Multi-Turing Instability and Spontaneous Spatial Fractals in Simple Optical Systems

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Abstract: We analyze the spontaneous fractal-generating properties of classic nonlinear optical systems. New results are presented for Fabry-Pérot cavities, along with the first predictions of multi-scale patterns in models beyond the paraxial approximation.
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1. Turing and multi-Turing instabilities

Alan Turing's groundbreaking ideas [1], published over half a century ago in 1952, have played a pivotal role in explaining the origins of pattern and form in Nature. In his seminal work, Turing discovered a universal mechanism for describing the birth of *simple patterns* in reaction-diffusion models: when a system is sufficiently stressed, arbitrarily-small disturbances to its uniform states can lead to spontaneous self-organization into finite-amplitude patterns. The emergent spatial structure is typically characterized by a single dominant scalelength that is directly related to the most-unstable Fourier mode. Familiar examples of this type of winner-takes-all dynamics include hexagons, honeycombs, squares, stripes, spirals, rings, and vortices [2].

Previously, we have proposed a generic mechanism for predicting a nonlinear wave-based system's capacity to generate *fractal patterns* (structures with comparable levels of detail spanning decimal orders of scale) [3]: *any* system whose Turing threshold spectrum comprises a hierarchy of comparable minima may be susceptible to spontaneous pattern-forming instabilities, where intrinsic nonlinear dynamics (harmonic generation and wave-mixing processes) generate new spatial scales. This route to fractality was investigated in a simple optical model: the classic diffusive Kerr slice with a single feedback mirror [4]. Emergent patterns were found to be examples of Berry's scale-dependent fractals [5]. Trends in the variation of dimension with material properties (e.g., diffusion length of carriers) and system parameters (e.g., mirror reflectivity and pump intensity) were also identified [3,6].

2. New contexts for spatial optical fractals: *cavities*

While ring cavities are well known to have multi-Turing threshold spectra [7,8], this inherent physical property tends to disappear in the mean field limit [9] – longitudinal averaging reduces the instability problem to, essentially, a quadratic characteristic equation (which is associated with a single minimum). Hence, profoundly new regimes of (multiple scale) pattern formation cannot be described with traditional mean-field models. In this presentation, we report on recent research developments that test the independence of our fractal-generating mechanism with respect to system nonlinearity and the details of external feedback by considering two new configurations: a diffusive Kerr slice and a saturable-absorber slice [10] inside a ring cavity.

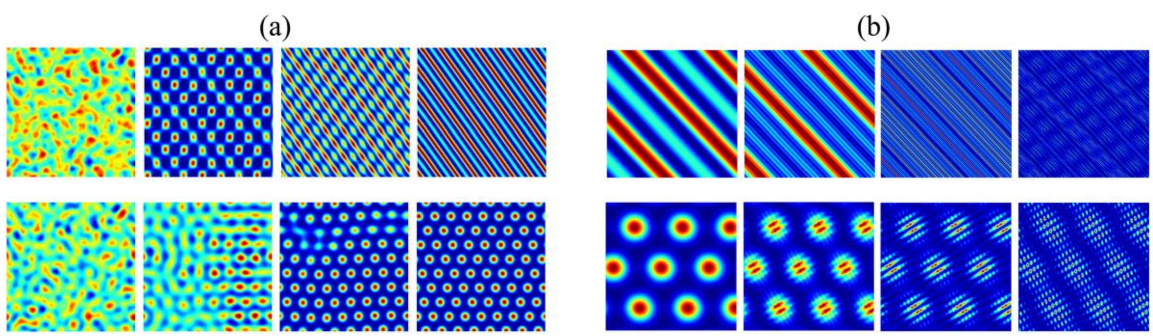

Fig. 1. (a) Spontaneous formation of simple Turing patterns in a ring cavity containing a thin slice of diffusive Kerr material (upper panes) and a diffusive Maxwell-Bloch saturable absorber (lower panes). The stripes and hexagons emerge when the stationary states of the system are initialized with a small level of background noise. (b) Transformation of simple Turing patterns towards spatial fractals.

Effective control of pattern formation can be achieved by placing a spatial filter in the free-space path of the cavity [3,11]. Simple patterns appear when only those spatial frequencies lying within the first instability band are allowed to propagate around the loop [see Fig. 1(a)]. Once the static patterns (e.g., stripes and hexagons) have taken over the system, the filter is instantaneously removed so that all (physical) spatial frequencies may contribute to the dynamics. The patterns subsequently evolve toward fractals [see Fig. 1(b)].

A new geometry to be discussed is the generalization of the single feedback-mirror system to a nonlinear Fabry-Pérot (FP) cavity, where one face of the slice is allowed to be partially reflecting [see Fig. 2(a)]. The FP cavity epitomizes optical complexity: it involves the interplay between diffraction of counterpropagating fields, diffusion of the medium photoexcitation density, and a range of cavity effects (round-trip time, mistuning, periodic pumping and losses). Analysis has uncovered multi-Turing instability minima that are precisely those proposed as indicative of spontaneous fractal generation [3] [see Fig. 2(b)].

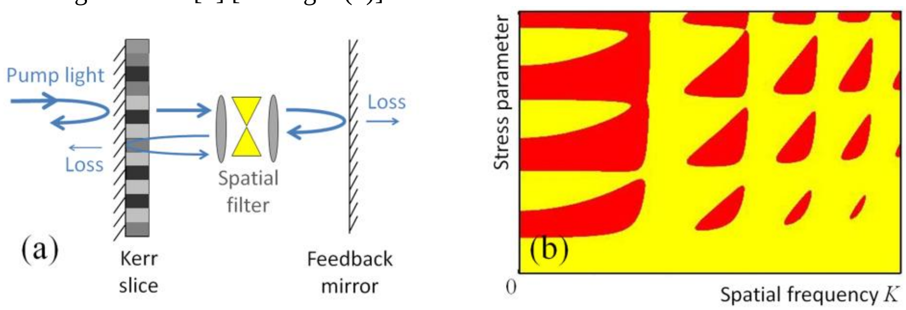

Fig. 2. (a) Schematic diagram of the nonlinear FP cavity. The spatial frequency filter controls the maximum K that can propagate around the loop. (b) Typical multi-Turing threshold spectrum for an FP cavity with a focusing diffusive Kerr nonlinearity. For increasing slice reflectivity, the classic instability 'lobes' for the single feedback-mirror system [3,4] break up into bands of discrete instability 'islands'.

3. New contexts for spatial optical fractals: *bulk media*

To date, fractal analyses have focused predominantly on classic slice-based systems, where diffraction inside the nonlinear material can be safely neglected. While thin-slice geometries certainly have a modern physical context (e.g., in terms of superlattice structures), it is also essential to address fractal pattern formation in bulk optical media. We will present some of our key findings for optical fractal formation in two such contexts: (i) counterpropagating beams in Kerr media (this simple cavityless configuration possesses multi-Turing threshold minima [12]), and (ii) a fully-nonparaxial model for propagation in a filled Kerr ring cavity.

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