SPONTANEOUS OPTICAL PATTERNS:
FROM SIMPLICITY TO COMPLEXITY IN NONLINEAR CAVITIES

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Introduction

The origin of our understanding of spontaneous pattern formation lies with the seminal work of Alan Turing in the early 1950s (Turing, 1952). Turing discovered a universal mechanism for describing the emergence of simple patterns in reaction diffusion models. He predicted that when such a system is sufficiently stressed, arbitrarily-small disturbances to the uniform states (i.e., those solutions that are stationary in time and homogeneous in space) can lead to spontaneous self-organization into finite-amplitude patterns (see Fig. 1). The emergent pattern tends to have a single dominant scalelength that is directly related to the most unstable spatial frequency in the system; well-known examples are hexagons, honeycombs, squares, stripes, rings, spirals, vortices, and single (or collections of) spots (Cross and Hohenberg, 1993). These patterns are familiar from everyday experience, appearing throughout Nature in activator-inhibitor chemical kinetics (animal coats, fish skins, shells), convective fluid flows (hydrodynamic vortices, Rayleigh-Bernard, Kelvin-Helmholtz, and Couette-Taylor instabilities), weather systems (tornadoes, hurricanes), and astrophysical phenomena (gravitational vortices, rotating spiral arms of galaxies).

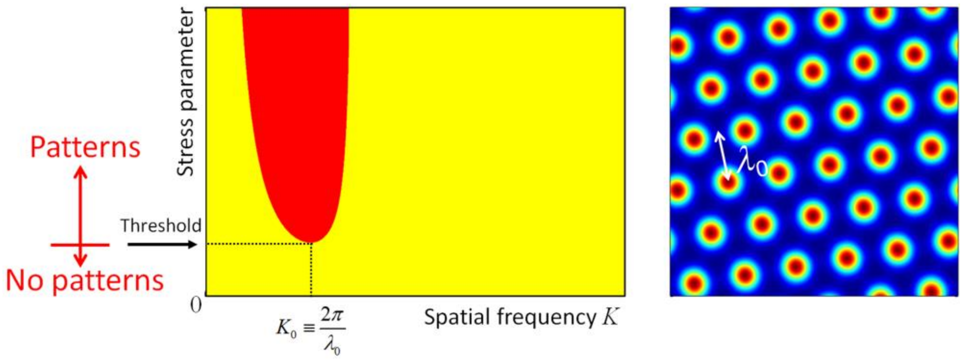

Figure 1: Schematic showing the relationship between a single Turing minimum and the dominant scalelenegth of the emergent pattern. In this case, the hexagonal array is static (i.e., once established, the pattern remains the same and does not change in time).

Nature also bombards us with a second class of pattern that is, in some sense, the antithesis of a 'simple' one. Fractals possess comparable levels of structure spanning many decades of scale. It is often remarked that patterns with many scales are equivalent to patterns with no natural scalelength. Crucially, it is important to note that the *existence* of many different scales in a given pattern is not sufficient for it to merit the term 'fractal'. Rather, the Fourier *amplitudes* associated with those scales (for example, in the power spectrum) must be comparable in order for the multi-scale characteristic to be truly meaningful.

A Route to Spontaneous Spatial Fractals

Turing considered models wherein the threshold instability spectrum comprises a single unique minimum (see Fig. 1). However, there also exists a class of system whose threshold instability spectrum comprises a discrete set of many distinct but comparable minima. When operating under highly stressed conditions (e.g., above several such minima), it seems plausible that Turing's route to spontaneous *simple* patterns (spatial structure on a single dominant scalelength) will also provide a mechanism for generating spontaneous *fractal* patterns (spatial structure on many scales). This universal signature of a system's fractal-generating capacity was proposed by our Group to have independence with respect to nonlinearity [which may be dispersive (Huang and McDonald, 2005) or absorptive (Huang, Christian, and McDonald, 2012)] and the details of the physical geometry.

From Single Feedback-Mirror to Fabry-Pérot

The Kerr slice with a single feedback mirror (SFM) is perhaps the simplest nonlinear optical system to consider. Originally proposed by Firth (1990), it comprises a thin slice of relaxing-diffusing Kerr-type material and a partially-reflecting feedback mirror placed some distance away in free space (see Fig. 2). Despite its simplicity, there are many different aspects to consider: the interplay between forward- and backward-propagating light waves, diffraction, nonlinearity, cavity effects (periodic pumping and losses), and the transverse diffusion of charge carriers that is driven by light intensity. One also has three different timescales to consider: time for light to traverse the slice, the carrier relaxation time, and the round-trip time of the light in the free-space path. This 'simple' system is, in reality, rather complicated and one can expect to find rich layers of spatiotemporal complexity lurking in its dynamics (D'Alessandro and Firth, 1991, 1992, Papoff *et al.*, 1993). Patterns similar to those predicted theoretically have also been observed experimentally (Grynberg *et al.*, 1994).

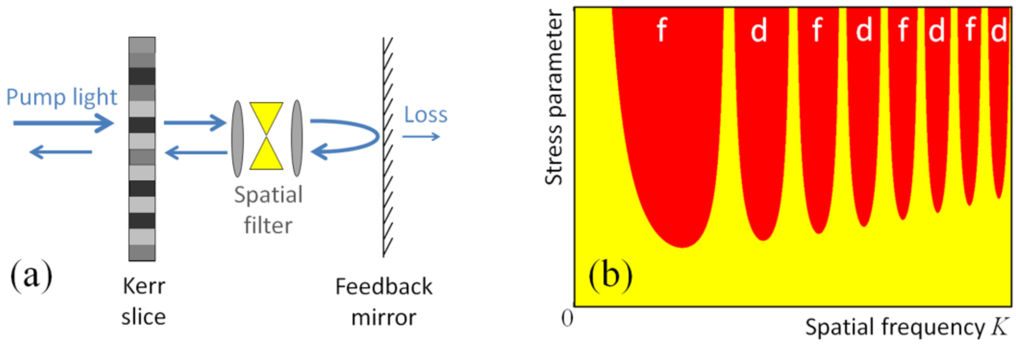

Figure 2: (a) Schematic diagram of the classic single feedback-mirror system (the filter allows for the control of pattern formation). (b) Typical threshold curves for static patterns in Kerr-type materials (f = focusing, d = defocusing). Diffusion modulates the spectrum, facilitating spatial frequency discrimination.

An intuitive generalization is the Fabry-Pérot (FP) cavity. The geometry of the FP cavity is identical to that of the SFM system, except that the input face of the slice is allowed to be partially reflecting. The nonlinear medium is then effectively sandwiched between two mirrors, so that the energy in the pump beam bounces back-and-forth periodically, passing through the slice each time (McDonald, Stephen, and Firth, 1990) [see Fig. 3(a)].

From Simplicity to Complexity

While the addition of a second mirror represents only a very small physical alteration to the system, the ensuing mathematical analysis is disproportionately increased. It is perhaps for this reason that, to date, there exists very limited published literature on nonlinear FP cavities. Historically, treatments have been in the plane-wave limit (Marburger and Felber, 1978, Firth, 1981), or predominantly computational in nature (Möller *et al.*, 1998).

The motivation behind the current programme of research is to develop a framework for predicting spatial instabilities in a nonlinear FP cavity, taking full account of transverse (diffraction and diffusion) effects. So far, we have found the uniform states of the cavity and deployed linearization techniques to quantify their susceptibility to spontaneous pattern-forming instabilities. Key results include the derivation of the threshold condition, with preliminary numerical calculations confirming theoretical predictions. For weak slice reflectivity, the instability minima resemble those for the SFM [see Fig. 2(b)] but with gentle modulations appearing in the lobes. For stronger reflectivities, the qualitative features of the spectrum change dramatically with the discrete and well-defined *bands* breaking-up into a sequence of instability *islands* [see Fig. 3(b)].

Unlike its SFM counterpart, the simple pattern-forming properties of the FP cavity remain largely unknown. Having isolated the Turing (static) patterns, the next key objective is to unravel the Hopf (oscillatory) patterns. We can also consider the spatiotemporal dynamics with both one and two transverse dimensions. A second

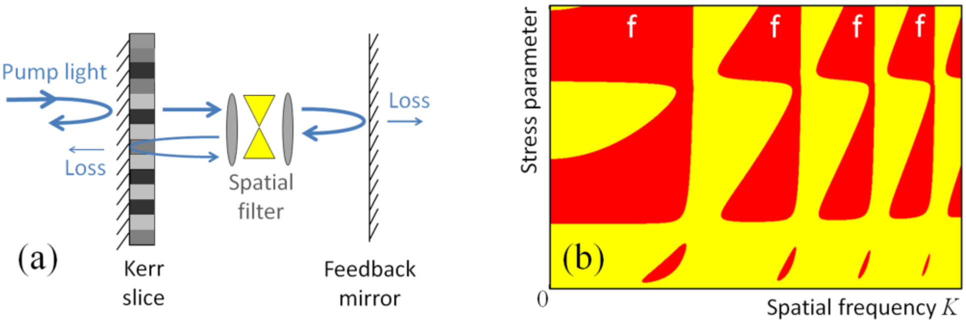

Figure 3: (a) Schematic diagram of the Fabry-Pérot cavity. (b) Typical threshold curves for static patterns in focusing Kerr-type materials [compare with the four f instability bands shown in Fig. 2(b)].

crucial (and computationally intensive) task is to consider the potential role of the FP cavity as a candidate fractal-generating device. The SFM system is already known to generate spatial fractals (Huang and McDonald, 2005) whose characteristics (e.g., dimension) depend upon parameters such as feedback-mirror reflectivity and diffusion length. Our intention is to undertake a similarly exhaustive analysis of the FP configuration.

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