Dipoles

Definition

A dipole is normally considered as a combination of 2 equal point electric charges, of opposite signs, separated by a small distance.

Dipole moment

The dipole moment, measured in coulomb metres, is the product of either charge (or pole) and the distance between them. This may also be expressed as the *couple* which would be required to maintain the dipole at right angles to an applied electric field.

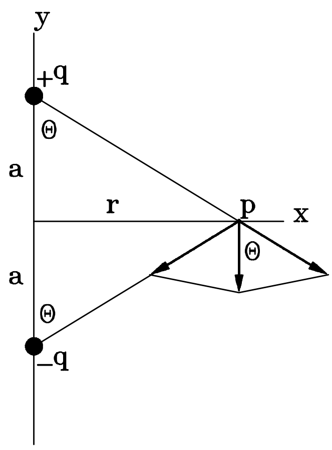
Note: Definition of a couple - Two equal and opposite parallel forces (not collinear) acting upon a body.

Molecules in which the centres of the positive and negative charges are separated, constitute a dipole. The dipole moments can provide evidence of the molecules shape. For example; water with a dipole of $6.1\text{x}10^{-30}$ Cm indicates a triangular shape with an angle of 105° between the two O-H bonds.

Q.1. A charged rod is brought near to a thin stream of tap water, what will be the observed result?.

A.1. The stream of water is attracted to the rod. This is due to the dipole of the molecule. The negative end (oxygen) is attracted to a positive rod causing the molecule to rotate with the net force on the molecule in the direction of the rod. The positive end (hydrogen) is attracted to a negative rod, again causing the molecule to rotate with the net force on the molecule in the direction of the rod.

If we now look back at the example from last week we will see that this is a dipole:



where the field at point P was found to be:

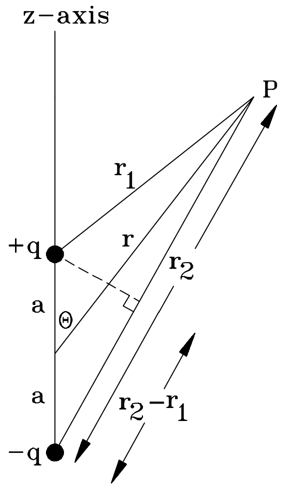
$$E = \frac{1}{4\pi\varepsilon_0}\frac{2aq}{r^3} \qquad \text{where } r>>a$$

The properties of the charge distribution i.e. the magnitude q and the separation 2a, are a product in the equation. This means that if we measure E at various distances from the dipole, we can never deduce q and 2a separately, but only as the product 2aq. If q was doubled and 2a was simultaneously halved, then E would not change. The product 2aq is the dipole moment p.

Therefore:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$$

Potential due to a dipole



From symmetry, the potential will not change as point P rotates about the z-axis (if r and Θ are fixed:

$$V = \sum_n V_n = V_1 + V_2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_1} - \frac{q}{r_2}\right) = \frac{q}{4\pi\varepsilon_0}\frac{r_2 - r_1}{r_1 r_2}$$

if r>>2a then:

$$r_2 - r_1 \approx 2a\cos\Theta \qquad \text{and} \qquad r_1 r_2 \approx r^2$$

and therefore:

$$V = \frac{q}{4\pi\varepsilon_0}\frac{2a\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

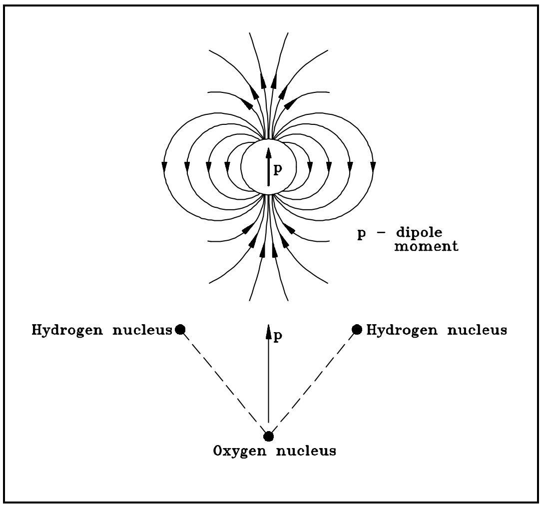
If Θ = 90° then V disappears and this shows that it takes zero work to bring in the test charge along this plane towards the dipole.

For a small set radius, V has its greatest positive value for Θ = 0°, and its greatest negative value for Θ = 180°.

The above equation describes the potential due to a simple dipole. It also holds for other charge configurations.

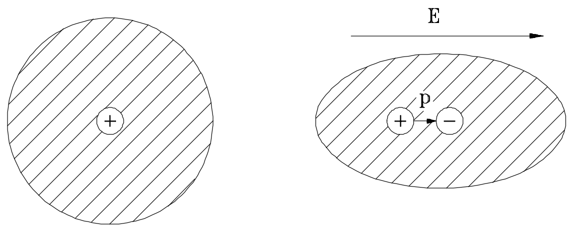
If we have an imaginary box containing a configuration of charges, which obey the above equation, then we can say that the charges within the box act as a dipole.

Water can be said to have a dipole of 6.1×10^{-30} Cm but it contains 3 nuclei and their associated electron clouds.



In this molecule, the effective centre of positive charge does not coincide with the centre of negative charge, and therefore the molecule can be treated as a dipole with an associated dipole moment.

Atoms and molecules do not always have permanent dipole moments, but they can be induced by placing them in an external electric field. The action of the field is to separate the centres of positive and negative charge (the molecule becomes polarised) and thus acquires an induced electric dipole moment.



+ve and −ve centres are coincident

dipole produced by external electric field

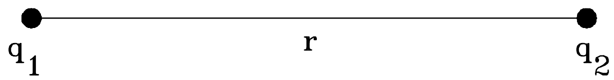
Electric Potential Energy

If 2 charges q_1 and q_2, initially separated by a distance r, are moved apart, then work must be done. If both the charges have the same sign then the work will be negative. If they are of opposite signs the work will be positive. The energy represented by this work is thought of as being stored by the system of charges as electric potential energy.

If the charges are of opposite signs (dipole) and they are released, they will accelerate towards each other transferring the potential energy into kinetic energy of the accelerating masses.

To define the electric potential energy for a system of point charges, we calculate the work required to bring them together from an infinite distance to their final positions in the system.

If q_2 is removed from the following system to infinity, then the electric potential caused by q_1 at the original position of q_2 is:



$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r}$$

If q_2 is now moved in to its original position the work done will be:

$$W = Vq_2$$

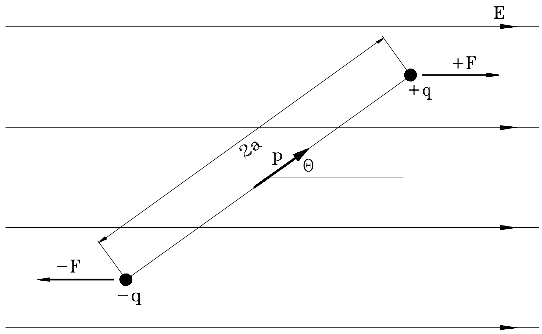
$$W = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}} = U \qquad = \text{electric potential energy}$$

For a dipole:

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1(-q_2)}{r_{12}}$$

Dipole in an electric field

The electric dipole moment can be considered as a vector **p** with a magnitude p. From the original dipole example, the magnitude of p is 2aq. the direction of **p** is from negative to positive:



In a uniform electric field **E**, the dipole with moment **p** makes an angle θ with the field. Two equal and opposite forces **F** and **-F** act on the charges (couple) and are equal to:

$$\mathbf{F} = q\mathbf{E}$$

The net force is zero, but there is a net torque about an axis through O given by:

$$T = 2F(a \sin\theta) = 2aF \sin\theta$$

Therefore:

$$T = 2qEa \sin\theta = pE \sin\theta$$

Thus, a dipole placed in an electric field **E** experiences a torque tending to align the dipole with the field.

$$T = \mathbf{p} \times \mathbf{E}$$

To change the orientation of a dipole in an electric field, work must be done (positive or negative). This work is stored as potential energy U.

if the dipole has an initial angle of θ_0, then the work required to turn the dipole to an angle θ will be:

$$W = \int_{\Theta_0}^{\Theta} dW = \int_{\Theta_0}^{\Theta} \mathrm{T} d\Theta = U$$

Substituting for T:

$$U = \int_{\Theta_0}^{\Theta} pE \sin\Theta . d\Theta = pE \int_{\Theta_0}^{\Theta} \sin\Theta . d\Theta$$

$$U = pE \left| -\cos\Theta \right|_{\Theta_0}^{\Theta}$$

If $\theta_0 = 90°$ then:

$$U = -pE\cos\theta$$