Turing instability: a universal route to spontaneous spatial fractals

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Summary

We present the *first predictions* of spontaneous spatial fractal patterns *in nonlinear ring cavities*. New analyses reveal multi-Turing spectra characteristic of susceptibility for spontaneous fractals. Extensive computer simulations confirm theoretical predictions.

Turing instability is the tendency of the uniform states of a system to become spontaneously patterned in the presence of any small fluctuation [1]. Archetypal Turing-instability patterns include hexagons, squares, stripes, and rings. These simple structures are universal in Nature, and characterized by a *single* dominant scale-length. Recently [2], we proposed that a *multi-Turing instability* may result in another type of universal pattern: fractals. Fractals possess proportional levels of detail spanning decades of scale-length, and are thus inherently *scaleless*. This prediction was confirmed in analysis of a classic photonic system (the diffusive Kerr slice with a single feedback mirror). The growth of such multi-scale patterns is entirely due to intrinsic nonlinear dynamics. They are thus physically distinct from fractal mode patterns of unstable-cavity lasers [3], and optical fractals that rely on system changes for introducing each scale-length [4].

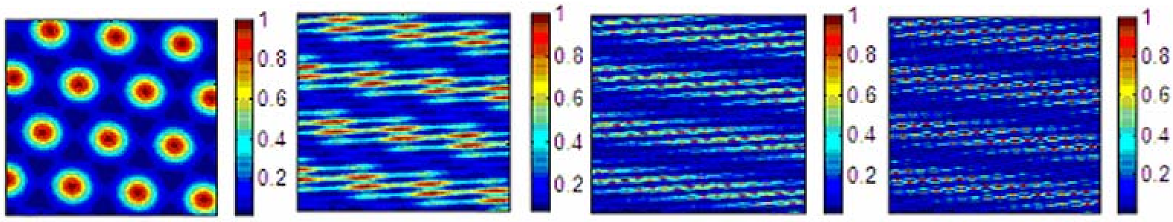

Fig1. Transverse intensity distributions. Transition from a simple Turing-instability pattern (hexagon) to a fractal mode in a pure-absorptive nonlinear cavity. Self-similar structure persists down to scale-lengths of the order of the optical wavelength.

Here, we present the *first predictions* of spontaneous spatial fractal patterns *in nonlinear ring cavities*. This includes the first reported spatial fractals arising from *purely-absorptive nonlinearity*. New analyses reveal multi-Turing spectra characteristic of susceptibility for spontaneous fractals. Computer simulations consider both one and two transverse dimensions (see Fig. 1), and quantify the fractal properties of the generated patterns. A range of results will be reported, including the proposal of a new kind of "fractal soliton".

References

[1] A. M. Turing, *Phil. Trans. Roy. Soc. London B*, **237**, 37,1952

[2] J. G. Huang and G. S. McDonald, *Phys. Rev. Lett.*, **94**, 174101, 2005

[3] G. P. Karman, G. S. McDonald, G. H. C. New and J. P. Woerdman, *Nature*, **402**, 138,1999; J. G. Huang, J. M. Christian and G. S. McDonald, *J. Opt. Soc. Am. A*, **23**, 2768, 2006

[4] S. Sears, M. Soljacic, M. Segev, D. Krylov and K. Bergman *et al.*, *Phys. Rev. Lett.*, **84**, 1902, 2000