Interaction of Kerr Spatial Solitons at Arbitrary Angles

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Helmholtz Nonparaxiality

- Helmholtz type of nonparaxiality.
  - Propagation of optical solitons at an arbitrary angle (rotation, steering or intrinsic).
  - Simultaneous propagation of multiplexed soliton beams (soliton collisions).
- Scalar Helmholtz equation.
- Bright and dark exact solitons, robustness, propagation and generation properties,…
  - P. Chamorro-Posada, G. S. McDonald and G. H. C. New, “Exact soliton solutions of the nonlinear Helmholtz equation: communication,” *J. Opt. Soc. Am. B* **19**, 1216 (2002).
  - P. Chamorro-Posada, G. S. McDonald and G. H. C. New, “Propagation properties of nonparaxial spatial solitons,” *J. Mod. Opt.* **47**, 1877 (2000).
  - P. Chamorro-Posada and G. S. McDonald, “Helmholtz dark solitons,” *Opt. Lett.* **28**, 825 (2003).
- Extension of soliton collisions for arbitrary angles (distinct from nearly exact co-propagation or counter-propagation).

Helmholtz Nonparaxiality

- The propagation of a CW optical beam at an arbitrary angle in a focusing Kerr medium, can be accurately described by a NHE which, when re-cast as a NNLS, becomes:

$$\kappa \frac{\partial^2 u}{\partial \zeta^2} + i \frac{\partial u}{\partial \zeta} + \frac{1}{2} \frac{\partial^2 u}{\partial \xi^2} + |u|^2 u = 0$$

where $\xi = \sqrt{2} x / w_0$ $\quad \zeta = z / L_D$ $\quad u(\xi, \zeta) = \left(k |n_2| L_D / n_0\right)^{1/2} B(\xi, \zeta)$

$E(x, z) = B(x, z) \exp(ikz)$ $\quad \kappa = 1/\left(k w_0^2\right)$

- $\kappa \rightarrow 0$:
  - $\kappa = 4\pi^2 (\lambda / w_0)^2$, $\kappa = (n_2 E_0^2)/(2 n_0)$, $\kappa = 1/2 (X_0/Z_0)^2$, $\tan\phi = V \rightarrow \tan\theta = (2\kappa)^{1/2} \tan\phi$.
- Rotational invariance vs Galilei invariance.
- Power conservation instead of "area" conservation.

$$P_\zeta = \frac{1}{2} \int_{-\infty}^{+\infty} |u|^2 \left(\frac{1}{2\kappa} + \frac{\partial \phi}{\partial \zeta} \right) d\xi = cte.$$

Helmholtz Soliton Collisions

- We consider two beams $u_1(\xi,\zeta)$ and $u_2(\xi,\zeta)$ propagating at angles θ and $-\theta$ to the ζ-axis and set $u(\xi,\zeta)=u_1(\xi,\zeta)+u_2(\xi,\zeta)$ in the NNLS equation.
- The simultaneous presence of the beams modulates the refractive index of the Kerr medium according to $|u|^2=|u_1|^2+|u_2|^2+u_1u_2^*+u_2u_1^*$ leading to three distinct effects:

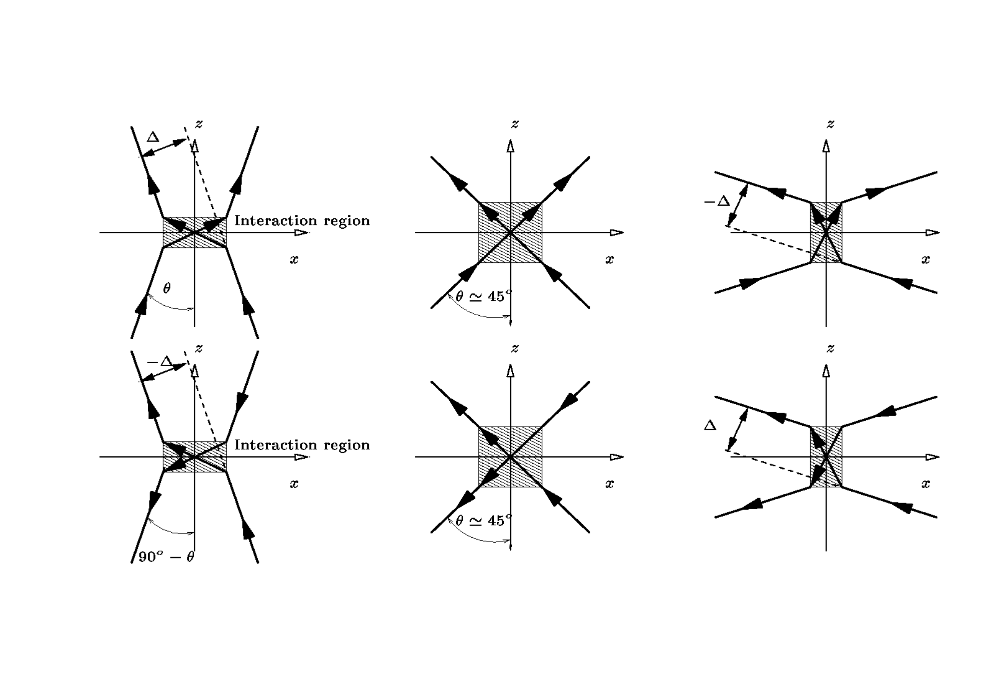
$|u_j|^2u_j$ (SPM) $2|u_{3-j}|^2u_j$ (XPM) $u_j^2u_{3-j}^*$ (Phase sensitive terms)

$$\kappa\frac{\partial^2 u_j}{\partial\zeta^2}+i\frac{\partial u_j}{\partial\zeta}+\frac{1}{2}\frac{\partial^2 u_j}{\partial\xi^2}+\left[|u_j|^2+(1+h)|u_{3-j}|^2\right]u_j=0$$

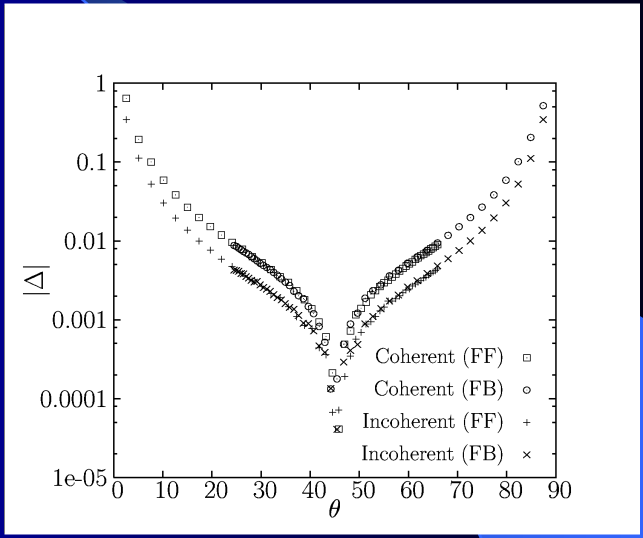
Helmholtz Soliton Collisions

Collision geometry

SPM and <u>XPM</u> effects



Numerical results



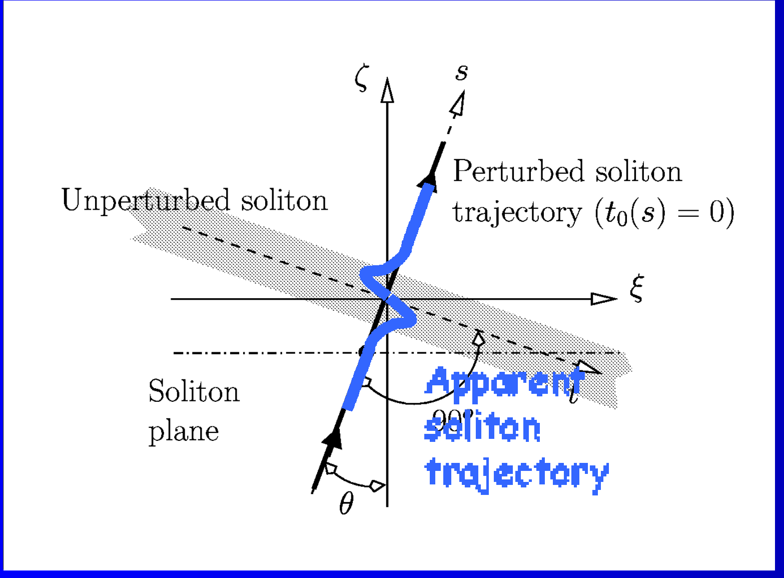
P. Chamorro-Posada, G. S. McDonald and G. H. C. New, "Nonparaxial beam propagation methods," *Opt. Commun.* **19**, 1 (2001).

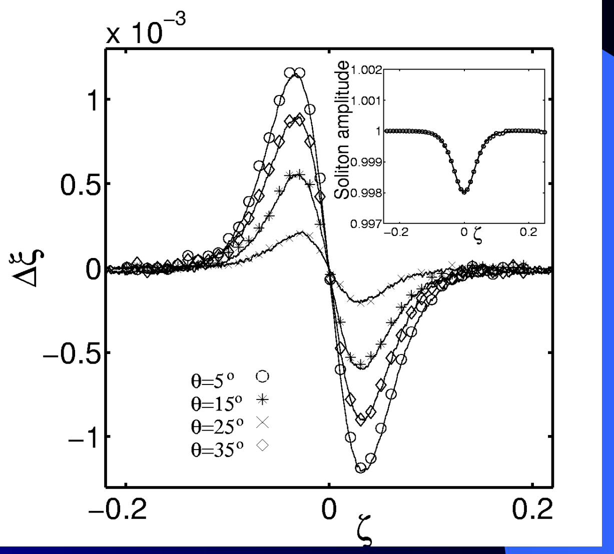
Helmholtz theory is <u>CONSISTENT</u> with the geometry of the problem

Quasi-particle Theory

- Non-integrability of the NHE: appoximate analytical approach.
- Adiabatic perturbation method.

A. Hasegawa and Y. Kodama, Sotions in optical communications, Oxford University Press, 1995.





Apparent Soliton Motion

- The conventional (NSE) approach fails to give a simple description of the soliton evolution even for the simplest case.
  - Additional effect due to the change of soliton amplitude.
    - NSE conservation of the area.
    - NHE (NNSE) power conservation.
  - Distortion of the soliton movement due to the different scalings for the x and z coordinates in the NSE and NNSE.

NHE Framework

- We reintroduce the phase reference $u' = u\exp(i\zeta/(2\kappa))$
- And re-scale the propagation coordinate $\zeta = (2\kappa)^{1/2}\zeta'$

$$\frac{1}{2}\frac{\partial^2 u_j}{\partial \zeta^2} + \frac{1}{2}\frac{\partial^2 u_j}{\partial \xi^2} + \frac{1}{4\kappa}u_j + |u_j|^2 u_j = R, \qquad R = -(1+h)|u_{3-j}|2u_j$$

- For R=0 the Helmholtz bright soliton reads

$$u(\xi,\zeta) = \eta\,\mathrm{sech}[\eta(\cos\theta\,\xi + \sin\theta\,\zeta)]\exp\left[i\sqrt{\frac{1+2\kappa\eta^2}{2\kappa}}(-\sin\theta\,\xi + \cos\theta\,\zeta)\right]$$

NHE Lagrangian Density

- For R=0 the NHE Lagrangian density

$$\mathcal{L}=\frac{1}{2}\left|\frac{\partial u}{\partial \zeta}\right|^2+\frac{1}{2}\left|\frac{\partial u}{\partial \xi}\right|^2-\frac{1}{4\kappa}|u|^2-\frac{1}{2}|u|^4$$

provides two propagation invariants

$$M_\xi=\int_{-\infty}^{+\infty}\left[\frac{1}{2}\frac{\partial u}{\partial \xi}\frac{\partial u^*}{\partial \zeta}+\frac{1}{2}\frac{\partial u}{\partial \zeta}\frac{\partial u^*}{\partial \xi}\right]d\xi \qquad M_\zeta=\int_{-\infty}^{+\infty}\left[\frac{1}{2}\left|\frac{\partial u}{\partial \zeta}\right|^2-\frac{1}{2}\left|\frac{\partial u}{\partial \xi}\right|+\frac{1}{2}|u|^4+\frac{1}{4\kappa}|u|^2\right]d\xi$$

which define a soliton vector in the propagation direction

$$\left(M_\xi, M_\zeta\right)=\eta\frac{3+4\kappa\eta^2}{3\kappa}\left(-\sin\theta,\cos\theta\right)$$

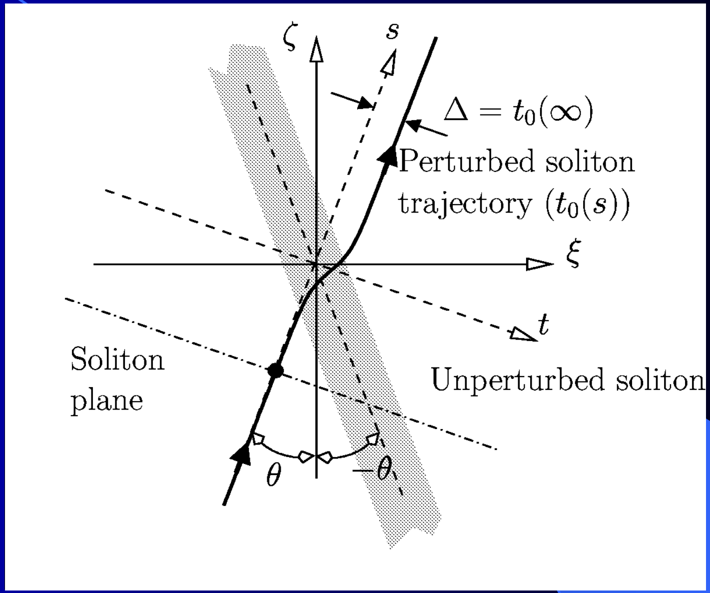
Adiabatic Perturbation Approach

- Power conservation:

$$P_\varsigma = \frac{1}{2}\int_{-\infty}^{+\infty} |u|^2 \frac{\partial \phi}{\partial \zeta} d\xi \quad \frac{\partial P_\varsigma}{\partial \zeta} = \frac{1}{2}\int_{-\infty}^{+\infty} \mathrm{Im}\{R\exp(-i\phi)\} d\xi = 0$$

- Soliton vector evolution:

$$\frac{\partial M_l}{\partial \zeta} = 2\int_{-\infty}^{+\infty} \mathrm{Re}\left\{R^* \frac{\partial u}{\partial l}\right\} d\xi = 0, \quad l = \xi, \zeta$$



- Ansatz

$$u(t,s) = a\,\mathrm{sech}[b(t+t0)]\exp[i\phi_0(-\delta\theta(t+t_0)+\sigma)]$$

$$a(s) = \eta_0 + \delta a(s), \quad b(s) = \eta_0 + \delta b(s), \quad \sigma(s) = s + \delta\sigma(s)$$

$$t_0 = \int_{-\infty}^{s} \delta\theta(r)dr \quad and \quad \phi_0 = \sqrt{\frac{1+2\kappa\eta^2}{2\kappa}}$$

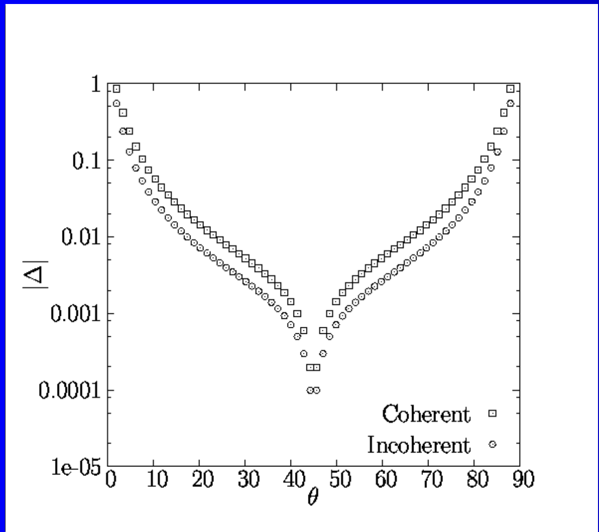
$$\delta\theta' \approx -2(1+h)\kappa\eta_0^2 \int_{-\infty}^{+\infty} |u_2(t,s)|^2 \mathrm{sech}^2[\eta_0(t+t_0)]\tanh[\eta_0(t+t_0)]dt$$

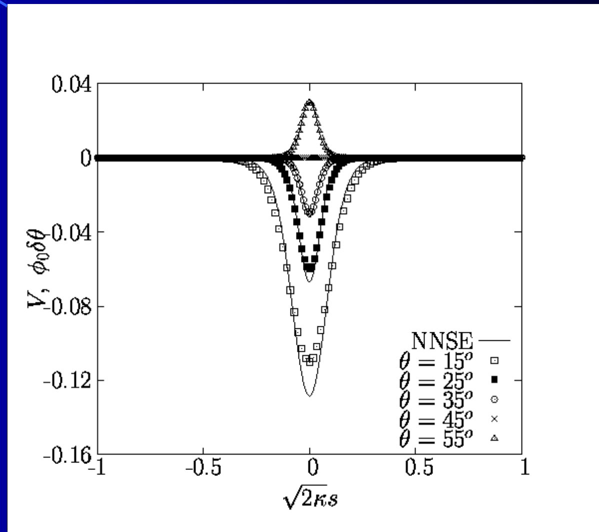
$$\delta\sigma' \approx -(1+h)\kappa\eta_0 \int_{-\infty}^{+\infty} |u_2(t,s)|^2 \mathrm{sech}^2[\eta_0(t+t_0)]dt$$

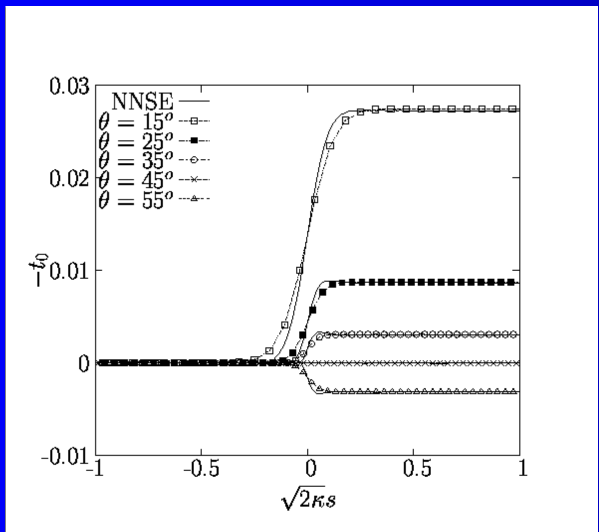
$$\delta\theta = O(\varepsilon) \quad \delta a = O(\kappa) \quad \delta b = O(\varepsilon^2) \quad \text{and} \quad \delta\sigma = O(\kappa)$$

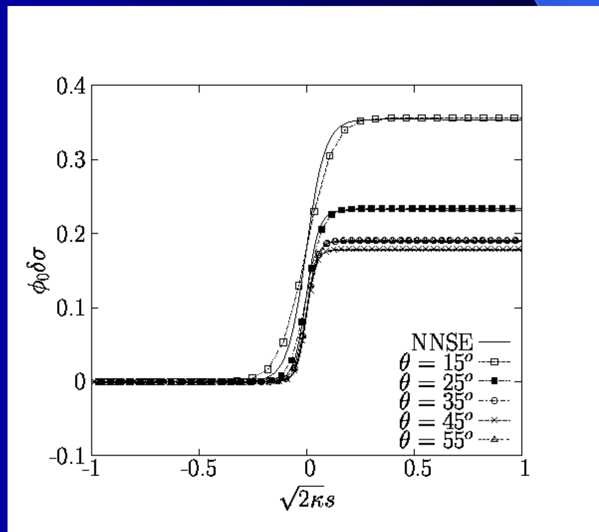
$$\delta a \approx -\frac{1}{2}\eta_0 \delta\sigma'$$

Adiabatic Perturbation Results









Conclusions

- The analysis based on the NSE fails to provide consistent results for soliton collisions at arbitrary angles.
- Soliton collisions theory has been extended by using Helmholtz nonparaxial theory.
- Helmholtz numerical results are in good agreement with the geometry of the problem.
- An approximate quasi-particle theory has been developed for Helmholtz soliton collisions.