Spatiotemporal Vector Solitons in Dispersive Systems with Kerr Nonlinearity

J. M. Christian, G. S. McDonald, and J. T. Ashley
Materials & Physics Research Centre, University of Salford, Greater Manchester M5 4WT, U.K.

P. Chamorro-Posada
ETSI Telecomunicación, Universidad de Valladolid, Campus Miguel Delibes Paseo Belén, E-47011 Valladolid, Spain
Author e-mail address: j.christian@salford.ac.uk

Abstract: A model is proposed for describing the evolution of coupled optical waves in nonlinear systems with both spatial and temporal dispersion. New results include exact analytical bright-bright solitons and an investigation of modulational instabilities.
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1. Modelling spatial dispersion in vector contexts

Menyuk's seminal analysis from 1987 [1] has undeniably helped lay the foundations of today's understanding of coupled optical modes in nonlinear systems. Rooted in the conventional "slowly-varying envelope approximation (SVEA) + Galilean-boost" formalism, a plethora of vector Schrödinger-type models have been developed and investigated exhaustively over the past two decades. While the SVEA remains a cornerstone of traditional photonics modelling, Bianacalana and Creatore [2] have pointed out that there exist modern contexts where its validity may be reassessed. In particular, they assert that spatial dispersion (for example, related to light-exciton coupling inside superlattice structures) is not necessarily well-described by the SVEA.

In a recent paper [3], we proposed a model for describing scalar pulses in the presence of spatiotemporal dispersion based on the more natural Helmholtz-type governing equation. In this presentation, we generalize that pproach to accommodate the simultaneous propagation of two optical modes. For a Kerr nonlinearity, the dimensionless wave envelopes u_j (with j = 1 and 2) satisfy

$$\kappa_j \frac{\partial^2 u_j}{\partial \zeta^2} + i\left(\frac{\partial u_j}{\partial \zeta} + \alpha_j \frac{\partial u_j}{\partial \tau}\right) + \frac{s_j}{2}\frac{\partial^2 u_j}{\partial \tau^2} + \left(\left|u_j\right|^2 + \sigma\left|u_{3-j}\right|^2\right)u_j = 0\,, \tag{1}$$

where (τ,ζ) are normalized laboratory time and (longitudinal) space coordinates. Here, $\kappa_j << O(1)$ and $s_j = O(1)$ parametrize spatial and temporal (i.e., group-velocity) dispersion, respectively, while α_j is related to the group velocity and σ determines the nonlinear (cross-phase) coupling between the two modes. As with its scalar counterpart [3], frame-of-reference considerations play a key role and space-time transformations dominate much of our analysis of Eq. (1). Of particular interest here are the exact stationary solutions and their stability, where the $\kappa_j\partial^2/\partial\zeta^2$ derivatives introduce surprising new parameter complexities into the problem.

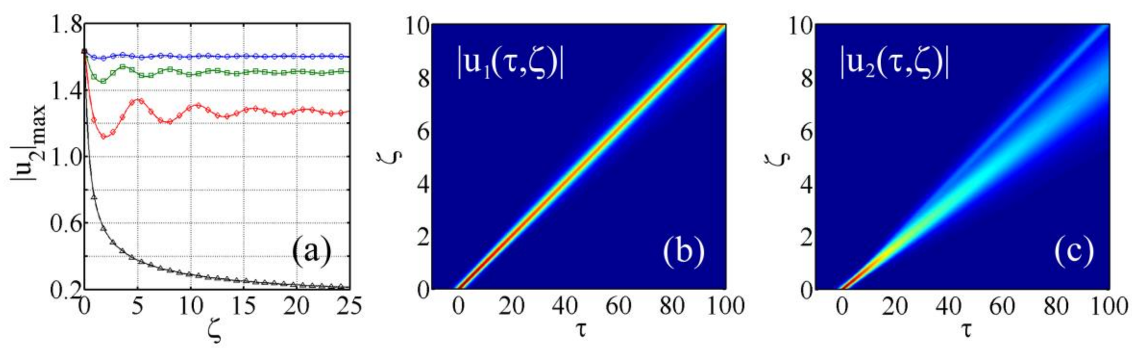

Fig. 1. Typical results from simulations using exact *conventional* bright-bright solitons as initial conditions in Eq. (1) with $\kappa_1 = 10^{-3}$ and $\kappa_2 = 2.5\times10^{-3}$. (a) Peak amplitude of u_2 with s_1 = 1.0 and s_2 = 1.2. (b) and (c): Evolution when s_1 = 1.0 and s_2 = 0.8.

2. Bright-bright solitons

Exact analytical bright-bright solitons of Eq. (1) have been derived by combining ansatz methods with geometrical transformations in the (τ,ζ) plane [3]. The two components form a bound stationary (i.e., propagation-invariant) state, with maximal overlap for cross-phase modulation occurring when they possess the same velocity and temporal duration. These two physical constraints mean that constituent pulse peak intensities and frequency shifts cannot be chosen independently. For any set of system parameters (i.e., κ_j, α_j, s_j, and σ), one typically specifies *two* peak intensities and *one* frequency shift; the remaining frequency shift is then uniquely determined in order to match velocities and pulse widths. Simulations have demonstrated that the new spatiotemporal solutions are often robust entities, propagating with invariant form over arbitrarily-long distances. Initial conditions for Eq. (1) corresponding to exact conventional solutions may also evolve into these stationary solutions as $\zeta \rightarrow \infty$ (see Fig. 1).

Asymptotic analysis of our new solutions has revealed that the predictions of the conventional model [obtained by neglecting the first term in Eq. (1)] can be recovered in an appropriate multiple limit. It is at this stage that one may introduce a Galilean-type boost [transforming to a local time frame moving at the average group speed: $\zeta_{loc} = \zeta$ and $\tau_{loc} = \tau - \alpha_{avg}\zeta$, where $\alpha_{avg} \equiv (\alpha_1 + \alpha_2)/2$]. The classic Menyuk model and its solutions [1] can then be found by setting $s_1 = s_2 = +1$. In this sense, conventional pulse theory emerges from our spatiotemporal model in much the same way that Newtonian mechanics corresponds to the low-speed limit of relativistic kinematics [4].

3. Modulational instabilities & dark solitons

Having derived families of exact analytical bright-bright solitons and investigated their stability properties, we are also interested in more sophisticated vector solutions with at least one dark (e.g., *tanh*-type) component [5,6]. Such solutions are of most physical interest (e.g., can be deployed in photonic device architectures) when the associated continuous-wave (cw) background fields are stable against small spontaneous fluctuations (e.g., from noise). Accordingly, we have performed a linear analysis of the vector cw solution of Eq. (1) to quantify the modulational instability (MI) spectrum and predict the most-unstable frequencies in the system [1,7]. Instabilities turn out to be described by the roots of an eighth-degree polynomial characteristic equation (which can only be solved numerically), and the results for long-wavelength perturbations map directly onto those predicted by the corresponding conventional model [see Figs. 2(a) and 2(b)]. Crucially, no MI has so far been found when the group-velocity dispersion (GVD) coefficients are both negative. It is in this regime where our current research efforts are focused to uncover exact dark-dark spatiotemporal solitons; like their bright-bright analogue, conventional dark-dark solitons [6] also appear to be unstable in the context of Eq. (1) [compare Figs. 1(b) and 2(c)].

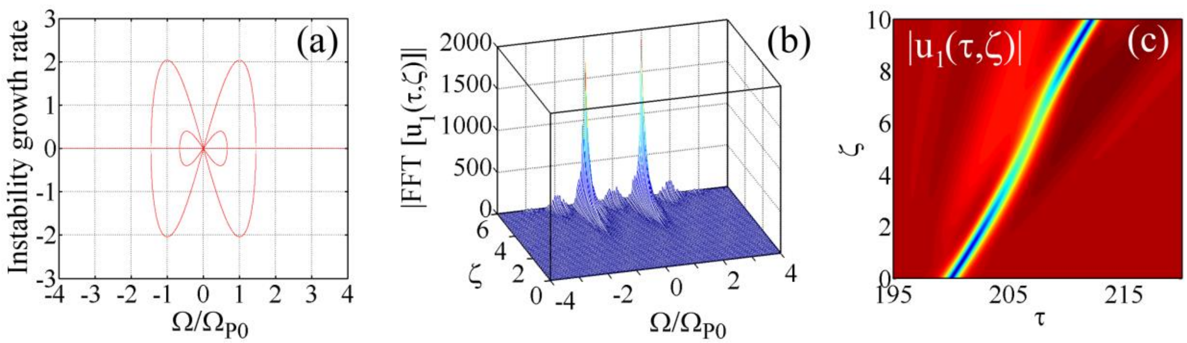

Fig. 2. (a) Typical long-wave MI spectrum for the cw vector solution in the GVD regime $s_1 = 1.0$ and $s_2 = 1.5$. (b) Extensive simulations have confirmed the theoretical prediction for the most-unstable frequency, denoted by Ω_{P0} [related to the dominant peak in part (a)]. (c) Instability of an exact conventional dark-dark soliton when used as an initial condition in Eq. (1).

4. References

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