# Helmholtz solitons at nonlinear interfaces

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For years, soliton evolution at nonlinear interfaces has been studied in terms of the nonlinear Schrödinger equation (NLS) [1], whose validity is restricted to paraxial propagation. Recently, the development of a new nonparaxial theory has presented a lucid generalisation of the existing theory [2] by considering the full nonlinear Helmholtz equation (NHE) allowing the study of soliton interaction at nonlinear interfaces for arbitrary angles of incidence.

A generalized NHE is employed to describe the evolution of Helmholtz solitons at the interface separating two Kerr-type media

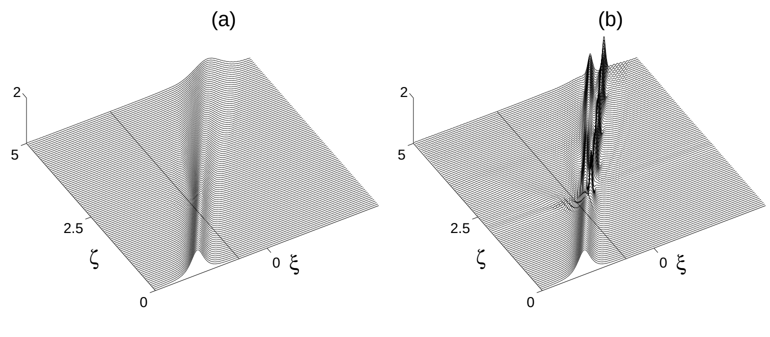
$$\kappa\frac{\partial^2 u}{\partial\zeta^2}+j\frac{\partial u}{\partial\zeta}+\frac{1}{2}\frac{\partial^2 u}{\partial\xi^2}+|u|^2 u \;=\left(\frac{\Delta}{4\kappa}+(1-\alpha^{-1})\,|u|^2\right)H(\xi)u \tag{1}$$

where $\kappa = 1/(kw_0)^2$ is a nonparaxial parameter, $H(\xi)$ is the Heaviside function and $u$ accounts for the complex envolope of a CW optical field in scaled units [2]. The relation between the linear and nonlinear parts of the refractive indexes at both sides of the interface is included trought $\Delta = (n_1^2 - n_2^2)/n_1^2$ and $\alpha = \alpha_1/\alpha_2$ respectively. A general solution to (1) has been proposed and studied numerically showing that when only linear mismatching in the refractive index is considered, Helmoltz solitons behave according to a Snell law for solitons.

In this work, we focus on analyzing the soliton behavior when the linear part of the refractive index is continuous across the interface, $\Delta = 0$. In that case, the solution to (1) in the second medium reads

$$u(\xi,\zeta)=\eta\mathrm{sech}\left(\frac{\eta'(\xi+V\zeta)}{\sqrt{1+2\kappa V^2}}\right)\exp\left[i\sqrt{\frac{1+2\kappa\eta'^2}{1+2\kappa V^2}}\left(-V\xi+\frac{\zeta}{2\kappa}\right)\right]\exp\left(\frac{-i\zeta}{2\kappa}\right) \tag{2}$$

where $\eta' = \eta\alpha^{-1/2}$. From (2), the soliton preserves the velocity and modifies both the width (depending on $\alpha^{-1/2}$) and the energy flow asociated, $2\eta\sqrt{1/\alpha^{-1}+2\kappa\eta^2}$. Therefore, when a soliton enters a medium with a weaker nonlinearity, $\alpha^{-1} < 1$, the resulting beam broadens without limit since the power of the incident soliton is not high enough to create a soliton in the second medium (a). On the other hand, when $\alpha^{-1} > 1$, the exceeding power associated with the incident soliton causes the soliton to break up into a series of narrower solitons (b). A great amount of numerical




work is under way to complete the characterization of the pattern formed in the second medium. However two important differences can be established with respect to the paraxial limit when $\alpha^{-1} > 1$. While in the paraxial limit, $\kappa\eta^2 \to 0$, the number of solitons grows depending on the strength of $\alpha^{-1/2}$, the nonparaxial framework becomes more restrictive in the number of solitons formed as $\kappa\eta^2$ and $1/\alpha^{-1}$ can be of the same order of magnitude. Moreover from the numerical integration of (1) the multi-soliton pattern created depends not only on $\alpha^{-1}$ as the paraxial theory states, but also depends on the angle of incidence.

[1] 1. A. B. Aceves, J. V. Moloney, A. C. Newell, *Theory of light-beam propagation at nonlinear interfaces*, Phys. Rev. A 39, 4, 1809 (1989).

[2] P. Chamorro-Posada, G. S. McDonald, G. H. C. New, *Exact soliton solutions of the nonlinear Helmholtz equation: communication*, J. Opt. Soc. Am. B 19, 1216 (2002).