# FROM MAXWELL'S EQUATIONS TO NEW FAMILIES OF HELMHOLTZ SOLITONS

*(Invited paper)*

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***Abstract –*** In this presentation, we give an overview of new results in Helmholtz soliton theory. Firstly, fundamental considerations are made in terms of new contexts for Helmholtz solitons that arise directly from Maxwell's equations. We will then explore applications involving a variety of different material interfaces and the role of Helmholtz solitons in these configurations. Finally, specific new families of solutions arising from the generalisation of the Manakov equation will be reported.
***Keywords*:** Spatial optical solitons, nonlinear Helmholtz equation, non-paraxiality, time-domain simulation

## INTRODUCTION

Previous research on the Non-Linear Helmholtz (NLH) equation has permitted the generalization of both bright [1] and dark [2] spatial solitons in Kerr media to the finite-angle regime, where oblique beam propagation may be at an *arbitrarily large angle* relative to the longitudinal axis. In this approach, the intrinsic angular limitations of conventional Non-Linear Schrödinger (NLS) analyses, imposed by the assumption of beam paraxiality, are eliminated. Our analytical investigations are complimented by well-tested numerical techniques, developed specifically for the accurate solution of the NLH equation [3].

The NLH equation is fully equivalent [1] to the Non-paraxial Non-Linear Schrödinger (NNLS) equation

$$\kappa\frac{\partial^2 u}{\partial\zeta^2}+i\frac{\partial u}{\partial\zeta}+\frac{1}{2}\frac{\partial^2 u}{\partial\xi^2}\pm|u|^2 u=0\,, \tag{1}$$

which describes the evolution of the normalized complex envelope $u$ of an optical beam. Here, $\zeta=z/L_D$, $\xi=2^{1/2}x/w_0$, $L_D=kw_0^2/2$, $\widetilde{E}(x,z)=E_0u(x,z)\exp(ikz)$ and $k=2\pi/\lambda$. $n_0$ is the linear refractive index, $\lambda=\lambda_0/n_0$ the optical wavelength, $E_0=(n_0/k|n_2|L_D)^{1/2}$, $n_2$ the Kerr coefficient and $\kappa=1/(k^2w_0^2)$ is the non-paraxiality parameter. The ± sign flags a focusing or defocusing Kerr non-linearity. Equation (1) retains the full spatial symmetry of the NLH model, and is a more convenient framework for comparing new results with those obtained from paraxial calculations. The NLS equation can be recovered from Eq. (1) when the Helmholtz term $\kappa\partial^2u/\partial\zeta^2$ is neglected.

In this presentation, we give an overview of some recent results in Helmholtz soliton theory. Three topics have been selected for this purpose: The modeling of the propagation properties of Helmholtz solitons directly using the full 2D Maxwell equations [4], the behaviour of solitons incident on non-linear interfaces at oblique angles [5], and new exact analytical vector solitons arising from the Helmholtz-Manakov (H-M) equation [6].

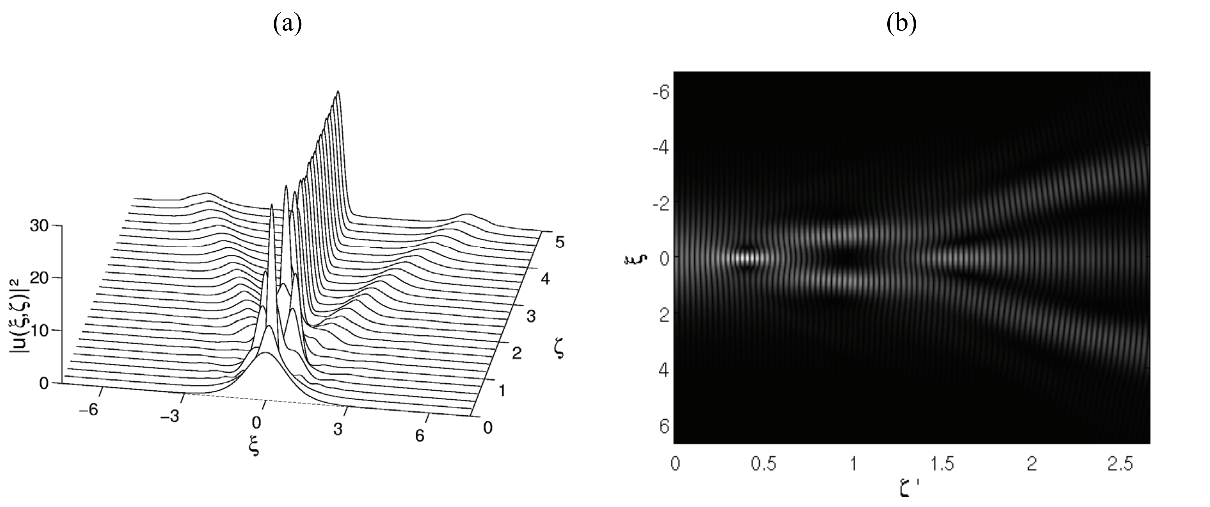


Fig.1 –Splitting of a third-order soliton. The result obtained from the numerical integration of the NNLS equation (a) is compared with the E-field magnitude $|E_y(\xi,\zeta,t_0)|$ computed from the time-domain Maxwell equations (b). In the calculations, equal scalings have been used for the $\xi=2^{1/2}\,x/w_0$ and $\zeta=2^{1/2}\,z/w_0$ coordinates. The $\zeta$ coordinate in plot (b) has been adjusted to that of (a) by using $\zeta'=(2\kappa)^{1/2}\zeta$.

## HELMHOLTZ SOLITONS AT NON-LINEAR INTERFACES

The evolution of Helmholtz solitons at the interface separating two Kerr-type media can be described by a generalized NNLS equation [5]. Numerical simulations show that when there is a mismatch only in the linear part of the refractive index, the incident solitons are governed by Snell's law. In general, it has been found that the reflection and refraction characteristics of optical solitons possess key features that cannot be adequately described by paraxial theory [7].

In this presentation, we will focus primarily on the analysis of soliton behaviour when the linear refractive index is continuous across the interface. The general solution shows that when a soliton enters a medium with a weaker non-linearity, the outgoing beam may suffer diffractive spreading without limit unless the input power exceeds some critical value (see Fig2.a). On the other hand, when the second medium is characterized by a stronger non-linearity, any excess power associated with the incident soliton causes the input beam to break up into a distribution of narrower solitons (see Fig2.b).

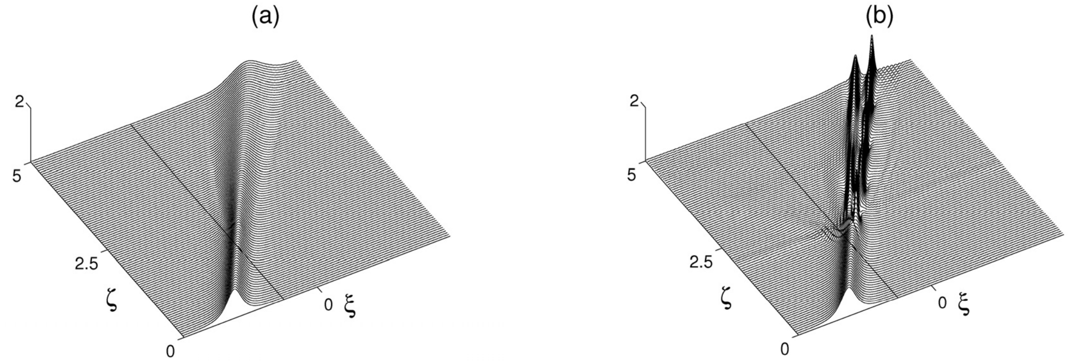


Fig. 2 – Numerical results corresponding to soliton evolution at the inteface between two Kerr-type media when the magnitude of the non-linear refractive index in the second medium is lower (a) and higher (b) than in the first one.

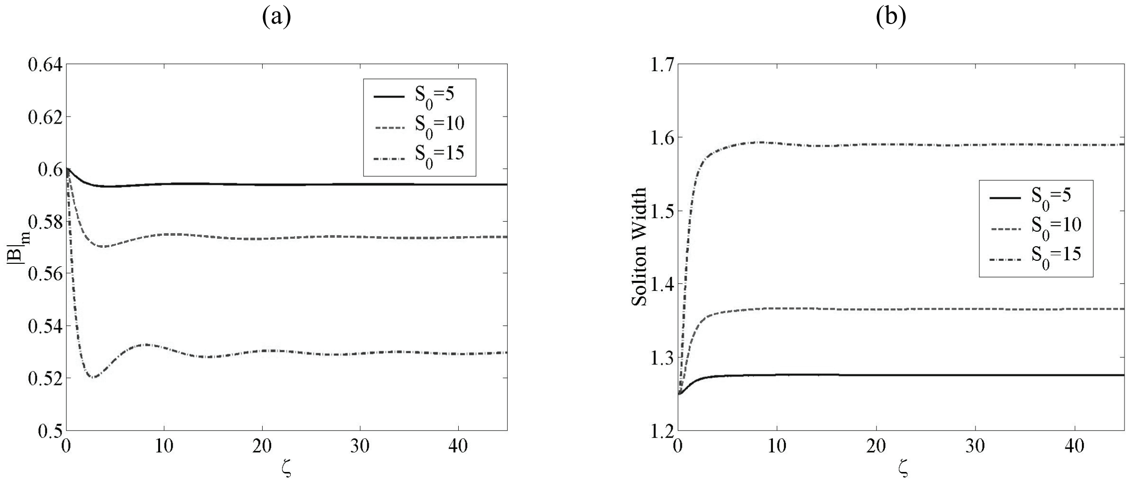


Fig. 4 – Beam self-reshaping oscillations in the (a) peak amplitude, and (b) width, of a perturbed canonical dark-bright soliton. The oscillations vanish in the asymptotic limit $\zeta \rightarrow \infty$, leaving a stationary beam with propagation-invariant characteristics. This is the signature of Helmholtz-Manakov soliton formation.

## ACKNOWLEDGEMENT

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