and optical fibres. The wave envelope u satisfies a dimensionless equation that is of the nonlinear Helmholtz type,

$$\kappa \frac{\partial^2 u}{\partial \zeta^2} + i\left(\frac{\partial u}{\partial \zeta} + \alpha \frac{\partial u}{\partial \tau} \right) + \frac{s}{2} \frac{\partial^2 u}{\partial \tau^2} + |u|^2 u = 0 \, ,$$

where the space/time coordinates are ζ / τ respectively, α is related to the group velocity, and $s = \pm 1$ flags the anomalous/normal temporal dispersion regime. This more general model retains the double-ζ term that is otherwise neglected in the literature of conventional pulses. Our governing equation and its solutions are thus endowed with some intriguing and mystical properties.

By considering the behaviour of Eq. (1) under various sets of coordinate transformations, we have found that our new model can be interpreted within the framework of Einstein's special theory of relativity. This leads to a range of new physical predictions that have no counterpart in conventional pulse theory. Exact analytical bright and dark solitons have been derived by applying various mathematical techniques, and the space-time geometry of these new self-localized stationary light structures has been explored in detail. Further analysis and simulations have confirmed the robustness of Helmholtz pulse solitons in the face of perturbations to their temporal shape (see figure 1), which is nice.
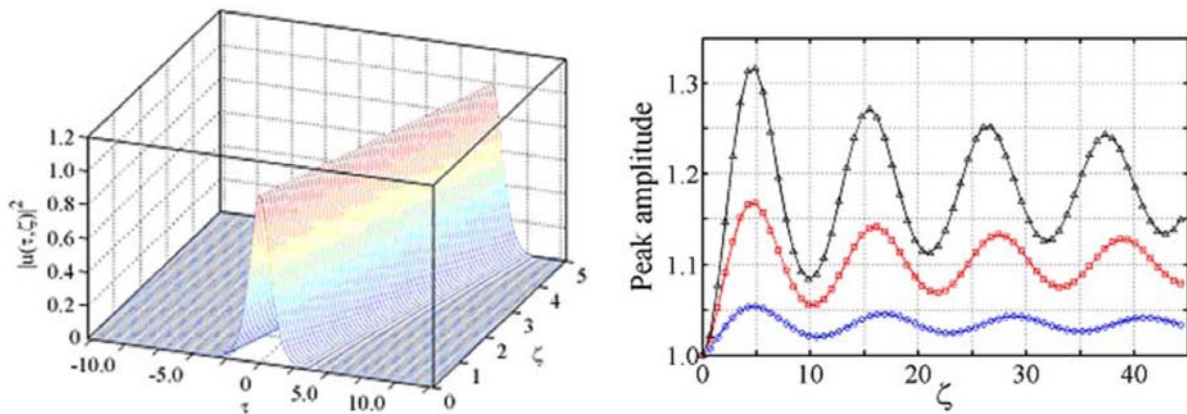

Fig. 1. Left: An exact Helmholtz bright soliton pulse. Right: A perturbed Helmholtz bright pulse injected into a waveguide (with $\kappa < 0$) evolving toward a stationary state of Eq. (1) as $\zeta \to \infty$.