

# SPATIAL SOLITONS AT OPTICAL INTERFACES: NEW ANGLES, PARADIGMS, & HORIZONS

Emily A. McCoy, James M. Christian, and Graham S. McDonald

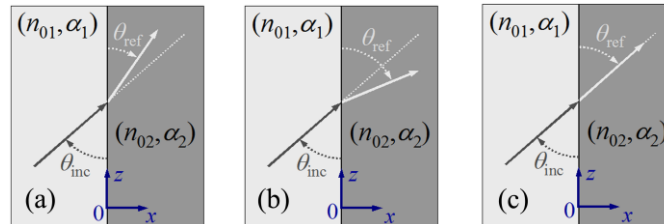
e.mccoy1@edu.salford.ac.uk & j.christian@salford.ac.uk

Materials & Physics Research Centre

## Introduction

The interaction of spatial solitons (self-collimating and self-stabilizing beams of light) with the planar interface between two nonlinear dielectric materials is perhaps an elementary problem in modern photonics. Seminal analyses by Aceves, Moloney, & Newell (1988a, 1988b) more than two decades ago considered the scattering of a single soliton incident on the boundary between two dissimilar Kerr-type materials. They developed a powerful and intuitive “equivalent particle” method (Aceves *et al.*, 1989a, 1989b) for quantifying a raft of interface-related phenomena, including: reflection, refraction, trapping, Goos-Hänchen (GH) shifting [where the reflected beam suffers a displacement *along the interface* relative to the path predicted by geometrical optics (Goos & Hänchen, 1947)], and beam splitting (where a single incident soliton breaks up into multiple filaments as it crosses the material boundary). Their impressive body of work, published in the late 1980s (Aceves *et al.*, 1988a, 1988b, 1989a, 1989b) and early 1990s (Aceves *et al.*, 1990, Varatharajah *et al.*, 1990, Aceves & Moloney, 1992) paved the way to 20 years of highly fruitful international research on the *solitons at interfaces* class of problem.

While Aceves and co-workers undeniably laid the theoretical foundations for the understanding of many scalar interface-type phenomena, their approach suffered from one fundamental limitation: the analysis is rooted firmly within paraxial wave optics. While the paraxial approximation simplifies the governing equation considerably (reducing it from the Helmholtz-type to the Schrödinger-type), it simultaneously restricts angles of incidence, reflection, and refraction (measured relative to the interface *in the laboratory frame*) to negligibly or near-negligibly small values (see Fig. 1). With this physical restriction in mind, it is not untrue to say that the intrinsic angular nature of soliton refraction is still not widely understood.

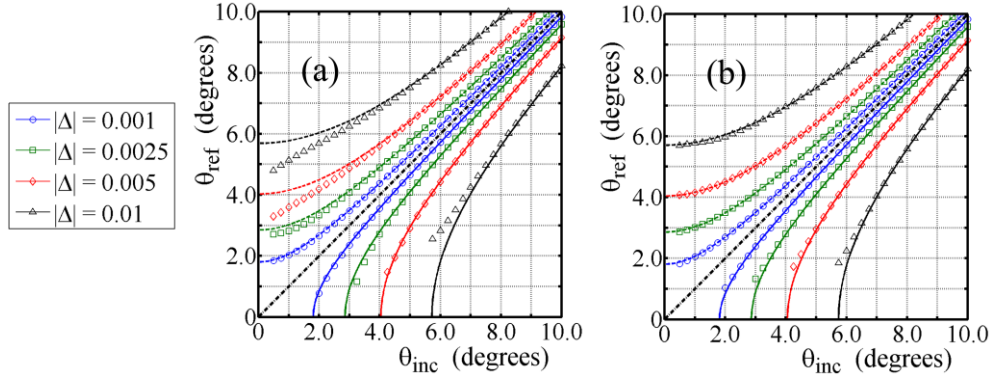


**Figure 1: Schematic showing (a) internal and (b) external refraction. Incident and refracted beams make angles  $\theta_{\text{inc}}$  and  $\theta_{\text{ref}}$ , respectively, relative to the interface. The straight-through condition is shown in (c), where there is no net change in the *total* effective refractive index. The linear and nonlinear material properties on each side of the interface are parametrized by  $n_{0j}$  and  $\alpha_j$  ( $j = 1$  and  $2$ ), respectively.**

In this presentation, we will give a brief overview of some key findings from recent research. Topics to disseminate include nonparaxial refraction, novel material considerations, and multi-layer geometries.

## New Angles: Nonparaxial Refraction

In collaboration with co-workers at the Universidad de Valladolid, Spain, our Group has been developing new models of spatial soliton refraction since 2007 (Sánchez-Curto *et al.*, 2007). By neglecting the classic assumption of beam paraxiality, we established the first mathematical framework capable of describing *arbitrary-angle refraction* at Kerr-type interfaces. This type of approach, based upon the underlying nonlinear



**Figure 2: Comparison of theoretical refraction angles [curves, obtained from Eq. (1)] with full numerical calculations (points) for an interface system with a power-law nonlinearity (the mismatch in the linear refractive index is quantified by  $\Delta = 1 - n_{02}^2/n_{01}^2$ ). (a) Relatively narrow beam. (b) Broader beam.**

Helmholtz equation, is crucial for capturing the angular characteristics of the problem at hand. Preliminary analyses of bright (Sánchez-Curto *et al.*, 2007, 2009) and dark (Sánchez-Curto *et al.*, 2010, 2011a) soliton refraction involved deriving a compact generalization of the famous Snell's law (Jackson, 1999), wherein the interplay between finite beam waists and mismatches in material properties is manifest within a single parameter  $\gamma$  (see Fig. 1):

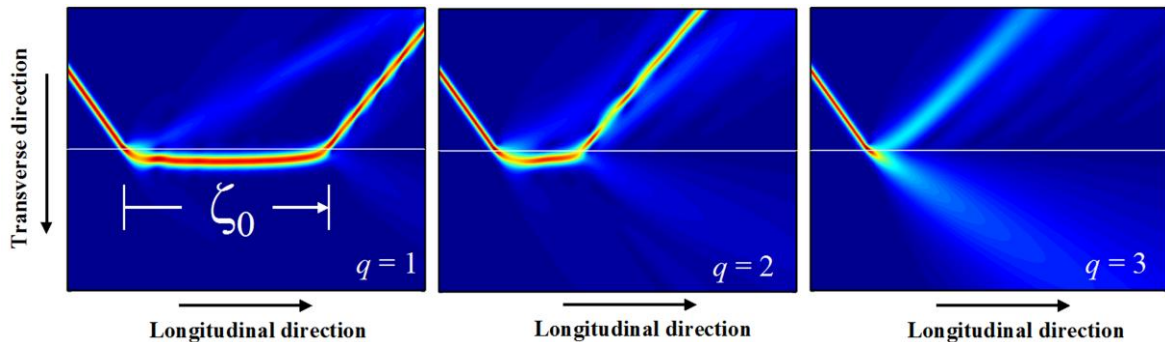
$$\gamma n_{01} \cos \theta_{\text{inc}} = n_{02} \cos \theta_{\text{ref}} \quad (1)$$

( $n_{01}$  and  $n_{02}$  are the linear refractive indexes on either side of the boundary). Subsequent numerical computations made some surprising new predictions about the nature of GH shifts (Sánchez-Curto *et al.*, 2011b). Simulations, combined with inverse-scattering methods, later quantified soliton splitting in nonparaxial regimes (Sánchez-Curto *et al.*, 2012).

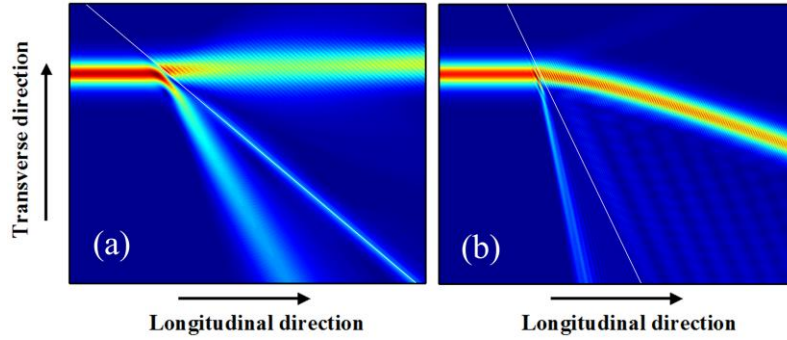
### New Paradigms: Non-Kerr Materials

Our attention has recently turned from the idealized Kerr-type response to other, more general, classes of host material. The ubiquitous the power-law nonlinearity (parametrized by a continuum exponent  $0 < q < 4$ ) includes the Kerr effect as a particular case (i.e.,  $q = 2$ ) (Mihalache *et al.*, 1989). By deploying established techniques, and using known power-law Helmholtz solitons (Christian *et al.*, 2007a) as basis functions, we have identified a wide range of new quantitative and qualitative phenomena that depend on the exponent  $q$  (Christian *et al.*, 2012). Excellent agreement has been uncovered between the theoretical predictions of Eq. (1) and full numerical computations. We have also discovered new qualitative phenomena allied to GH shifts in non-Kerr materials (see Fig. 3).

Another classic material system that has received very little attention in the interfaces literature is the cubic-quintic nonlinearity (Pushkarov *et al.*, 1979). This universal model incorporates leading-order contributions



**Figure 3: Simulations showing GH shifting (denoted by  $\zeta_0$ ) at the (linear) interface between two power-law materials (Christian *et al.*, 2012). The shift can be augmented in sub-Kerr regimes ( $q = 1$ ), but may disappear in super-Kerr regimes ( $q = 3$ ), where the reflected beam eventually disintegrates into radiation.**



**Figure 4: Simulations of a spatial soliton injected obliquely from a Kerr continuum into a coupled-waveguide array at (a) a quasi-paraxial angle of incidence ( $\theta_{\text{inc}} = 4.5$  degrees), and (b) a nonparaxial angle of incidence ( $\theta_{\text{inc}} = 10.0$  degrees). In part (a), one can see the excitation of a nonlinear surface wave.**

from both the  $\chi^{(3)}$  and  $\chi^{(5)}$  electric susceptibility tensors. We will present some of our more recent results for the cubic-quintic governing equation (Christian *et al.*, 2007b), which includes a Snell's-law type of analysis [deriving the most general  $\gamma$  parameter to date for Eq. (1)] and many new numerical results. For instance, we have probed the extreme sensitivity of the GH effect to variations in  $\theta_{\text{inc}}$  and have uncovered what are, to the best of our knowledge, the largest shifts ever reported.

### New Horizons: Coupled Waveguide Arrays

We will conclude with a brief summary of our preliminary simulations of the side-coupling of spatial solitons from a nonlinear continuum into a periodic array of waveguides. This class of problem has received much attention from the photonics community over the past decade, but nearly all analyses have been restricted by the paraxial approximation (Mandelik *et al.*, 2004, Sukhorukov *et al.*, 2004). Here, we take the first steps toward understanding the behaviour of nonlinear light beams when they travel obliquely (i.e., at arbitrary angles) across patterned optical structures such as coupled-waveguide arrays and photonic crystals (see Fig. 4).

### References

- Aceves, A. B., Moloney, J. V., & Newell, A. C. (1988a). Reflection and transmission of self-focused channels at nonlinear dielectric interfaces, *Optics Letters*, 13(11), pp. 1002–1004.
- Aceves, A. B., Moloney, J. V., & Newell, A. C. (1988b). Snell's law at the interface between nonlinear dielectrics, *Physics Letters A*, 129(4), pp. 231–235.
- Aceves, A. B., Moloney, J. V., & Newell, A. C. (1989a). Theory of light-beam propagation at nonlinear interfaces. I. Equivalent-particle theory for a single interface, *Physical Review A*, 39(4), pp. 1809–1827.
- Aceves, A. B., Moloney, J. V., and Newell, A. C. (1989b). Theory of light-beam propagation at nonlinear interfaces. II. Multiple-particle and multiple-interface extensions, *Physical Review A*, 39(4), pp. 1828–1840.
- Aceves, A. B., Varatharajah, P., Newell, A. C., Wright, E. M., Stegeman, G. I., Heatley, D. R., Moloney, J. V., & Adachihra, H. (1990). Particle aspects of collimated light channel propagation at nonlinear interfaces and in waveguides, *Journal of the Optical Society of America B*, 7(6), pp. 963–974.
- Aceves A. B. & Moloney, J. V. (1992). Effect of two-photon absorption on bright spatial soliton switches, *Optics Letters*, 17(21), pp. 1488–1490.
- Christian, J. M., McDonald, G. S., & Chamorro-Posada, P. (2007a). Helmholtz solitons in power-law optical materials, *Physical Review A*, 76(3), art. no. 033834.
- Christian, J. M., McDonald, G. S., and Chamorro-Posada, P. (2007b). Bistable Helmholtz solitons in cubic-quintic materials, *Physical Review A*, 76(3), art. no. 033833.

- Christian, J. M., McCoy, E. A., McDonald, G. S., Sánchez-Curto, J., & Chamorro-Posada, P. (2012). Helmholtz bright spatial solitons and surface waves at power-law optical interfaces, *Journal of Atomic, Molecular, and Optical Physics*, in press.
- Goos F. & Hänchen H. (1947). Ein neuer und fundamentaler Versuch zur Totalreflexion, *Annals der Physik*, 1(1), pp. 333–346.
- Jackson, J. D. (1999). *Classical Electrodynamics* (3rd ed.). New York: John Wiley & Sons.
- Mandelik, D., Morandotti, R., Aitchison, J. S., & Silberberg, Y. (2004). Gap solitons in waveguide arrays, *Physical Review Letters*, 92(9), art. no. 093904.
- Mihalache, D., Bertolotti, M., & Sibilia, C. (1989). Nonlinear wave propagation in planar structures, *Progress in Optics*, 27(1), 229–313.
- Pushkarov, K. I., Pushkarov, D. I., & Tomov, I. V. (1979). Self-action of light beams in nonlinear media: soliton solutions, *Optical and Quantum Electronics*, 11, pp. 471–478.
- Sánchez-Curto, J., Chamorro-Posada, P., & McDonald, G. S. (2007). Helmholtz solitons at nonlinear interfaces, *Optics Letters*, 32(9), pp. 1126–1128.
- Sánchez-Curto, J., Chamorro-Posada, P., & McDonald, G. S. (2009). Nonlinear interfaces: intrinsically nonparaxial regimes and effects, *Journal of Optics A*, 11, art. no. 054015.
- Sánchez-Curto, J., Chamorro-Posada, P., & McDonald, G. S. (2010). Dark solitons at nonlinear interfaces, *Optics Letters*, 35(9), pp. 1347–1349.
- Sánchez-Curto, J., Chamorro-Posada, P., & McDonald, G. S. (2011a). Black and grey Helmholtz Kerr soliton refraction, *Physical Review A*, 83(1), art. no. 013828.
- Sánchez-Curto, J., Chamorro-Posada, P., & McDonald, G. S. (2011b). Giant Goos-Hänchen shifts and radiation-induced trapping of Helmholtz solitons at nonlinear interfaces, *Optics Letters*, 36(18), pp. 3605–3607.
- Sánchez-Curto, J., Chamorro-Posada, P., & McDonald, G. S. (2012). Helmholtz bright and black soliton splitting at nonlinear interfaces, *Physical Review A*, 85(1), art. no. 013836.
- Sukhorukov, A. A., Neshev, D., Krolikowski, W., & Kivshar, Y. S. (2004). Nonlinear Bloch-wave interaction and Bragg scattering in optically induced lattices, *Physical Review Letters*, 92(9), art. no. 093901.
- Varatharajah, P., Newell, A. C., Moloney, J. V., & Aceves, A. B. (1990). Transmission, reflection, and trapping of collimated light beams in diffusive Kerr-like nonlinear media, *Physical Review A*, 42(3), pp. 1767–1774.