Spontaneous optical fractal pattern formation*

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Typical generic signatures of complexity

Spontaneous occurrence of …

1. **SIMPLE PATTERN** (e.g. stripes, hexagons)

– associated with *one* characteristic scale

2. **COMPLEX PATTERN** (e.g. fractals)

– associated with *many* decades of scales, ie scale-less

Mandelbrot suggested that …

a fractal is a shape made of parts similar to the whole in some way

Kerr slice with single-feedback mirror system

For optical fields , F and B ,

and medium excitation n ...

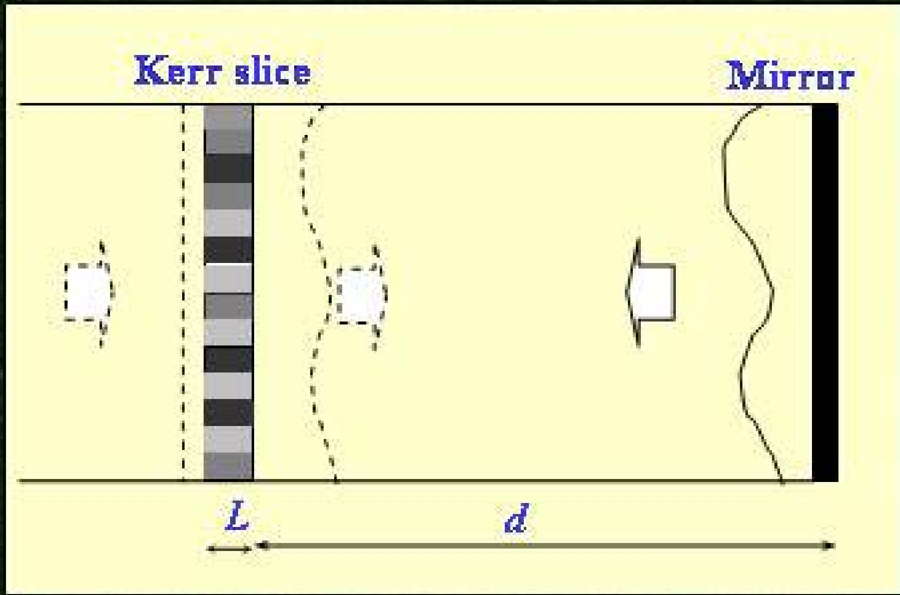
$$\frac{\partial F}{\partial z} = i\chi nF \qquad \frac{\partial B}{\partial z} = -i\chi nB$$

$$-l_D^2 \nabla_{\perp}^2 n + \tau \frac{\partial n}{\partial t} + n = |F|^2 + |B|^2$$

where diffraction of spatial transforms of the optical fields, $B(K)$ and $F(K)$, gives ...

$$B(K) = e^{-i\theta} F(K)$$

$$\theta = \frac{2d}{k_0} \frac{K^2}{\sqrt{1 + (1 - \frac{K^2}{k_0^2})}}$$



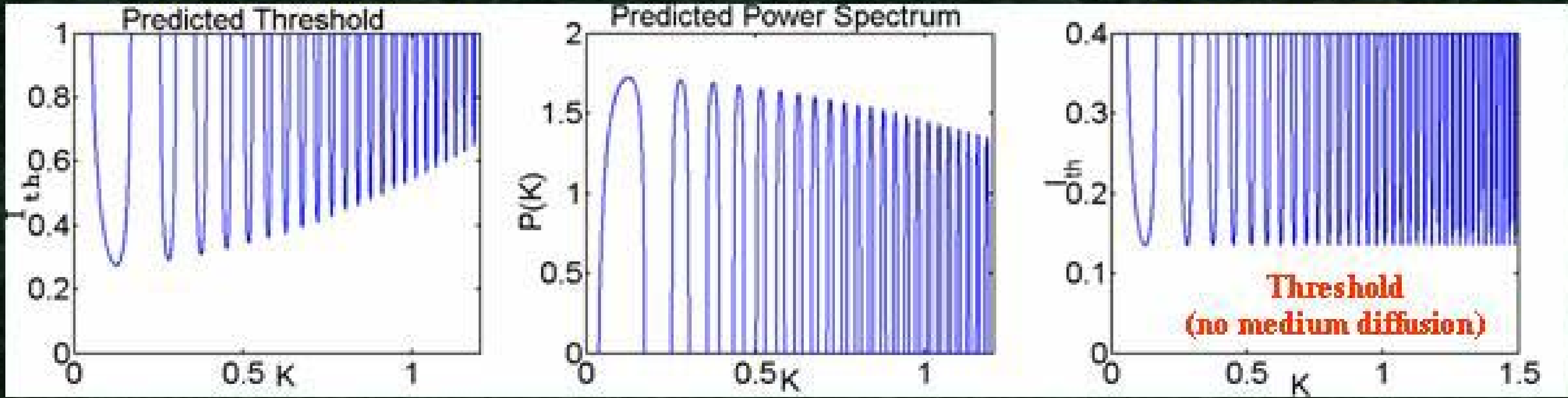
Spatial fluctuations in the medium modulate the optical field phases

(dashed line)

Diffraction produces amplitude modulation

(solid line)

Instability threshold and expected power spectrum

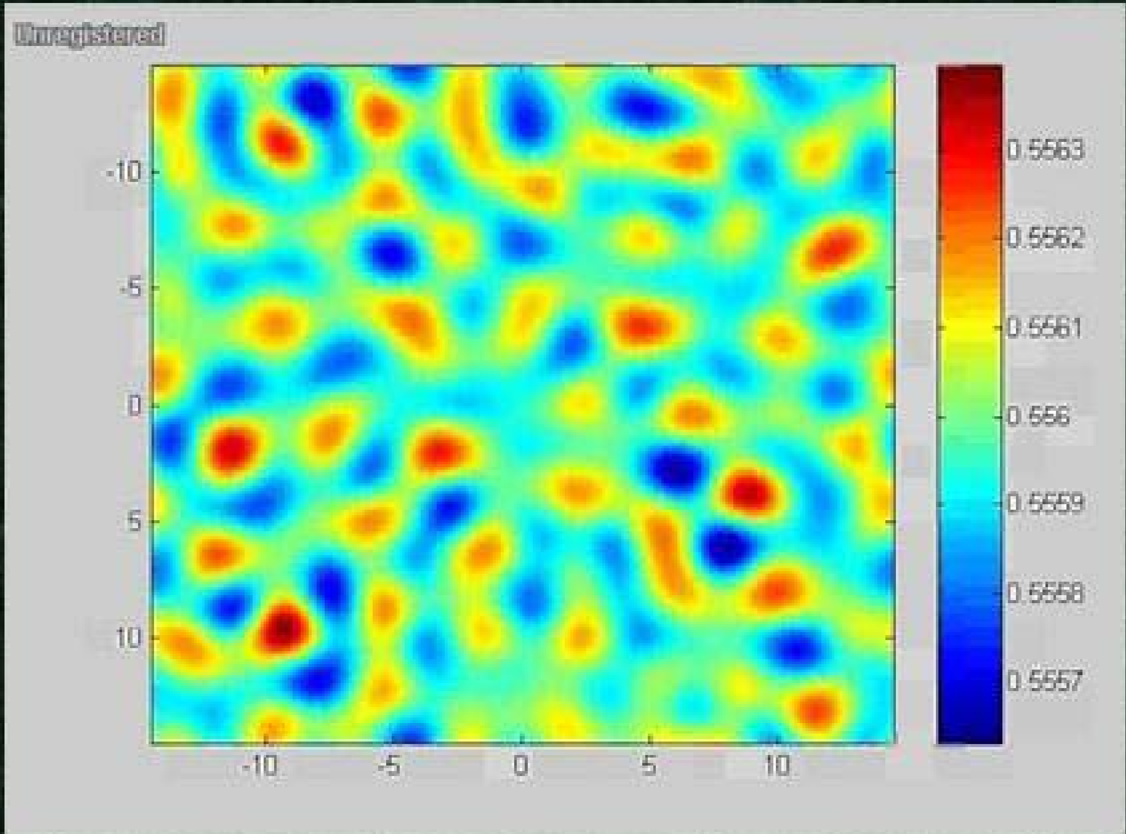


$l_D = 0, \tau = 0$

Linear stability analysis gives intensity threshold:

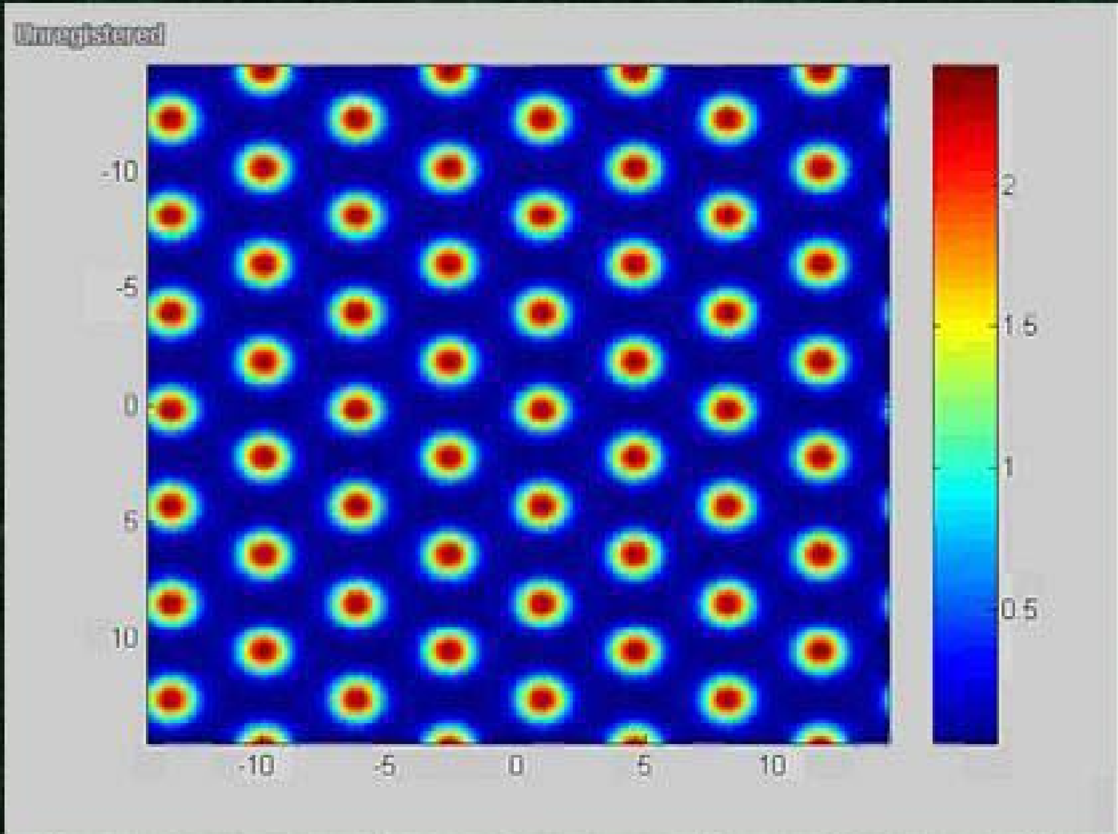
$$I_{th} = \frac{1 + K^2 l_D^2}{2RL\chi \sin(K^2 d / k_0)}$$

Conventional pattern formation



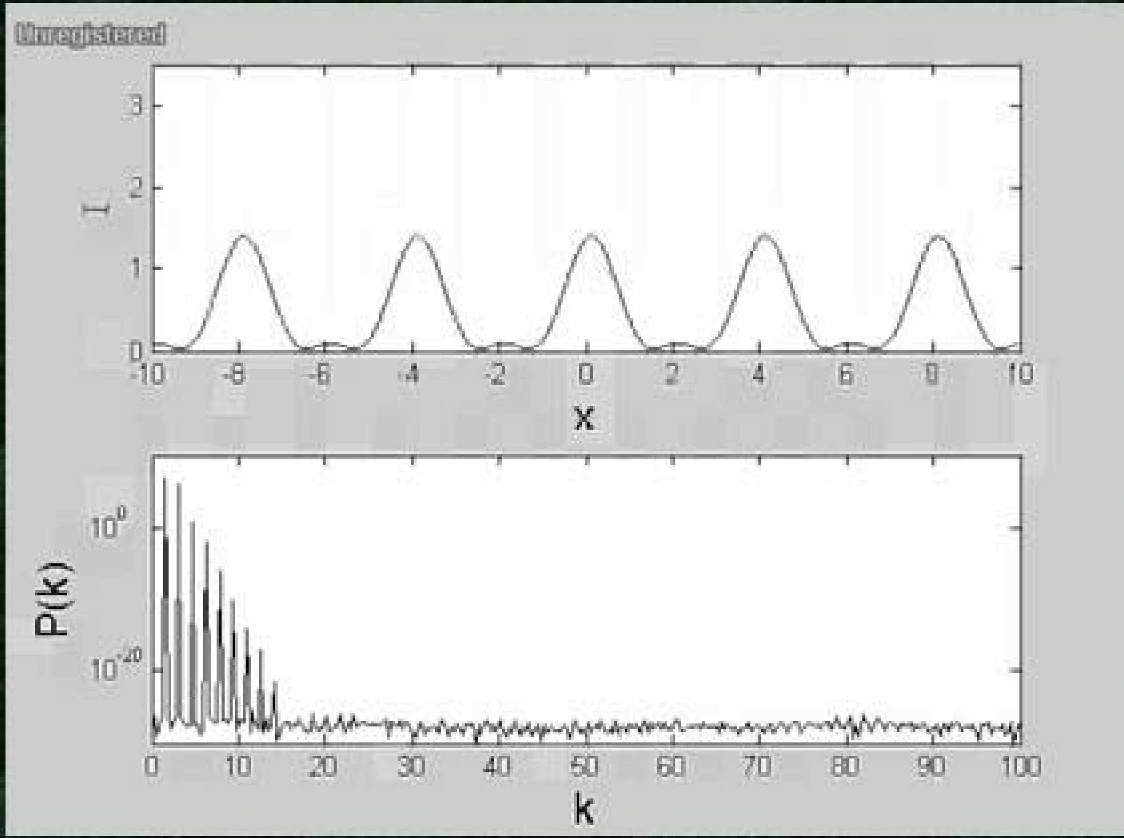
Systems with spatial filtering allow stable hexagonal patterns to appear spontaneously from background noise

Spatial optical fractals

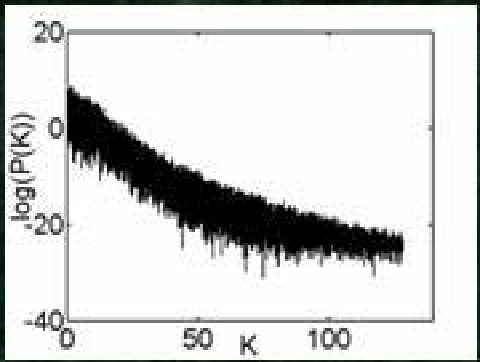


with removal of such filtering, SIMPLE spatial patterns can evolve to extremely COMPLEX spatial patterns

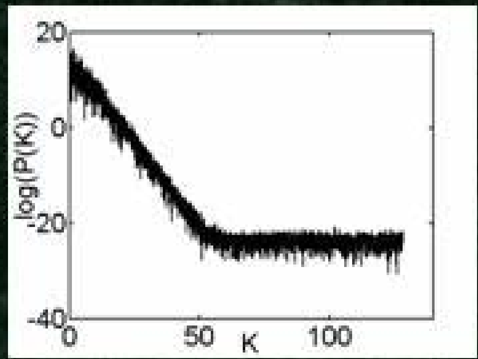
Pattern and power spectrum evolution in one transverse dimension (x)



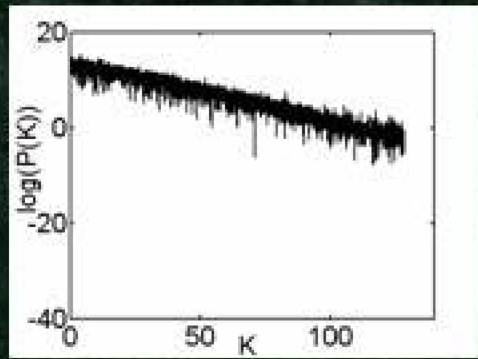
Power spectrum evolution in time
(with medium diffusion $l_D \neq 0$)



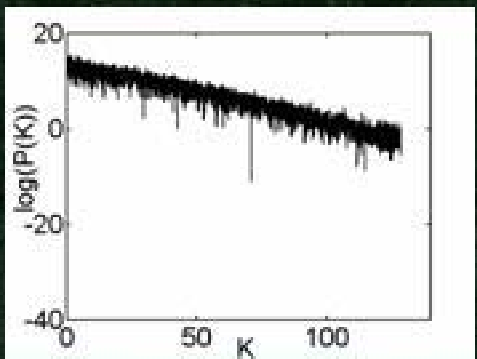
(a) t=2T_R



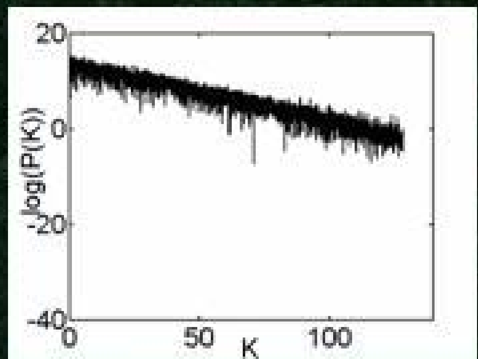
(b) t=5T_R



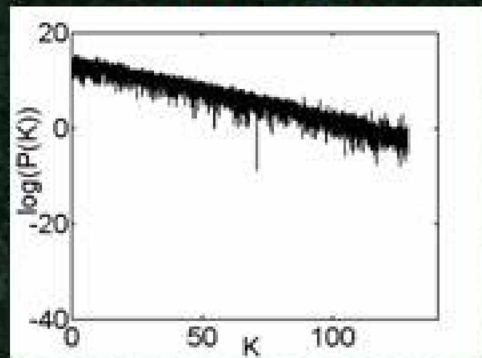
(c) t=10T_R



(d) t=50T_R



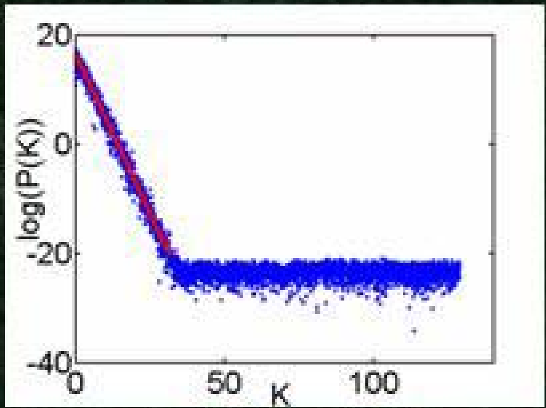
(e) t=500T_R



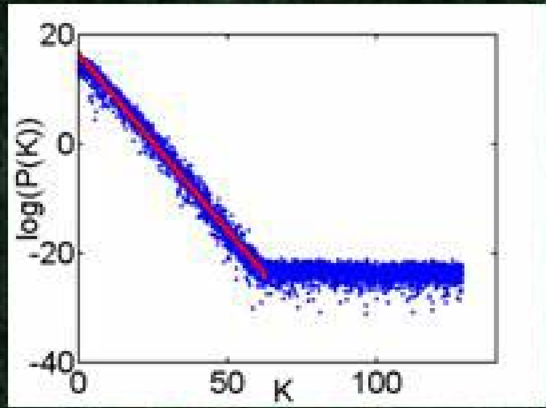
(f) t=2000T_R

Light travels: Kerr slice → feedback mirror → Kerr slice in time T_R

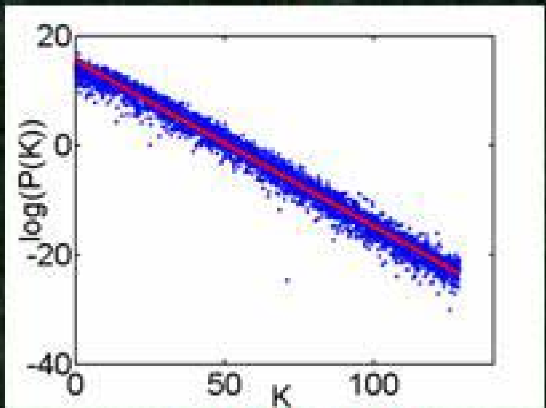
Variation of equilibrium power spectra with diffusion length (l_D)



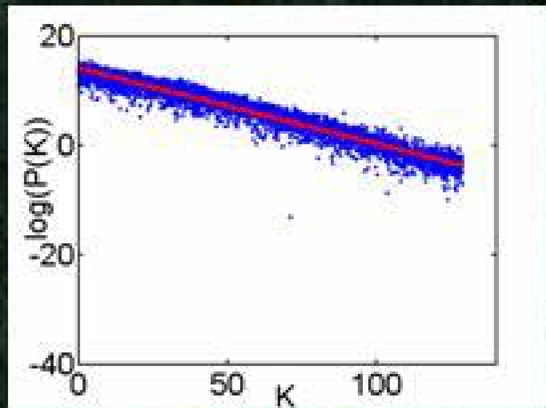
(a) l_D=0.8



(b) l_D=0.4



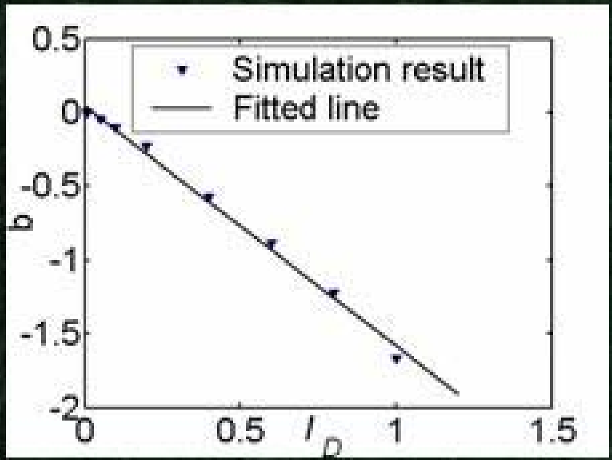
(c) l_D=0.2



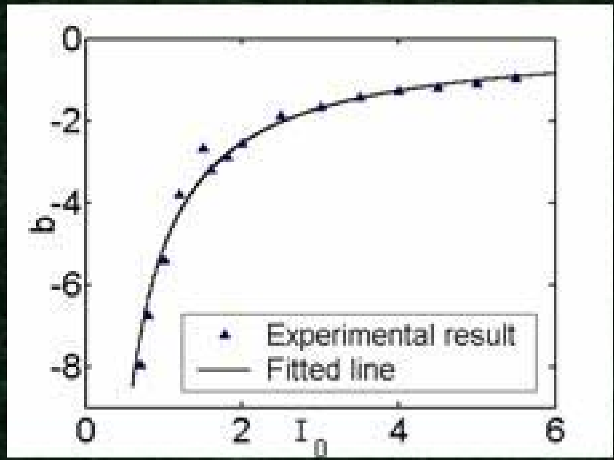
(d) l_D=0.1

Equilibrium power spectrum has characteristic slope b

Variation of slope *b* of equilibrium power spectrum



(a) ***b*** vs diffusion length, l_D



(b) ***b*** vs incident field intensity, I_0

Results show dependence of slope *b*

on l_D and I_0 given by

$$b = b_0 l_D / I_0$$

Variation of fractal dimension versus space frequency *K*

Average trend of each equilibrium power spectrum:

$$log(P(K)) = a + bK$$

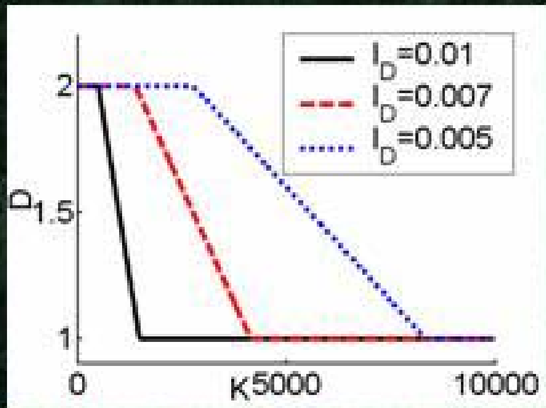
where *a* and *b* are constants dictated by system parameters.

Definition of power spectrum fractal dimension:

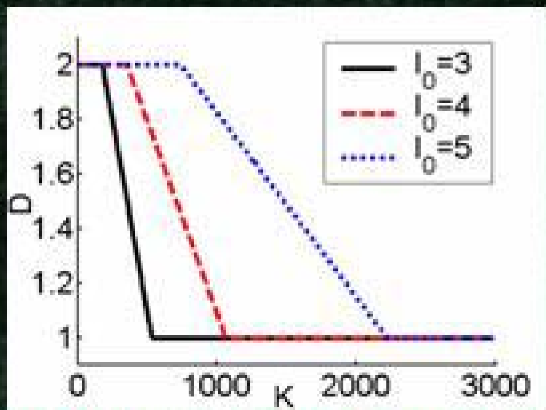
$$D = \frac{1}{2}[5 + \frac{d(logP)}{d(logK)}],$$

gives fractal dimension:

$$D(K) = \begin{cases} 2 & \\ \frac{5}{2} + \frac{b}{2}K & \text{when} \\ 1 & \end{cases} \quad \begin{matrix} \frac{5}{2} + \frac{b}{2}K > 2 \\ 1 \le D(K) \le 2 \\ \frac{5}{2} + \frac{b}{2}K < 1 \end{matrix}$$



(a) Changing diffusion length, l_D



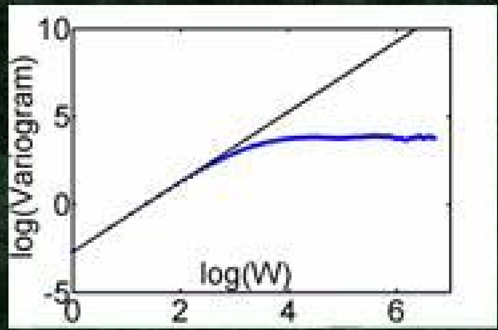
(b) Changing light intensity, I_0

Variogram dimension of optical field intensity

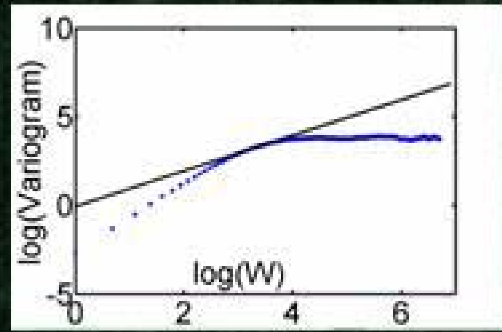
The variogram dimension of the generated light patterns:

$$D_v = 2 - \frac{1}{2}\frac{d(\log V)}{d(\log W)}$$

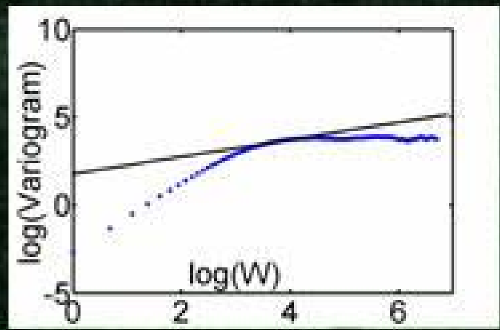
Variogram V is expected value of squared difference of intensities separated in space by distance W



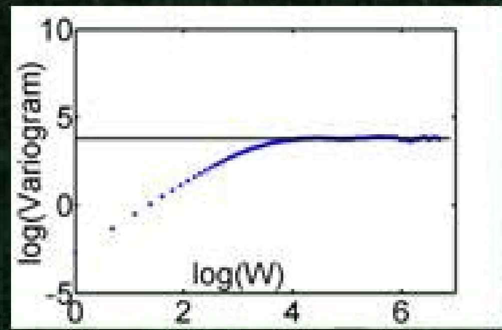
(a) slope=1.996, D_v=1.002



(b) slope=1.002, D_v=1.499



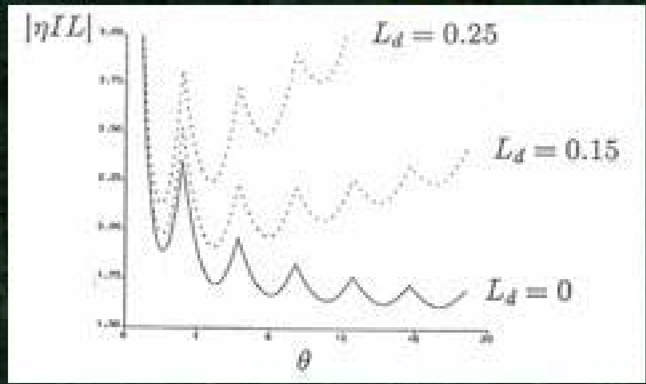
(c) slope=0.49, D_v=1.755



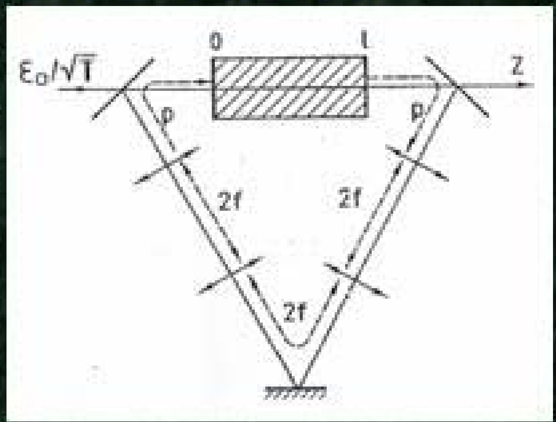
(d) slope=0, D_v=2

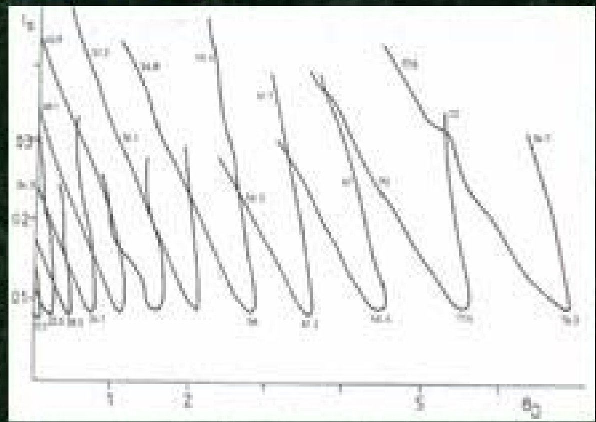
Other example non-linear systems

(i) Counter-propagating beams – no cavity [1]



(ii) Ring cavity with 2-level atoms [2]



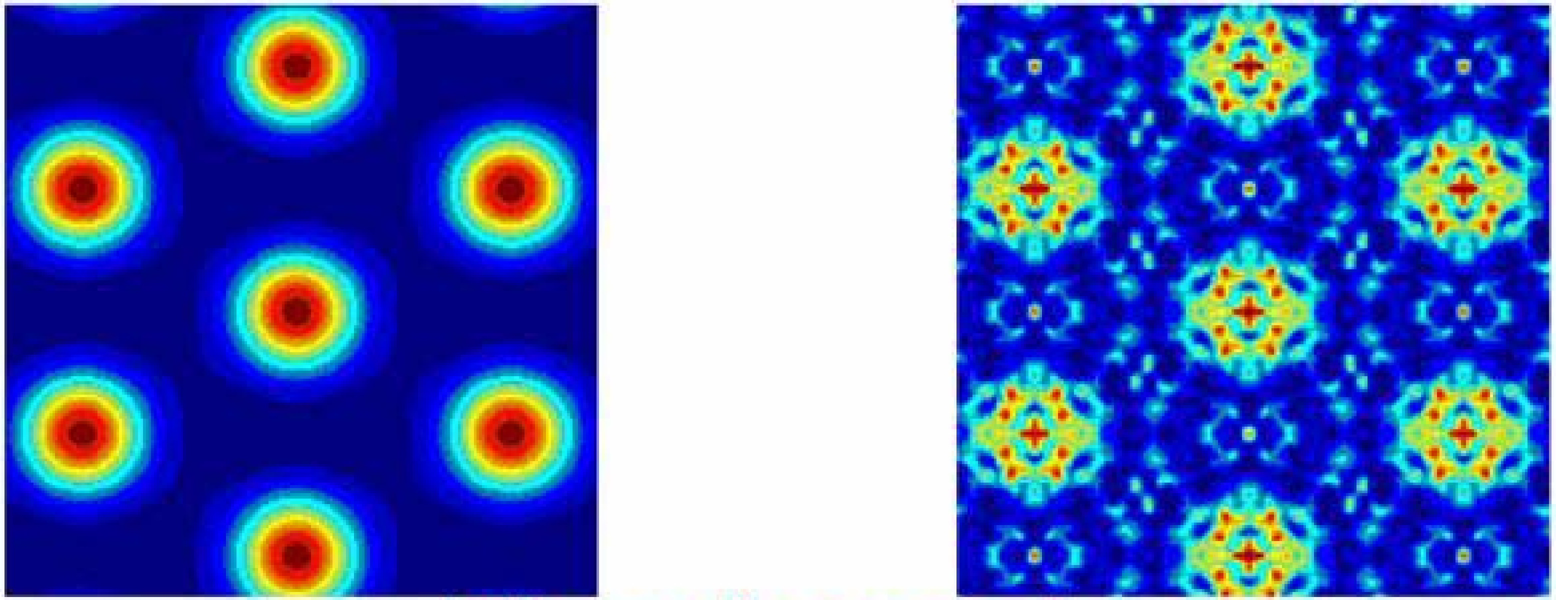


[1] W J Firth, C Pare, Opt Lett 13 (1988) 1096

[2] A S Patrascu C Nath, M Le Berre, E Ressayre, A Tallet, Opt Commun 91 (1992)

Conclusions

- *New generic mechanism* for *spontaneous fractal pattern formation* (expected in wide variety of non-linear systems)
- Spatial filtering allows demonstration of *both* SIMPLE (conventional) pattern formation and COMPLEX (fractal) pattern formation in *same* optical system
- This system: dependence of spectral characteristics (on diffusion and intensity) given by *rather simple law*
- *Analytical form* derived for *scale-dependent fractal dimension* (predictions confirmed by variogram analysis)



Thank you!