# DIELECTRICS

## Dielectrics and the parallel plate capacitor

When a dielectric is placed between the plates of a capacitor q is larger for the same value of voltage. From the relation $C = q/V$ it can be seen that the capacitance must also increase.

The ratio of the capacitance of the capacitor with the dielectric to the capacitance of the capacitor without the dielectric is called the *dielectric constant* $\kappa$ of the material.

If the same charge is maintained on the capacitor with and without the dielectric then the potential difference between the plates of the capacitor with the dielectric, $V_d$ will be less than that without the dielectric $V_0$ by a factor of $1/\kappa$.

$$V_d = V_0/\kappa$$

$$C = \frac{q}{V_0} = \frac{EA\varepsilon_0}{Ed} = \frac{A\varepsilon_0}{d}$$

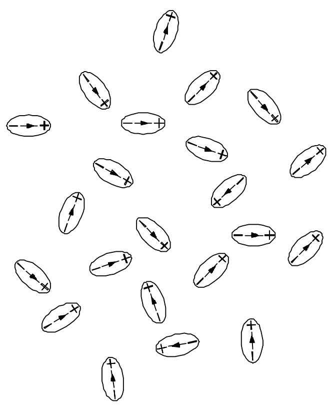
Where A is the plate area and d is the plate separation.

Therefore: $$C = \frac{A\kappa\varepsilon_0}{d}$$

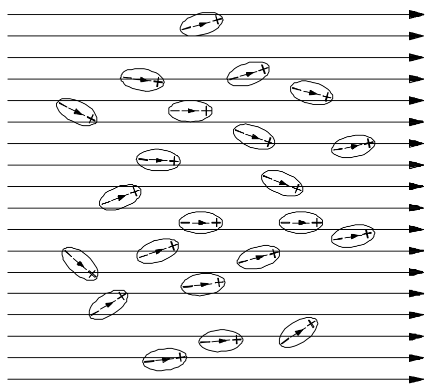
## Dielectric materials - a description

Dielectrics can be of two types, those which possess permanent dipole moments such as water and those which obtain an induced dipole.

When an electric field is applied to these materials the dipoles tend to align themselves to the field - this is not a perfect alignment due to thermal effects. The alignment is improved by either increasing the field or by decreasing the temperature. The alignment is due to the electric dipole moment **p** which is proportional to the electric field.



Molecules with a permanent electric dipole moment (no external electric field)



Molecules with a permanent electric dipole moment (with an external electric field)

To illustrate the dipole effects taking place within a slab of dielectric, we can take a charged parallel plate capacitor (battery disconnected) which has a fixed charge q and provides a uniform electric field $\mathbf{E_0}$.

When the dielectric is placed between the plates then the dipoles align with the electric field and the centre of positive charge separates from the centre of negative charge i.e. the dielectric becomes polarised while remaining electrically neutral. This separation of charge is on the atomic scale and it should be noted that the charge does not move as it would if the slab were made from a conductor - no charge movement over macroscopic distances.

The effect of this charge separation is the introduction of a electric field **E'** which opposes the external field $\mathbf{E_0}$**,** the resultant field **E** is therefore the vector sum of these two fields:

$$\mathbf{E} = \mathbf{E'} + \mathbf{E_0}$$

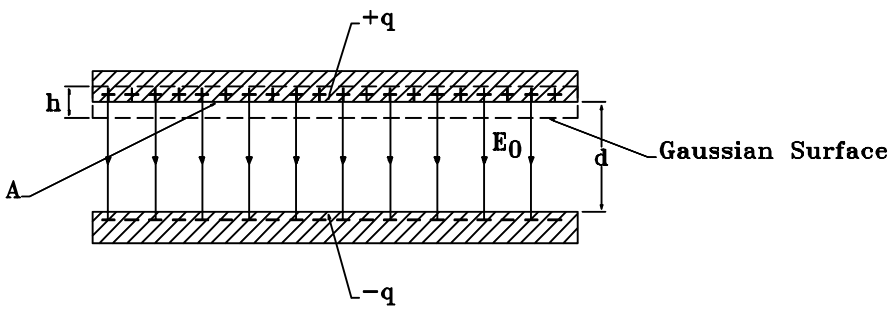
which is smaller than the original field.

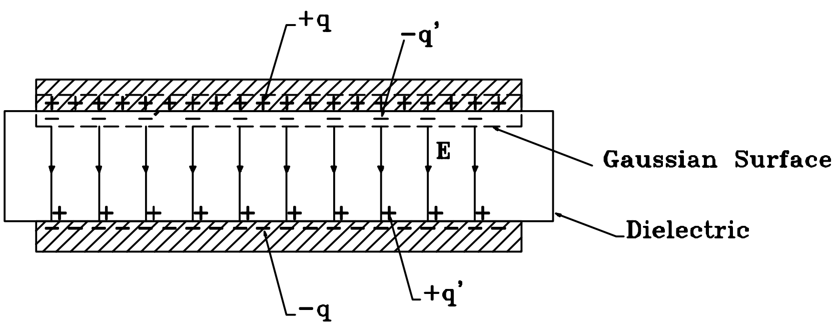
From the equation for a parallel plate resistor (V = Ed) it can be seen that the field is directly proportional to the potential difference and therefore the reduction in the overall field results in a reduction in the potential difference between the plates, and:

$$E_0/E = V_0/V_d = \kappa$$

If the battery is left connected during the introduction of the dielectric, then the above equation does not hold. The potential difference now remains constant but the charge q on the plates increases by a factor of κ.

## The use of Gauss's law for capacitors with a dielectric



If no dielectric is present then Gauss's law gives:

$$\varepsilon_0 \oint \mathrm{E.dA} = \varepsilon_0 E_0 A = q$$

$$E_0 = \frac{q}{\varepsilon_0 A} \qquad \text{D1}$$

With the dielectric present then Gauss's law gives:

$$\varepsilon_0 \oint \mathrm{E.dA} = \varepsilon_0 E A = q - q'$$

$$E_0 = \frac{q}{\varepsilon_0 A} - \frac{q'}{\varepsilon_0 A} \qquad \text{D2}$$

Using $E = E_0/\kappa$ and substituting in D1 we get:

$$E = \frac{E_0}{\kappa} = \frac{q}{\kappa \varepsilon_0 A}$$

By combining this equation with D2 we find:

$$\frac{q}{\kappa \varepsilon_0 A} = \frac{q}{\varepsilon_0 A} - \frac{q'}{\varepsilon_0 A}$$

$$q' = q\left(1 - \frac{1}{\kappa}\right)$$

q' the surface induced charge, is shown to be always less than the magnitude of the free charge and is equal to zero when there is no dielectric i.e. $\kappa = 1$.

Returning to the integral for the case with a dielectric it can be shown that:

$$\varepsilon_0 \oint \kappa \mathrm{E.dA} = q$$

This equation generally holds for all capacitors and is used when a dielectric is present.

**Energy within a capacitor**

Work must be done to separate two equal and opposite charges and this energy can be stored in the system i.e. on the capacitor plates. The energy can be recovered if the charges are allowed to come back together. If a capacitor is initially uncharged, then the work W done to charge the capacitor is equal to the electric potential energy U stored by the charged capacitor. This can be visualised as pulling electrons from one plate and depositing them onto the other plate.

If at time t a charge q' has been transferred from one plate to the other plate of a capacitor, then the potential difference V will be equal to q'/C. If now, an extra small amount of charge dq' is transferred, then the extra work needed to do this will be equal to:

$$dW = Vdq' = (q'/C)dq'$$

If this process is continued until the total charge is q, then the total work done will be:

$$W = \int dW = \int_0^q \frac{q'}{C} dq' = \frac{1}{2}\frac{q^2}{C}$$

Substituting for q using the standard relation, q = CV we obtain:

$$W = U = \frac{1}{2} CV^2$$

The energy stored in a capacitor is said to reside in the electric field.

In a parallel plate capacitor, if we neglect fringing at the edges, then the electric field has the same value for all points between the plates. Therefore the energy stored per unit volume (the energy density) is uniform and is given by:

$$u = \frac{U}{Ad} = \frac{\frac{1}{2} CV^2}{Ad}$$

where *A* is the area of the plates and *d* is the plate separation: *Ad* is therefore the volume

If we remember that the capacitance C for a parallel plate capacitor is:

$$C = \frac{\kappa \varepsilon_0 A}{d}$$

then by substitution:

$$u = \frac{\kappa \varepsilon_0}{2}\left(\frac{V}{d}\right)^2$$

Now the electric field E = V/d so:

$$u = \frac{1}{2}\kappa\varepsilon_0 E^2$$

This equation was derived for the parallel plate capacitor but it also holds true for all capacitors.

In general, if we have an electric filed E at any point in space, we can think that at that point there is a site of stored energy of magnitude $u = \frac{1}{2}\kappa\varepsilon_0 E^2$ per unit volume.