COULOMB'S LAW

Charles Coulomb (1736-1806) was one of the early scientists to study electrostatics. Coulomb knew that charged objects attracted or repelled each other and therefore they must exert a force on each other. He discovered that :

a) the force F depended on how far apart the charged objects were and falls inversely as the square of the distance r: $F \propto 1/r^2$

b) the force depended on the amount of charge on each of the objects and is proportional to the product of the charges. $F \propto q_1 q_2$

These statements can be summarised in Coulomb's Law, which shows the force F between two point charges q_1 and q_2, separated by a distance r is:

$$F \propto q_1 q_2 / r^2$$

If we now introduce a proportionality constant k, then:

$$F = k q_1 q_2 / r^2$$

k is known as Coulomb's constant and is equal to:

$$k = 1/4\Pi\varepsilon_0$$

Therefore the force F in Newtons is:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$ Coulomb's Law

ε_0 is the permittivity of free space = $8.854 \times 10^{-12}\ C^2N^{-1}m^{-2}$
q1 and q2 are the amounts of charge in Coulombs
r is the distance between the charges in metres.

Question: Two point charges exert a force of F on each other when at a distance d apart. What will the force be if the distance between them is now 3d

Answer: F/9

Question: Two point charges exert a force F on each other. If both the charges are doubled, what will the new force be.

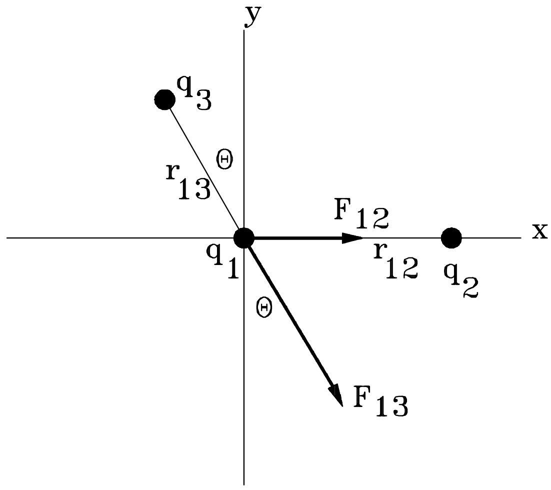
Answer: 4F

Coulomb's law only holds for charged objects whose sizes are much smaller than the distance between them i.e only point charges.

If more than 2 charges are present then Coulomb's law holds for each pair of charges and we can calculate the forces exerted on only one charge by using the vector equation:

Force exerted on q_1 = $\mathbf{F_1} = \mathbf{F_{12}} + \mathbf{F_{13}} + \mathbf{F_{14}} + \ldots\ldots\ldots\ldots\ldots\ldots$

where F_{12} is the force exerted on q_1 by q_2



Assume $q_1 = -1.0\text{x}10^{-6}$ C

$q_2 = +3.0\text{x}10^{-6}$ C
$q_3 = -2.0\text{x}10^{-6}$ C
$r_{12} = 15$ cm
$r_{13} = 10$ cm
$\theta = 30°$

The magnitude of the force is calculated using Coulomb's law (ignore the signs on the charges)

$$F_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2} \qquad F_{13} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}^2}$$

$F_{12} = 1.2$ N $\qquad$ $F_{13} = 1.8$ N

The direction of the forces are shown on the diagram.

The components of the resultant force F_1 acting on q_1 are:

$F_{1x} = F_{12x} + F_{13x} = F_{12x} + F_{13x} \sin\theta$
$F_{1x} = 2.1$ N

$F_{1y} = F_{12y} + F_{13y} = 0 + F_{13y} \cos\theta$
$F_{1y} = -1.6$ N

The magnitude of the resultant force F_1 on q_1 is:

$$F_1^2 = F_{1x}^2 + F_{1y}^2$$
$$F_1 = (F_{1x}^2 + F_{1y}^2)$$
$$F_1 = 2.64 \text{ N}$$

$$\tan\theta = 1.6/2.1$$
$$\theta = 37.3°$$

Note: Coulomb's law resembles the inverse square law for gravitation - Newton where the masses have been replaced with charge and the universal gravitational constant G is replaced by the Coulomb's constant k.

$$F = G\frac{m_1 m_2}{r^2} \qquad G = 6.67\text{x}10^{-11}\ \text{Nm}^2\text{kg}^{-2}$$

EXAMPLE

The distance r between the electron and the proton in the hydrogen atom is approximately $5.3\text{x}10^{-11}$*m. What are the magnitudes of :*

(a) the electrical force between the two particles

(b) the gravitational force between the two particles

Mass of proton = $1.7\text{x}10^{-27}$

Mass of electron = $9.1\text{x}10^{-31}$

(a)
$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

$$F_e = \frac{(9.0\text{x}10^9)(1.602\text{x}10^{-19})^2}{(5.3\text{x}10^{-11})^2}$$

$$F_e = 8.1\text{x}10^{-8}N$$

(b)
$$F = G\frac{m_1 m_2}{r^2}$$

$$F_g = \frac{(6.67\text{x}10^{-11})(9.1\text{x}10^{-31})(1.7\text{x}10^{-27})}{(5.3\text{x}10^{-11})^2}$$

$$F_g = 3.7\text{x}10^{-47}N$$

Electric Field

As we have seen, two point charges separated by some distance will interact, but how does one particle sense the other? An electric charge creates an electric field in the space around it. A second particle does not interact directly with the first, it responds to the field it encounters.

To define the electric field E created by a point charge q_1 we place a small test charge q_0 (assumed +ve by convention) at a point in space which is to examined. We now measure the force that acts on this test charge. The electric field **E** at the point is defined:

$$\mathbf{E} = \mathbf{F}/q_0$$

The direction of the electric field is the direction of the force.

Units: N/C = volts/metre

Using Coulomb's law we can now evaluate the electric field due to a point charge q_1. The force on q_0 placed a distance r from q_1 is:

$$F_{01} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_0}{r^2}$$

$$E_1 = \frac{F_{01}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_0}{r^2}$$

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2}$$

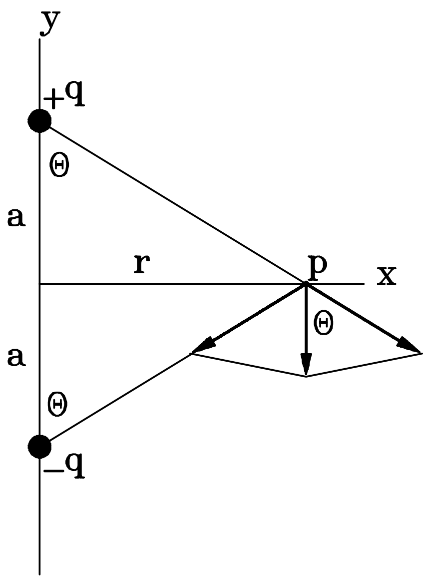
The electric field points away from a +ve charge and points towards a -ve charge.

To find the resultant field **E** due to a number of point charges: first, calculate the field at the point of interest for each charge as if it were the only charge present; second, add these values vectorially to find the resultant field E at the point:

$$\mathbf{E_1} = \mathbf{E_1} + \mathbf{E_2} + \mathbf{E_3} + \mathbf{E_4} + \ldots\ldots\ldots\ldots\ldots\ldots$$

EXAMPLE

A positive and a negative charge of equal magnitude q are placed a distance 2a apart. What is the field E due to these charges at a point P, a distance r along the perpendicular bisector of the line joining the charges. Assume r>>a



$\boldsymbol{E} = \boldsymbol{E_1} + \boldsymbol{E_2}$

$$E_1 = E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2 + r^2}$$

The vector sum of $\boldsymbol{E_1}$ *and* $\boldsymbol{E_2}$ *points vertically down and has a magnitude of:*

$E = 2E_1\cos\theta$

Now $\cos\theta = \dfrac{a}{\surd(a^2+r^2)}$

by substitution we get:

$$E = \frac{2}{4\pi\varepsilon_0}\frac{q}{(a^2 + r^2)}\frac{a}{\sqrt{a^2 + r^2}} = \frac{1}{4\pi\varepsilon_0}\frac{2aq}{(a^2 + r^2)^{3/2}}$$

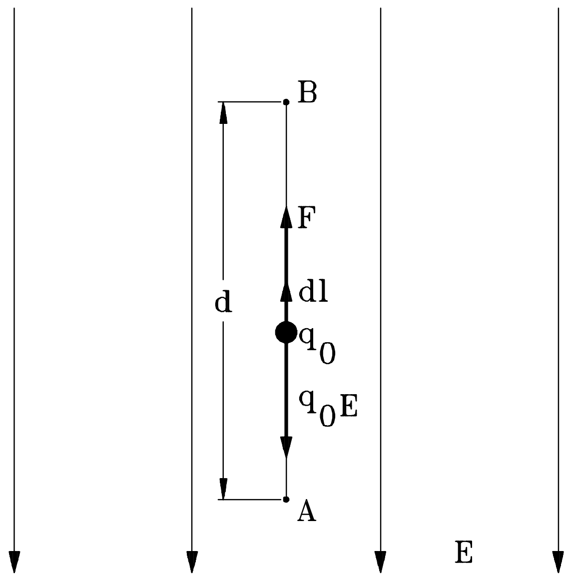
If $r >> a$ *then we can ignore a in the denominator and the equation reduces to:*

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2aq}{r^3}$$

Electric potential energy

when a charged particle is placed in an electric field then it must have an associated potential energy as the field does work to move the particle from one place to another. If we wish to move a test charge q_0 through a uniform electric field **E** from point A to point B (a distance d) then external work W_{AB} must be done to overcome the electric force of q_0**E** on the charge.

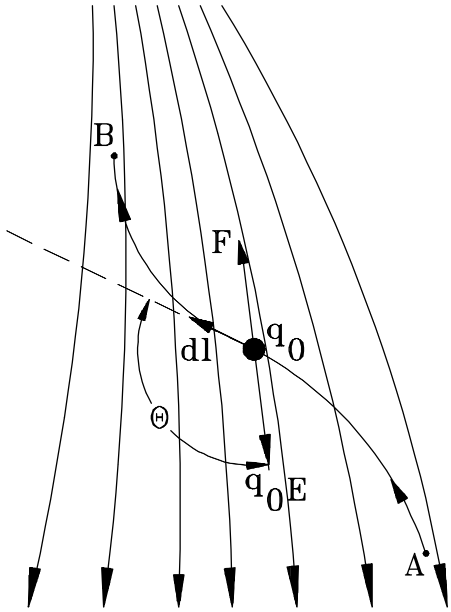
$$W_{AB} = Fd = q_0Ed = \text{potential energy } U$$



Now the electric
potential V at a point in an electric field is defined as the potential energy per unit charge at the point. The electrical potential difference between two points will therefore be:

$$V_B - V_A = W_{AB}/q_0 = Ed$$

This gives the relationship between potential difference and electric field for a simple case. A more general case involves a non-uniform field and a test charge which does not move in a straight line.



The electric field exerts a force on the test charge of $q_0\mathbf{E}$. To stop the test charge from accelerating an external force **F** must be applied which is exactly equal and opposite to the electrical force. If this external force moves the charge a short distance dl along the path from A to B then an element of work will have been done equal to **F**.dl. To find the total work done we have to integrate along the path A to B:

$$W_{AB} = \int_A^B \mathbf{F}.\,d\mathbf{l} \;=\; -q_0\int_A^B \mathbf{E}.\,d\mathbf{l}$$

$$V_B - V_A = \frac{W_{AB}}{q_0} = -\int_A^B \mathbf{E}.\,d\mathbf{l}$$

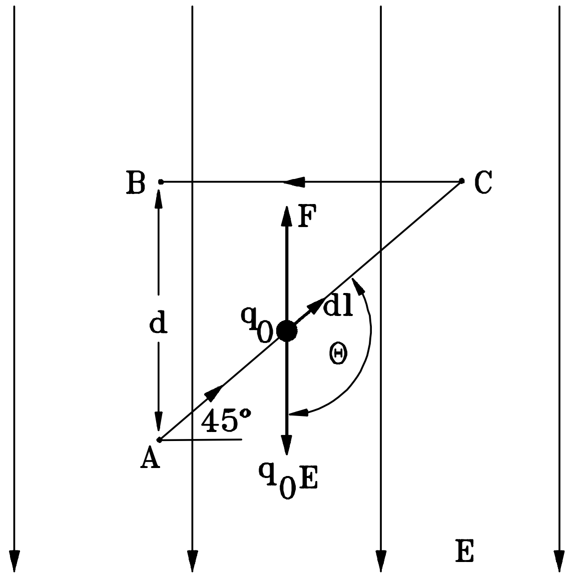
If A is at infinity and V_A is zero then the potential at point B can be found:

$$V_B = -\int_\infty^B \mathbf{E}.\,d\mathbf{l}$$

These two equations allow the calculation of potential difference between two points or at a single point if the electric field is known.

EXAMPLE

A test charge q_0 is moved from A to B over the path shown. Calculate the potential difference between A and B.



For the path AC, $\theta = 135°$

$$V_C - V_A = -\int_A^C \mathbf{E}.\mathbf{dl} = -\int_A^C \mathbf{E}\cos 135 \, dl = \frac{E}{\sqrt{2}}\int_A^C dl$$

The integrall is the length of the line AC which is $d\sqrt{2}$, thus:

$V_C - V_A = Ed$

Points B and C have the same potential as no work is done in moving from C to B (E and dl are at right angles on the line CB) and so B and C lie on the same equipotential surface which is at right angles to the lines of force.

$V_B - V_A = V_C - V_A = Ed$

This is the same answer as that derived for a direct path connecting A to B - the potential difference between two points is path independent.