Refraction of power-law spatial solitons – the Helmholtz-Snell law

J. M. Christian[1], J. Sánchez-Curto[2], P. Chamorro-Posada[2], G. S. McDonald[1], E. A. McCoy[1]
[1]Materials & Physics Research Centre, University of Salford, U.K.
[2]Universidad de Valladolid, ETSI Telecomunicación, Spain

Abstract: The behaviour of a scalar optical beam at the boundary between two dissimilar nonlinear media is of fundamental interest in nonlinear photonics. Here, we report the first systematic generalization of our Kerr analyses to a wider class of power-law materials. Universal refraction laws will be given, and theory-simulation agreement demonstrated.

I. INTRODUCTION

A light beams impinging on the interface between two dissimilar dielectric materials is a fundamental optical geometry. After all, the single-interface configuration is a key "building block" structure that facilitates more sophisticated device designs and architectures for a diverse range of applications. The seminal papers of Aceves, Moloney and Newell [1,2] considered perhaps the simplest scenario, where a spatial soliton was incident on the boundary between two different Kerr-type materials. Their intuitive approach reduced the full complexity of the electromagnetic interface problem to something far more tractable – namely, the solution a scalar equation of the inhomogeneous nonlinear Schrödinger (NLS) type. Over the past two decades, investigations of single and double-layer interface geometries have paved the way to deeper understandings of how light behaves inside patterned nonlinear structures such as coupled waveguide arrays and photonics crystals.

It is true to say that the analyses of Aceves and coworkers have provided an enormous level of insight into soliton behavior at interfaces [1–4], and that they have heralded new research fields in nonlinear photonics. However, there is an intrinsic limitation in the use of paraxial (i.e., NLS-based) models to describe geometries that are, by their very nature, highly nonparaxial. For instance, the paraxial approximation restricts their domain of validity to regimes were the angles of incidence, reflection and refraction are negligibly small. Hence, there is an obvious gap that needs to be filled – it is desirable to be able to describe angles of *any* size (see Fig. 1), while keeping an intuitive scalar model.

Since the "interface" class of problem is intrinsically nonparaxial – for instance, impinging beams can be arbitrarily oblique with respect to the interface – the limitations of conventional (paraxial) theory must be observed with care. Recently, we proposed the first Helmholtz model of Kerr spatial soliton refraction that is valid across the entire range of incidence, reflection and refraction angles. Analyses have considered the refraction of spatial solitons incident on the boundary between dissimilar Kerr-type materials at *arbitrary angles* [5–7]. By deploying the formalism of Helmholtz soliton theory, the angular limitation of paraxial models was lifted, and a manageable envelope equation emerged (by retaining the full generality of the in-plane Laplacian). Applying appropriate field continuity conditions at the interface led to a Snell's law for Kerr spatial solitons. At first glance, this new law strongly resembles the classic refraction rule for *plane waves* at the interface between *linear* media. However, there appears a universal factor (denoted by γ) that captures the general interplay between finite-beam effects and (linear and nonlinear) medium discontinuities.

In this presentation, we report the first steps toward extending our Kerr analyses to regimes involving wider classes of materials. In our systematic approach, we first consider media whose nonlinear refractive index n_{NL} has a generic power law-type dependence on the electric field amplitude E,

$$n_{NL}\left(|E|\right) = \frac{\alpha}{2n_0}|E|^q, \qquad (1)$$

where α is a (small) positive coefficient, n_0 is the linear index, and the exponent $q > 0$. The single power-law model (1) is perhaps the simplest non-Kerr nonlinearity one might care to consider [8,9]. Materials that fall within this category include some semiconductors (e.g., InSb and GaAs / GaAlAs), doped filter glasses (e.g., CsS_xSe_{x-2}) and liquid crystals. Non-Kerr regimes (i.e., where q deviates from the value of 2) have been
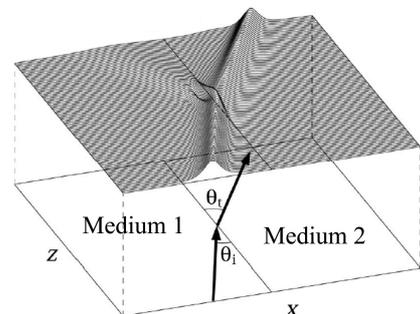
Fig. 1. Arbitrary-angle spatial soliton refraction. The refraction process here is *external* ($\theta_t > \theta_i$; see text in Sec. III).

found to give rise to a diverse range of new qualitative phenomena.

II. POWER-LAW INTERFACES MODEL

We consider the TE-polarized continuous-wave scalar electric field

$$\tilde{E}(x,z,t) = E(x,z)\exp(-i\omega t) + E^*(x,z)\exp(+i\omega t), \quad (2)$$

where x and z are the spatial coordinates in the medium, t is the time coordinate and ω is the angular frequency. This representation makes sure that the field remains real, as should be the case. If the spatial part of the field varies slowly (in the transverse direction) on the scale of the free-space optical wavelength λ, then $E(x,z)$ satisfies a nonlinear Helmholtz equation on each side of the material boundary:

$$\frac{\partial^2 E}{\partial z^2} + \frac{\partial^2 E}{\partial x^2} + \frac{\omega^2}{c^2}n_j^2(|E|)E = 0, \quad (3)$$

where $j = 1, 2$ denotes the medium and c is the vacuum speed of light. The total refractive index n_j is routinely taken to be the sum of two terms: $n_j = n_{0j} + n_{\mathrm{NL}j}(|E|)$, where n_{0j} is the linear index of medium j ($= 1, 2$) at frequency ω, and $n_{\mathrm{NL}j}(|E|)$ is a small field-dependent correction. Since Eq. (3) is quadratic in n_j, one may make the approximation $n_j^2 \simeq n_{0j}^2 + 2n_{0j}n_{\mathrm{NL}j}(|E|) = n_{0j}^2 + \alpha_j|E|^q$. The carrier wave component of E can be factored out according to $E(x,z) = E_0 u(x,z)\exp(ik_1 z)$, so that z and x are the longitudinal and transverse coordinates, respectively. Here, $E_0 = (n_0/\alpha_1 k L_D)^{1/q}$ is a (real) constant, $k_1 = (\omega/c)n_{01}$, and $u(x,z)$ is the dimensionless complex envelope. Arbitrarily, the carrier wave in medium 1 has been factored out of E [equally, one could have factored out the complex-exponential factor $\exp(ik_2 z)$]. After substitution into Eq. (3), it can be shown that u satisfies the dimensionless inhomogeneous equation,

$$\kappa\frac{\partial^2 u}{\partial\zeta^2} + i\frac{\partial u}{\partial\zeta} + \frac{1}{2}\frac{\partial^2 u}{\partial\xi^2} + |u|^q u = \left[\frac{\Delta}{4\kappa} + (1-\alpha)|u|^q\right]H(\xi)u. \quad (4)$$

The longitudinal and transverse coordinates are $\zeta = z/L_{D1}$ and $\xi = 2^{1/2}x/w_0$, where $L_{D1} = k_1 w_0^2/2$ and w_0 are the diffraction length and waist of a reference Gaussian beam. The nonparaxial parameter $\kappa = 1/(k_1 w_0)^2 = \varepsilon^2/4\pi^2 n_{01}^2$ quantifies the (inverse) beam width. The validity of the Helmholtz modelling approach requires $\varepsilon \equiv \lambda/w_0 << O(1)$, so that beam waists are much larger than the free-space light wavelength. Hence, κ is always taken to be a small parameter throughout: $\kappa << O(1)$. The Heaviside unit function $H(\xi)$ is defined so that $H(\xi < 0) = 0$ and $H(\xi > 0) = +1$ (see Fig. 1 – in this configuration, the interface is aligned along the z axis). Equation (4) contains interface parameters that describe the mismatch between the linear and nonlinear characteristics of the two media:

$$\Delta \equiv 1 - \left(\frac{n_{02}}{n_{01}}\right)^2 \quad \text{and} \quad \alpha \equiv \frac{\alpha_2}{\alpha_1} \quad (5a,b)$$

From these relations, three distinct scenarios emerge: (*i*) linear interfaces (defined by $\alpha = 1$ so that there is no mismatch in the nonlinear coefficients), (*ii*) nonlinear interfaces (defined by $\Delta = 0$, so the two media have the same linear index), (*iii*) mixed interfaces (with arbitrary choices of α and Δ).

Model (4) in medium 1 is just the conventional power-law Helmholtz equation, for which the exact analytical bright soliton solutions are now known [10]. The forward-propagating solutions have a $\mathrm{sech}^{2/q}$ profile,

$$u(\xi,\zeta) = \eta_0\mathrm{sech}^{2/q}\left(a\frac{\xi - V\zeta}{\sqrt{1+2\kappa V^2}}\right)$$
$$\times\exp\left[i\sqrt{\frac{1+4\kappa\beta}{1+2\kappa V^2}}\left(V\xi + \frac{\zeta}{2\kappa}\right)\right]\exp\left(-i\frac{\zeta}{2\kappa}\right), \quad (6)$$

where η_0 is the peak amplitude and V is the conventional transverse velocity. Here, V is related to the propagation angle θ in the laboratory (x,z) frame (measured with respect to the longitudinal, i.e., z, axis) through the relation,

$$\tan\theta = \sqrt{2\kappa}V. \quad (7)$$

Propagation angles are thus bounded by $-90° \leq \theta \leq +90°$, and they may be arbitrarily large. The additional parameters in solution (6) are $\beta \equiv 2\eta_0^q/(2+q)$ and $a \equiv q[\eta_0^q/(2+q)]^{1/2}$.

III. UNIVERSAL HELMHOLTZ-SNELL LAW

By deploying the power-law Helmholtz solitons [10] in tandem with the Helmholtz interfaces formalism developed in Refs. [5–7], one can arrive at the following universal Helmholtz-Snell law predicting soliton refraction:

$$\gamma n_{01}\cos\theta_i = n_{02}\cos\theta_t. \quad (6)$$

Here, θ_i and θ_t are the angles of incidence and transmission, respectively (see Fig. 1). The factor γ is a universal function that allows for the interplay between finite-width optical beams, system nonlinearity, and mismatched medium properties.

The critical angle θ_C is defined to be the value of θ_i at which $\theta_t = 0$ (i.e., where the refracted beam, in principle, travels along the material boundary). From Eq. (6), this condition leads to

$$\tan\theta_c = \left[\frac{\Delta + 4\kappa\beta(1-\alpha)}{1-\Delta + 4\kappa\beta\alpha}\right]^{1/2}. \quad (7)$$

The critical angle clearly depends upon the interplay between the mismatch parameters (both α and Δ), the nonparaxial parameter κ, and (through β) the peak amplitude and the exponent q.

When analyzing refraction phenomena, it is convenient to introduce the dimensionless parameter δ through [5–7]

$$\delta \equiv \Delta + 4\kappa\beta(1-\alpha). \quad (8)$$

Refraction thus tends to fall into two regimes: it may be either *external* (where $\delta < 0$, which implies $\theta_t > \theta_i$) or *internal* (where $\delta > 0$, which implies $\theta_t < \theta_i$). The boundary between these two regimes is determined by $\delta = 0$, in which case $\theta_t = \theta_i$ and the incident beam passes undeviated across the interface. Thus, $\delta = 0$ defines the *transparency condition*, in which linear and nonlinear mismatches in refractive index exactly cancel each other. One can also note that the "no interface" scenario, where $\Delta = 0$ and $\alpha = 1$, satisfies the transparency condition, as one would expect intuitively.

IV. SIMULATIONS

Extensive simulations have been used to test the analytical predictions of the Helmholtz-Snell law (6) for $q = 1, 2$ and 3 against direct numerical integration of Eq. (4) [11]. The Kerr case of $q = 2$ is "sandwiched" between two regimes where the focusing properties of the two media are weaker and stronger than the Kerr type ($q = 1$ and 3, respectively). Various sets of simulations have considered linear interfaces (defined by $\alpha = 1$), nonlinear interfaces (defined by $\Delta = 0$), and mixed interfaces (with arbitrary values of α and Δ). Computations have considered arbitrary incidence angles and regimes where $\Delta < 0$, neither of which are accessible to paraxial theory [1–4]. A full account of new phenomena in various parameter regimes (see Fig. 2) will be presented.
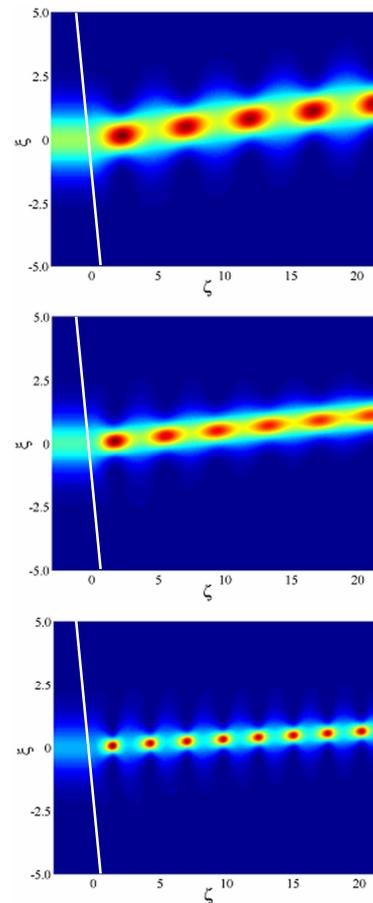

Fig. 2. External nonparaxial refraction ($\theta_i = 30°$) for unit amplitude solitons at a nonlinear interface (marked by the white line) when $q = 1$ (top), 2 (middle), and 3 (bottom).

REFERENCES

[1] A. B. Aceves, J. V. Moloney, and A. C. Newell, "Theory of light-beam propagation at nonlinear interfaces. I. Equivalent-particle theory for a single interface," Phys. Rev. A vol. 39, no. 4, pp. 1809-1827, 1989.

[2] A. B. Aceves, J. V. Moloney, and A. C. Newell, "Theory of light-beam propagation at nonlinear interfaces. II. Multiple-particle and multiple-interface extensions," Phys. Rev. A vol. 39, no. 4, pp. 1828-1840, 1989.

[3] P. Varatharajah, A. C. Newell, J. V. Moloney, and A. B. Aceves, "Transmission, reflection, and trapping of collimated light beams in diffusive Kerr-like nonlinear media," Phys. Rev. A vol. 42, no. 3, pp. 1767-1774, 1989.

[4] A. B. Aceves et al., "Particle aspects of collimated light channel propagation at nonlinear interfaces and in waveguides," J. Opt. Soc. Am. B vol. 7, no. 6, pp. 963-974, 1990.

[5] J. Sánchez-Curto, P. Chamorro-Posada, and G. S. McDonald, "Helmholtz solitons at nonlinear interfaces," Opt. Lett. vol. 32, no. 9, pp. 1126-1129, 2007.

[6] J. Sánchez-Curto, P. Chamorro-Posada, and G. S. McDonald, "Nonlinear interfaces: intrinsically nonparaxial regimes and effects," J. Opt. A: Pure Appl. Opt. vol. 11, art. no. 054015, 2009.

[7] J. Sánchez-Curto, P. Chamorro-Posada, and G. S. McDonald, "Helmholtz dark solitons at nonlinear interfaces," Opt. Lett. vol. 35, no. 9, 1347-1349, 2010.

[8] A. W. Snyder and D. J. Mitchell, "Spatial solitons of the power-law nonlinearity," Opt. Lett. vol. 18, no. 2, pp. 101-103, 1993.

[9] D. Mihalache, M. Bertolotti, and C. Sibilia, "Nonlinear wave propagation in planar structures," Progress in Optics 27, ed. E. Wolf (Amsterdam: Elsevier), pp. 229-313, 1989.

[10] J. M. Christian, G. S. McDonald, and P. Chamorro-Posada, "Helmholtz solitons in power-law optical materials," Phys. Rev. A vol. 76, art. no. 033834 (2007).

[11] P. Chamorro-Posada, G. S. McDonald, and G. H. C. New, "Non-paraxial beam propagation methods," Opt. Commun. vol. 192, pp. 1-12, 2001.