Electricity

Revision Notes

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DIRECT CURRENTS

Introduction

Current:	Rate of charge flow,	I = dq/dt	Units: amps
Potential an	d potential difference:	work done to to point B. Potential=W Units: volts	o move unit +ve charge from point A /ork/unit charge,
Electric field	<i>d:</i> Force which a	acts on a unit	+ve charge at a particular point.
	Force = field F = Eq	x charge E = F/q	
Since	e work = force x distanc	$e = F \times L$	

potential = work/charge = (force x distance)/charge = field x distance = EL

Drude theory of metals - Free electron model

In metals the valence electrons are no longer associated with a single atom but are free to move under the influence of external forces. The metal is therefore considered to be a container of free electrons.

Electrical Current

i = dq/dt

Units of current: Ampere = Coulomb/second

The direction of the current is always defined as the direction that positive charges move. For electrical current this will be in the opposite direction to the electron flow.

Current density

Uniform current J = i/A Units: Am^{-2}

The direction of \mathbf{J} is defined as the direction of the net flow of positive charges at the particular element.

In the general case where the current is not uniform:

$$di = J.dA = J dA \cos \Theta$$
 Θ is the angle made between J and dA

The total current passing through the total area A is therefore the sum of the differential elements:

$$I = \int J. dA$$

For a number of electrons each with charge q, the number in a unit volume is the number density n_q . If the electrons have a velocity v then in time Δt then the total charge passing through an area A will be ΔQ . This is the amount of charge which is contained in the volume A(v Δt). Therefore the amount of charge will be:

$$\Delta \mathbf{Q} = (\mathbf{n}_{q}\mathbf{q})(\mathbf{A}\mathbf{v}\Delta\mathbf{t})$$

The current is then given by the charge passing through the area in unit time:

$$\mathbf{I} = \Delta \mathbf{Q} / \Delta \mathbf{t} = \mathbf{n}_{q} \mathbf{q} \mathbf{A} \mathbf{v}$$

As the current density **J** is I/A then:

$$\mathbf{J} = n_q q \mathbf{v}$$

Resistance and Ohms Law

Resistance is defined as a measure of the ease of current flow, and is the ratio of the potential difference to the current - **Ohm's law**.

R = V/I

Resistivity

When looking at electrical current we can say that:

the resistance of a wire is proportional to the length L

and

the resistance of a wire is inversely proportional to its cross sectional area A

 $R \alpha L/A$

$R = \rho L/A$	Where ρ is the resistivity, units Ohm-meters
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Thus resistivity is a characteristic of the material and not on the dimensions of the material. Resistivity is temperature dependent and this can be complicated, but for most metals the relationship between temperature and resistivity is given by:

$$\rho(t) = \rho^* [1 + \alpha(t - t^*)]$$

where: ρ^* is the resistivity at a reference temperature t^{*} α is the temperature coefficient of resistivity $\rho(t)$ is the resistivity at temperature t The reciprical of resistivity is **conductivity** σ

$$\sigma = 1/\rho$$

Ohms law can now be written in terms of resistivity or conductivity:

$$V = IR = I\rho L/A = \rho L I/A$$

 $V/L = \rho I/A$

V/L is the magnitude of the electric field E, and I/A is the magnitude of the current density J; therefore:

 $\mathbf{E} = \mathbf{\rho} \mathbf{J}$

Electric power - Energy transfer

When charges move along a conductor the potential energy of the charge decreases. If a potential difference of V volts is applied across a conductor, then the work done to maintain the flow of a charge q will be:

W = qV

Power is defined as the rate at which work is done i.e. P = dW/dt, so we can write: P = Vdq/dt

Now dq/dt is the current I, so:

 $P = Vi = i^2 R = V^2/R$ Watts

Electomotive force

The emf is the potential difference produced by a device when no current is drawn from it. Examples of these devices are:

Battery:	emf produced by chemical reactions
Solar cells:	emf produced by light energy
Thermocouples:	emf produced by thermal energy
Generator	emf produced by work

DIRECT CURRENT CIRCUITS

Resistors in Series

 $R = R_1 + R_2 + R_3 + R_4$

Resistors in parallel

 $1/R = 1/R_1 + 1/R_2 + 1/R_3$

Compound resistor circuits

The rules used to calculate series and parallel equivalent resistances can be used repeatedly to find the equivalent resistance of circuits containing both series and parallel elements.

If a resistor R_1 is in series with a parallel combination of two resistors R_2 and R_3 then the equivalent resistance with be:

 $1/R_{P} = 1/R_{2} + 1/R_{3}$ $R_{P} = R_{2}R_{3}/(R_{2}+R_{3})$ $R_{E} = R_{1} + R_{P}$ $R_{E} = R_{1} + R_{2}R_{3}/(R_{2}+R_{3})$

Simple DC circuits

General rules

In a series circuit it is the current which remains constant through each element of the circuit.

In a parallel circuit it is the voltage which remains constant through each arm of the circuit.

Divider circuits

A circuit which is particularly useful involves the division of a voltage between two resistors connected in series.

If we have a voltage V applied across two series resistors R_1 and R_2 then the voltage drop V_1 across R_1 is:

 $V_1/V = R_1/(R_1 + R_2)$

KIRCHOFF'S LAWS

Definitions

- Loops: All circuits can be divided into a number of closed loops
- Nodes: A node is any point where 3 or more conductors come together#
- **Branch:** A path between two nodes which does not contain any other node.

1st law - The node rule

The total current flowing towards a node is equal to the total current flowing away from the node i.e. the algebraic sum of the currents flowing towards a node is zero. The current is said to be positive if it flows into a node and negative if it flows away from a node.

A general rule: the number of independent equations obtained by applying the node rule is one less than the number of nodes in the circuit.

2nd Law - The loop rule

The algabraic sum of the emf's in any closed loop is equal to the algabraic sum of the potential drops. This rule is states that the algabraic sum of the potential differences across all the circuit elements in a loop is zero.

A general rule: the number of independent equations obtained by applying the loop rule is one less than the total number of loops in the circuit.

Thevenin's theorem

The venin's theorem states: The current through a resistor R connected across any two points A and B of an active network (i.e. a network containing one or more sources or emf) is obtained by dividing the potential difference between A and B (with R disconnected) by R+r, where r is the resistance of the network measured between A and B with R disconnected and the sources of emf replaced by their values of internal resistance.

ELECTRICAL MEASUREMENT

The indicating analogue measuring device contains three essential features:

a) A deflecting device: a mechanical force is produced by the current, voltage or power.
 b) A controlling device: the value of the deflection is dependent on the magnitude of the quantity being measured.
 c) A damping device: to prevent oscillation of the moving system and allow it to reach its final value quickly.

POTENTIOMETER

Measures potential difference, current and resistance.

Measuring potential difference

The principle action is that an unknown potential difference is measured by balancing it against a known potential difference. The slider is moved along a uniform wire (connected across a known potential difference) until the deflection in the galvanometer is zero.

$$V_u/V_0 = R_1/(R_1 + R_2)$$

or if the wire is uniform then the resistance is proportional to the length, and so:

 $V_u/V_0 = L_1/(L_1 + L_2)$

WHEATSTONE BRIDGE

This is used to measure unknown resistance's.

 $R_2/R_x = R_1/R_3$

GALVANOMETER/AMMETER/VOLTMETER

The galvanometer measures small currents or by adding a resistor can be made to measure large currents or volts.





Galvonometer as an ammeter Galvonometer as a voltmeter

ELECTROSTATICS

ELECTRIC CHARGE

"Like charges repel each other, unlike charges attract each other."

Although charge is an intrinsic feature of every atom, and although we know how charges interact, the nature of charge is still a mystery and there is still no theory to explain the nature of charge.

The net charge of a body is the algebraic sum of the individual charges.

SI unit of charge is the COULOMB (C) Definition: The coulomb is defined in terms of the ampere. If 1 ampere of charge current flows in 1 second, then the total charge moved is 1 coulomb.

Magnitude of charge q or $e = 1,602 \times 10^{-19} C$

Charge in matter

In an atom, it is the electric forces which exist between the electrons and protons which hold the atom together.

All electrons have the same charge and therefore in bodies the net charge is the integral multiple of this charge - charge is therefore said to be *quantised*.

In a closed system the total charge is always conserved. Individual charges can be created or destroyed so long as the net change in charge is zero - *conservation of charge*.

COULOMB'S LAW

- a) the force F depended on how far apart the charged objects were and falls inversely as the square of the distance r: $F \alpha 1/r^2$
- b) the force depended on the amount of charge on each of the objects and is proportional to the product of the charges. F α q₁q₂

$$F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}$$
 Coulomb's Law

Coulomb's law only holds for charged objects whose sizes are much smaller than the distance between them i.e only point charges.

If more than 2 charges are present then Coulomb's law holds for each pair of charges and we can calculate the forces exerted on only one charge by using the vector equation:

Force exerted on $q_1 = F_1 = F_{12} + F_{13} + F_{14} + \dots$

where F_{12} is the force exerted on q_1 by q_2

ELECTRIC FIELD

An electric charge creates an electric field in the space around it. A second particle does not interact directly with the first, it responds to the field it encounters.

The electric field **E** at the point is defined:

 $\mathbf{E} = \mathbf{F}/\mathbf{q}_0$

The direction of the electric field is the direction of the force.

Units: N/C = volts/metreUsing Coulomb's law we can now evaluate the electric field due to a point charge.

The electric field points away from a +ve charge and points towards a -ve charge.

To find the resultant field E due to a number of point charges: first, calculate the field at the point of interest for each charge as if it were the only charge present; second, add these values vectorially to find the resultant field E at the point:

$$E_1 = E_1 + E_2 + E_3 + E_4 + \dots$$

ELECTRIC POTENTIAL ENERGY

when a charged particle is placed in an electric field then it must have an associated potential energy as the field does work to move the particle from one place to another.

External work W_{AB} must be done to overcome the electric force of $q_0 E$ on the charge.

 $W_{AB} = Fd = q_0Ed = potential energy U$

The electrical potential difference between two points will therefore be:

$$V_B - V_A = W_{AB}/q_0 = Ed$$

If this external force moves the charge a short distance dl along the path from A to B then an element of work will have been done equal to **F**.dl. To find the total work done we have to integrate along the path A to B:

$$W_{AB} = \int_{A}^{B} \mathbf{F} \cdot d\mathbf{l} = -q_{0} \int_{A}^{B} \mathbf{E} \cdot d\mathbf{l}$$
$$V_{B} - V_{A} = \frac{W_{AB}}{q_{0}} = -\int_{A}^{B} \mathbf{E} \cdot d\mathbf{l}$$

If A is at infinity and V_A is zero then the potential at point B can be found:

$$\mathbf{V}_{\mathrm{B}} = -\int_{\infty}^{\mathrm{B}} \mathbf{E} \cdot \mathbf{d} \mathbf{l}$$

These two equations allow the calculation of potential difference between two points or at a single point if the electric field is known.

DIPOLES

A dipole is normally considered as a combination of 2 equal point electric charges, of opposite signs, separated by a small distance.

Dipole moment

The dipole moment, measured in coulomb metres, is the product of either charge (or pole) and the distance between them.

The field at point P due to a dipole is found to be:

$$E = \frac{1}{4\pi\varepsilon_0} \frac{2aq}{r^3} \qquad \text{where } r \gg a$$

The properties of the charge distribution i.e. the magnitude q and the separation 2a, are a product in the equation. This means that if we measure E at various distances from the dipole, we can never deduce q and 2a separately, but only as the product 2aq. If q was doubled and 2a was simultaneously halved, then E would not change. The product 2aq is the dipole moment p.

Therefore:

$$E = \frac{1}{4\pi\varepsilon_0} \frac{p}{r^3}$$

Potential due to a dipole

$$V = \frac{q}{4\pi\varepsilon_0} \frac{2a\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0} \frac{p\cos\theta}{r^2}$$

If $\Theta = 90^{\circ}$ then V disappears and this shows that it takes zero work to bring in a test charge along this plane towards the dipole.

For a small set radius, V has its greatest positive value for $\Theta = 0^{\circ}$, and its greatest negative value for $\Theta = 180^{\circ}$.

The above equation describes the potential due to a simple dipole. It also holds for other charge configurations.

ELECTRIC POTENTIAL ENERGY

If 2 charges q_1 and q_2 , initially separated by a distance r, are moved apart, then work must be done. If both the charges have the same sign then the work will be negative. If they are of opposite signs the work will be positive. The energy represented by this work is thought of as being stored by the system of charges as electric potential energy.

$$W = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{12}} = U \qquad = \text{electric potential energy}$$

For a dipole:

$$U=\frac{1}{4\pi\varepsilon_0}\frac{q_1(-q_2)}{r_{12}}$$

Dipole in an electric field

In a uniform electric field **E**, the dipole with moment **p** makes an angle θ with the field. Two equal and opposite forces **F** and -**F** act on the charges (couple) and are equal to:

$$\mathbf{F} = \mathbf{q}\mathbf{E}$$

The net force is zero, but there is a net torque about an axis through O given by:

Therefore:

$$\Gamma = 2F(a \sin \theta) = 2aF \sin \theta$$

 $T = 2qEa \sin\theta = pE \sin\theta$

Thus, a dipole placed in an electric field **E** experiences a torque tending to align the dipole with the field.

$$T = p \times E$$

if the dipole has an initial angle of θ_0 , then the work required to turn the dipole to an angle θ will be:

$$W = \int_{\Theta_0}^{\Theta} dW = \int_{\Theta_0}^{\Theta} T d\Theta = U$$

Substituting for T:

$$U = \int_{\Theta_0}^{\Theta} pE \sin \Theta \cdot d\Theta = pE \int_{\Theta_0}^{\Theta} \sin \Theta \cdot d\Theta$$
$$U = pE |-\cos \Theta|_{\Theta_0}^{\Theta}$$

Gauss's Law

The electric field and field lines

Electric field lines provide a convenient way of visualising the electric field.

The field lines are drawn according to the following rules:

- 1. The field lines point in the same direction as the electric field at every point in space. Therefore the electric field at a point is always at a tangent to the field lines at that point.
- 2. Field lines always start on the positive charge and end on the negative charge.
- 3. The strength of the field is represented by the density of the lines. The number of lines, in 3 dimensions, leaving a positive charge or approaching a negative charge is proportional to the magnitude of the charge.
- 4. No two lines touch or cross.

Electric Flux **\phi**

Definition of flux: The flux of any vector quantity through an area is the product of the area and the component of the vector at right angles to the area.

For a flat surface in a uniform electric field **E** the electric flux is:

$$\phi = \text{EAcos } \theta$$

$$\Phi = \int_{surface} \mathbf{E} \cdot d\mathbf{A} \qquad \text{general equation}$$

Gaussian surfaces

A Gaussian surface is an imaginary surface drawn so as to be able to use Gauss's law.

dA represents a small area of the surface and is known as the surface vector. The surface is chosen to represent the symmetry of the system.

Gauss's Law

The total electric flux of a closed surface in an electric field is 4π times the electric charge within that surface.

or

The total electric flux through a closed surface is proportional to the charge enclosed by that surface.

 $\phi = q_n / \epsilon_0$

 $q_n = \mathcal{E}_0 \oint \mathbf{E}_{\bullet} d\mathbf{A}$

where: q_n is the net charge enclosed by the surface **E.dA** is the flux through a small surface element $\oint E.dA$ is the total flux through a closed surface

Application to a point charge (zero dimension)

$$\Phi = \frac{q}{\varepsilon_0} = 4\pi r^2 E$$

Uniform line of charge (1 dimension)

$$\frac{\mathbf{q}}{\varepsilon_0} = \mathbf{E} \int \mathbf{dA} = \mathbf{E} 2\pi \mathbf{T} \mathbf{L}$$

This can be rearranged to give the magnitude of the electric field:

$$\mathbf{E} = \frac{\mathbf{q}}{\mathbf{L}} \frac{1}{2\pi\varepsilon_0} \frac{1}{\mathbf{r}} = \frac{1}{2\pi\varepsilon_0} \frac{\lambda}{\mathbf{r}}$$

The net charge enclosed by the surface = λL

A uniform sheet of charge (2 dimension)

$$\frac{q}{\varepsilon_0} = E(A + A) = 2EA$$

Substituting the charge density into this equation gives:

$$\mathbf{E} = \frac{\sigma}{2\varepsilon_0}$$

Spherical symmetric charge distribution (3 dimension)

Assign a Gaussian spherical surface of radius r where r > R. Applying Gauss's law:

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \qquad q \text{ is the total charge}$$

Assign a Gaussian spherical surface of radius r where r < R. Applying Gauss's law:

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q^*}{r^2}$$
 where q* is the part of q contained within
the sphere of radius r

For points inside the sphere, then

$$q^* = q \frac{\frac{4}{3}}{\frac{4}{3}} \frac{\pi r^3}{\pi R^3}$$
$$q^* = q \left(\frac{r}{R}\right)^3$$

the expression for E becomes:

$$E = \frac{1}{4\pi\varepsilon_0} \frac{qr}{R^3}$$

Relationship between distance r and dimensionality

An electric field's dependance on distance r depends on the dimensionality of the charge distribution that creates it.

0-dimension: the electric field for point charge falls as $1/r^2$

1-dimension: the electric field for a line of charge falls as 1/r

2-dimension: the electric field for plane of charge is constant everywhere in space

3-dimension: the electric field for a spherical distribution of charge rises with r

From the above it can be seen that:

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THE ELECTRIC FIELD CHANGES AS r^{D\mathchar`2}
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CAPACITORS AND DIELECTRICS

Definitions and Units

Capacitance:

The property of a system of electrical conductors and insulators which enables it to store electric charge when a potential difference exists between the conductors.

Units: farad (Coulomb per volt)

Defined as the capacitance of a capacitor between the plates of which there appears a potential drop of 1 volt when it is charged with one coulomb of electricity

Dielectric:

A non-conductor of electricity, insulator. A substance in which an electric field gives rise to no net flow of electric charge but only to a displacement of charge.

Dielectric constant or relative permittivity:

The ratio of the capacitance of a capacitor with the given substance as the dielectric, to the capacitance of the same capacitor with air (or a vacuum) as the dielectric. Symbol κ .

Dielectric strength:

The maximum voltage which can be applied to a dielectric material without causing it to break down. Usually expressed in volts per mm.

Capacitors

Spheres

The potential of a positively charged conducting sphere is given by:

$$V_+' = \frac{1}{4\pi\varepsilon_0} \frac{q}{R}$$

If another sphere of the same radius R, carrying a negative charge -q, is located at a distance >>R from the first sphere then it can be said that both spheres are electrically isolated. The potential of the second sphere is therefore given by:

$$V_{-}' = \frac{1}{4\pi\varepsilon_0} \frac{q}{R}$$

The potential difference between the two spheres is thus:

$$V' = V_{+}' - V_{-}' = \frac{1}{4\pi\varepsilon_0} \frac{2q}{R}$$

The potential difference is therefore proportional to the charge on either sphere.

This equation can now be written:

$$q = (2\pi\varepsilon_0 R)V' = C'V'$$

C', the proportionality constant, is called the capacitance of the two spheres.

when the two spheres are brought closer together, the positive sphere will have the effect of raising the potential of the negative sphere from V_{-} ' to V_{-} and the negative sphere will lower the potential of the positive sphere from V_{+} ' to V_{+} .

It is therefore simple to deduce that, although the charge on each sphere has remained constant, the potential difference between the spheres has been reduced and the capacitance has been increased.

q = CV where C>C' and V<V'

Parallel plate capacitor

$$C = \frac{q}{V} = \frac{\varepsilon_0 EA}{Ed} = \frac{\varepsilon_0 A}{d}$$
 Parallel plate capacitor

A cylindrical capacitor

$$\mathbf{E} = \frac{\mathbf{q}}{2\pi \mathbf{rl}\boldsymbol{\varepsilon}_0}$$

The potential difference between the plates is:

$$V = \int_{a}^{b} E dr = \int_{a}^{b} \frac{q}{2\pi\varepsilon_{0}l} \frac{dr}{r} = \frac{q}{2\pi\varepsilon_{0}l} \ln \frac{b}{a}$$

The capacitance is given by:

$$C = \frac{q}{V} = \frac{2\pi\varepsilon_0 l}{\ln(b/a)}$$

The capacitance only depends on the geometry of the capacitor.

Capacitors in Series

$$1/C = 1/C_1 + 1/C_2 + 1/C_3$$

Capacitors in Parallel

 $C = C_1 + C_2 + C_3$

DIELECTRICS

Dielectrics and the parallel plate capacitor

When a dielectric is placed between the plates of a capacitor q is larger for the same value of voltage. From the relation C = q/V it can be seen that the capacitance must also increase.

The ratio of the capacitance of the capacitor with the dielectric to the capacitance of the capacitor without the dielectric is called the *dielectric constant* κ of the material.

If the same charge is maintained on the capacitor with and without the dielectric then the potential difference between the plates of the capacitor with the dielectric, V_d will be less than that without the dielectric V_0 by a factor of $1/\kappa$.

$$V_{d} = V_{0}/\kappa$$
$$C = \frac{A\kappa\varepsilon_{0}}{d}$$

The use of Gauss's law for capacitors with a dielectric

$$\frac{q}{\kappa\varepsilon_0 A} = \frac{q}{\varepsilon_0 A} - \frac{q'}{\varepsilon_0 A}$$
$$q' = q \left(1 - \frac{1}{\kappa}\right)$$

q' the surface induced charge, is shown to be always less than the magnitude of the free charge and is equal to zero when there is no dielectric i.e. $\kappa = 1$.

Returning to the integral for the case with a dielectric it can be shown that:

$$\mathcal{E}_0 \oint \kappa \mathbf{E} \cdot \mathbf{dA} = q$$

This equation generally holds for all capacitors and is used when a dielectric is present.

Energy within a capacitor

$$W = \int dW = \int_{0}^{q} \frac{q'}{C} dq' = \frac{1}{2} \frac{q^{2}}{C}$$

Substituting for q using the standard relation, q = CV we obtain:

$$W = U = \frac{1}{2}CV^2$$

The energy stored in a capacitor is said to reside in the electric field.

In a parallel plate capacitor, if we neglect fringing at the edges, then the electric field has the same value for all points between the plates. Therefore the energy stored per unit volume (the energy density) is uniform and is given by:

$$u = \frac{U}{Ad} = \frac{\frac{1}{2}CV^2}{Ad}$$

C for a parallel plate capacitor is:

$$C = \frac{\kappa \varepsilon_0 A}{d}$$

then by substitution:

$$u=\frac{\kappa\varepsilon_0}{2}\left(\frac{V}{d}\right)^2$$

Now the electric field E = V/d so:

$$u=\frac{1}{2}\kappa\varepsilon_0 E^2$$

This equation was derived for the parallel plate capacitor but it also holds true for all capacitors.

In general, if we have an electric filed E at any point in space, we can think that at that point there is a site of stored energy of magnitude $u = \frac{1}{2} \kappa \varepsilon_0 E^2$ per unit volume.