

Resistance and Ohms Law

When we have a fixed potential difference then the electric field will also be fixed and therefore there will be a steady electric force on the charge carriers (electrons) - this implies that the carriers should accelerate. In reality, we find that a steady current is maintained. When charge flows in a wire (due to the potential difference) then we have already seen that the velocity of this charge is limited by the interaction of the charge with the ions /atoms which make up the wire and it is found that the amount of current flow is dependant on the properties and dimensions of the conductor.

Resistance is defined as a measure of the ease of current flow, and is the ratio of the potential difference to the current - **Ohm's law**.

$$R = V/I$$

This is not really a law as it is an empirical statement based on the observation that *resistance is constant over a wide range of potential differences*. This is not always the case as certain classes of material are non-ohmic

Question: Can you think of an example of a non-ohmic material (device)

Answer: devices called diodes (explain IV characteristics - see electronics notes).

Resistivity

When charge flows through a wire then it can be compared to the flow of water in a pipe. This water flow will be due to the pressure difference maintained by the pump which is comparable with the electrical potential difference. The flow of electrical current in a wire can therefore be compared to the flow of water in a pipe.

With water it is easy to see that by keeping the pressure difference constant and doubling the length of the pipe then the water flow is halved i.e the water flow is proportional to the length of the pipe. If the cross section of the pipe is now doubled then twice as much water will flow through the pipe. When looking at electrical current we can say that:

the resistance of a wire is proportional to the length L

and

the resistance of a wire is inversely proportional to its cross sectional area A

$$R \propto L/A$$

$$R = \rho L/A$$

Where ρ is the resistivity, units Ohm-meters

Thus resistivity is a characteristic of the material and not on the dimensions of the material.

Resistivity is temperature dependant and this can be complicated, but for most metals the relationship between temperature and resistivity is given by:

$$\rho(t) = \rho^* [1 + \alpha(t - t^*)]$$

where: ρ^* is the resistivity at a reference temperature t^*
 α is the temperature coefficient of resistivity
 $\rho(t)$ is the resistivity at temperature t

Material	Resistivity	Temperature Coeff
Copper	1.72×10^{-8}	3.9×10^{-3}
Gold	2.44×10^{-8}	3.4×10^{-3}
Aluminium	2.82×10^{-8}	3.9×10^{-3}
Iron	10×10^{-8}	5×10^{-3}
Lead	22×10^{-8}	3.9×10^{-3}
Brass	7×10^{-8}	2×10^{-3}
Carbon	5×10^{-5}	-5×10^{-4}
Glass	$10^{10} - 10^{14}$	
Polyethylene	10^{16}	

The reciprocal of resistivity is **conductivity** σ

$$\sigma = 1/\rho$$

Ohms law can now be written in terms of resistivity or conductivity:

$$V = IR = I\rho L/A = \rho L I/A$$

$$V/L = \rho I/A$$

V/L is the magnitude of the electric field E , and I/A is the magnitude of the current density J ; therefore:

$$E = \rho J$$

Question:	Some materials show a property known as superconductivity - what happens to the resistance and hence the current in a material when it becomes superconducting.
Answer:	The resistance goes to a value of zero; and therefore currents once established in a conducting circuit persist for weeks at their original value. If the temperature is raised (only slightly) then the current very quickly reduces to zero.

Electric power - Energy transfer

When charges move along a conductor the potential energy of the charge decreases. This energy is transferred to the conductor and work must be done to maintain the flow of the charge.

If a potential difference of V volts is applied across a conductor, then the work done to maintain the flow of a charge q will be:

$$W = qV$$

Power is defined as the rate at which work is done i.e. $P = dW/dt$, so we can write:

$$P = Vdq/dt$$

Now dq/dt is the current I , so:

$$P = Vi = i^2R = V^2/R \text{ Watts}$$

Electromotive force

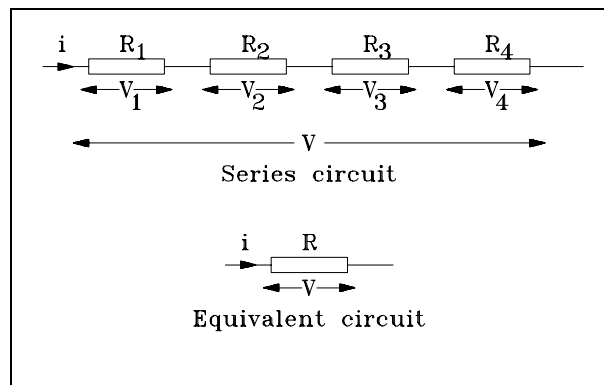
The emf is the potential difference produced by a device when no current is drawn from it. Examples of these devices are:

Battery:	emf produced by chemical reactions
Solar cells:	emf produced by light energy
Thermocouples:	emf produced by thermal energy
Generator	emf produced by work
etc. etc.	

DIRECT CURRENT CIRCUITS

In a simple circuit with only a current i passing through and a resistor R then the resulting potential difference across the resistor is iR . If the circuit contains a number of resistors then they can be connected in a number of different ways which if reduced to a simple value makes the overall analysis of the circuit easier.

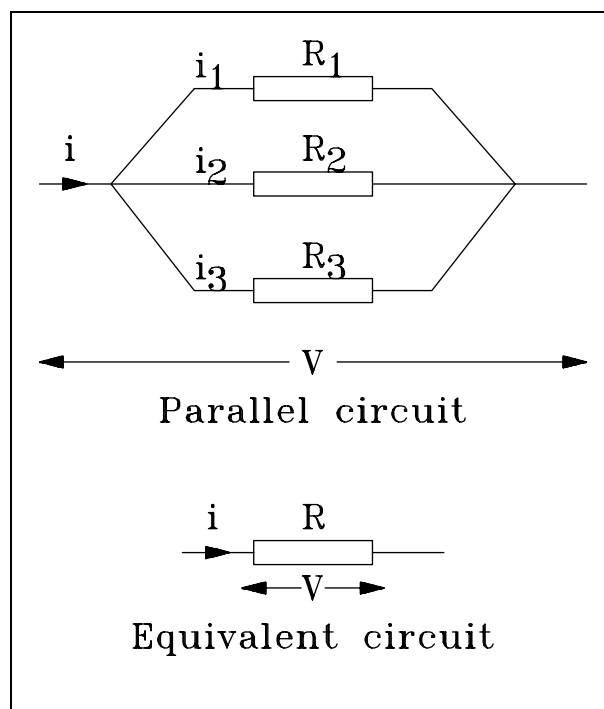
Resistors in Series



$$\begin{aligned} V_1 &= iR_1, V_2 = iR_2, V_3 = iR_3, V_4 = iR_4 \\ V &= V_1 + V_2 + V_3 + V_4 \\ V &= iR_1 + iR_2 + iR_3 + iR_4 \\ V &= i(R_1 + R_2 + R_3 + R_4) \end{aligned}$$

$$V/i = R = R_1 + R_2 + R_3 + R_4$$

Resistors in parallel



$$\begin{aligned}
i &= i_1 + i_2 + i_3 \\
V &= i_1 R_1 & i_1 &= V/R_1 \\
V &= i_2 R_2 & i_2 &= V/R_2 \\
V &= i_3 R_3 & i_3 &= V/R_3 \\
i &= i_1 + i_2 + i_3 = V/R_1 + V/R_2 + V/R_3 \\
i/V &= 1/R_1 + 1/R_2 + 1/R_3
\end{aligned}$$

$$1/R = 1/R_1 + 1/R_2 + 1/R_3$$

$$\begin{aligned}
1/R &= (R_2 R_3 + R_1 R_3 + R_1 R_2) / R_1 R_2 R_3 \\
R &= R_1 R_2 R_3 / (R_2 R_3 + R_1 R_3 + R_1 R_2)
\end{aligned}$$

Question: A circuit is made up of 12 resistors in parallel, whose values range between $10\text{K}\Omega$ and $100\text{K}\Omega$. If a single 10Ω resistor is included by mistake what will be the approximate value of the the equivalent resistance.

Answer: 10Ω (work it out!!)

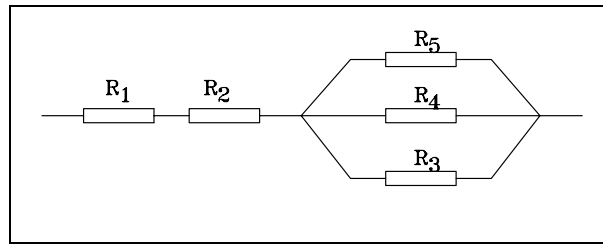
Compound resistor circuits

The rules used to calculate series and parallel equivalent resistances can be used repeatedly to find the equivalent resistance of circuits containing both series and parallel elements.

If a resistor R_1 is in series with a parallel combination of two resistors R_2 and R_3 then the equivalent resistance will be:

$$\begin{aligned}
1/R_p &= 1/R_2 + 1/R_3 \\
R_p &= R_2 R_3 / (R_2 + R_3) \\
R_E &= R_1 + R_p \\
R_E &= R_1 + R_2 R_3 / (R_2 + R_3)
\end{aligned}$$

EXAMPLE: A circuit contains 3 resistors which form a parallel network connected to 2 series resistors. Calculate the overall resistance of the resistor combination explaining each step.



Step 1. Calculate the equivalent resistance of the parallel network:

$$1/R_p = 1/R_3 + 1/R_4 + 1/R_5$$

$$1/R_p = (R_4R_5 + R_3R_5 + R_3R_4) / R_3R_4R_5$$

$$R_p = R_3R_4R_5 / (R_4R_5 + R_3R_5 + R_3R_4)$$

Step 2. Calculate the equivalent resistance of the series network:

$$R_s = R_1 + R_2$$

Step 3. Add the two equivalent resistances to find the total equivalent resistance:

$$R_T = R_p + R_s$$

$$R_T = [R_3R_4R_5 / (R_4R_5 + R_3R_5 + R_3R_4)] + R_1 + R_2$$