

SCHOOL OF COMPUTING, SCIENCE AND ENGINEERING

SEMESTER TWO EXAMINATION

PROGRAMMES:

BSc (HONS) PHYSICS
BSc (HONS) PHYSICS WITH COMPUTING
BSc (HONS) PURE & APPLIED PHYSICS
BSc (HONS) PHYSICS WITH SPACE TECHNOLOGY
BSc (HONS) PHYSICS WITH PHOTONICS
MPhys (HONS) PHYSICS
MPhys (HONS) PHYSICS WITH A YEAR IN NORTH AMERICA
MPhys (HONS) PHYSICS WITH COMPUTING
MPhys (HONS) PHYSICS WITH SPACE TECHNOLOGY
MPhys (HONS) PHYSICS WITH PHOTONICS
MPhys (HONS) PHYSICS WITH MEDICAL PHYSICS
MPhys (HONS) PHYSICS WITH ACOUSTICS

BLOCK CODES:

S/P/F1, MP/P/F1,
MP/PN/F1
S/PCO/F1, MP/PCO/F1
S/PAP/F1
S/PST/F1, MP/PST/F1
S/PPT/F1, MP/PPT/F1
MP/PMP/F1
MP/PAT/F1

ELECTRICITY

ANSWERS

2005

Instructions to Candidates

Time allowed 1 hour 30 minutes

Answer the question in **SECTION A** **AND** any **TWO** questions from **SECTION B**

The question in Section A carries 20 marks
Each question in Section B carries 40 marks

You are advised to spend about **20 minutes on Section A**
and **70 minutes on Section B**.

STANDARD LIST OF PHYSICAL CONSTANTS

Acceleration due to gravity,	g	=	9.81 m s^{-2}
Atomic mass unit,	u	=	$1.66 \times 10^{-27} \text{ kg}$
Avogadro constant,	N	=	$6.02 \times 10^{23} \text{ mol}^{-1}$
Bohr magneton,	μ_B	=	$9.27 \times 10^{-24} \text{ J T}^{-1}$
Boltzmann constant,	k_B	=	$1.38 \times 10^{-23} \text{ J K}^{-1}$
Electron charge,	e	=	$1.60 \times 10^{-19} \text{ C}$
Electron rest mass,	m_e	=	$9.11 \times 10^{-31} \text{ kg}$
Faraday constant,	F	=	$9.65 \times 10^4 \text{ C mol}^{-1}$
Gas constant,	R	=	$8.3 \text{ J mol}^{-1} \text{ K}^{-1}$
Gravitational constant,	G	=	$6.672 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$
Permeability of free space,	μ_0	=	$4\pi \times 10^{-7} \text{ H m}^{-1}$
Permittivity of free space,	ϵ_0	=	$8.85 \times 10^{-12} \text{ F m}^{-1}$
Planck constant,	h	=	$6.63 \times 10^{-34} \text{ J s}$
Proton rest mass,	m_p	=	$1.67 \times 10^{-27} \text{ kg}$
Rydberg constant,	R_∞	=	$1.0974 \times 10^7 \text{ m}^{-1}$
Solar constant	S_0	=	$1.37 \times 10^3 \text{ W m}^{-2}$
Stefan-Boltzmann constant	σ	=	$5.670 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$
Velocity of light in vacuo,	c	=	$3.00 \times 10^8 \text{ m s}^{-1}$
	1 eV	=	$1.60 \times 10^{-19} \text{ J}$

SECTION A

1. Answer **ALL** parts of the question:

- a) Give an alternative name for the *Drude theory* of metals and then a brief explanation of how it can be used to describe electron conduction processes within a metal.

(4 marks)

The *Drude theory* of metals is better known as the free electron model.

In metals the valence electrons are no longer associated with a single atom but are free to move under the influence of external forces. The metal is therefore considered to be a container of free electrons.

- b) Give 4 devices which are capable of producing an *emf* and briefly state how each produces the *emf*.

(4 marks)

Battery:	emf produced by chemical reactions
Solar cells:	emf produced by light energy
Thermocouples:	emf produced by thermal energy
Generator	emf produced by work

- c) Give a brief definition of a *dipole* and its associated *dipole moment*.

(2 marks)

A dipole is normally considered as a combination of 2 equal point electric charges, of opposite signs, separated by a small distance.

The dipole moment, measured in coulomb metres, is the product of either charge (or pole) and the distance between them.

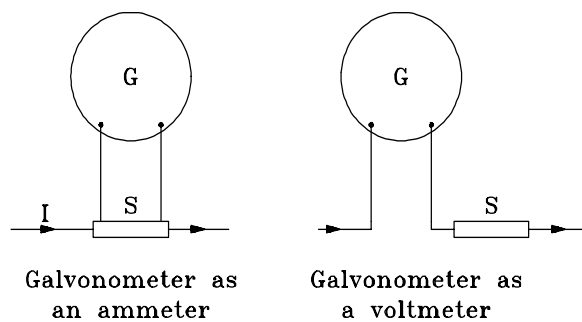
- c. Electric field lines provide a convenient way of visualising the electric field. State the 4 rules that are used to construct the Electric field lines.

(4 marks)

1. The field lines point in the same direction as the electric field at every point in space. Therefore the electric field at a point is always at a tangent to the field lines at that point.
2. Field lines always start on the positive charge and end on the negative charge.
3. The strength of the field is represented by the density of the lines. The number of lines, in 3 dimensions, leaving a positive charge or approaching a negative charge is proportional to the magnitude of the charge.
4. No two lines touch or cross.

- e. Draw two simple diagrams to show how you can convert a galvanometer into firstly an ammeter and secondly a voltmeter.

(4 marks)



- f. Two point charges exert a force of F on each other when at a distance d apart. What will the force be if the distance between them is changed to $3d$ and the magnitude of each charge is doubled?

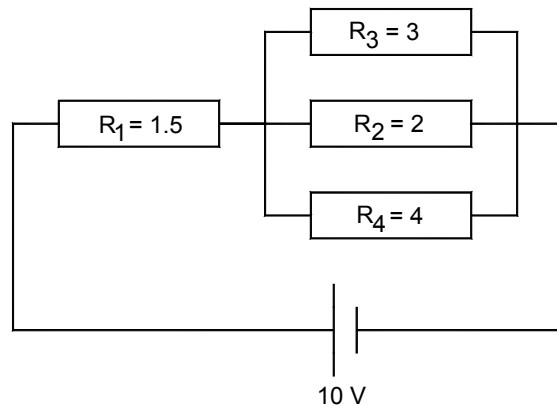
(2 marks)

$$4F/9$$

SECTION B

2. Answer **ALL** parts of the question:

- a) Consider the combination of resistors shown in the diagram. Determine:
- the current through each resistor
 - the power dissipated by each resistor



(15 marks)

- i. First calculate the resistance R^* of the parallel network:

$$1/R^* = 1/R_2 + 1/R_3 + 1/R_4$$

$$1/R^* = 1/2 + 1/3 + 1/4$$

$$R^* = 0.923 \text{ ohms}$$

The total resistance R is thus $0.923 + 1.5 = 2.423$ ohms

The total current I supplied by the battery = $V/R = 10/2.423 = 4.13$ A (this is the current through R_1).

The voltage drop across R_1 is thus $1.5 \times 4.13 = 6.2$ V. Therefore 3.8 V must be dropped across the parallel network.

$$I_2 R_2 = I_3 R_3 = I_4 R_4 = 3.8 \text{ V}$$

Therefore:

$$I_2 = 1.9 \text{ A}, \quad I_3 = 1.27 \text{ A}, \quad I_4 = 0.95 \text{ A}$$

- ii The power dissipated by each resistor $P = I^2 R$

$$P_1 = 25.58 \text{ W}, \quad P_2 = 7.22 \text{ W}, \quad P_3 = 4.84 \text{ W}, \quad P_4 = 3.61 \text{ W}$$

- b) A wire of length 5 m and uniform diameter 1.5 mm has a resistance of 2.5 Ω . What will be the new resistance if the length is increased to 20 m and the diameter reduced to 0.5 mm?

(5 marks)

$$R = \rho L/A$$

The resistivity will remain constant and therefore:

$$\rho = R_1 A_1 / L_1 = R_2 A_2 / L_2$$

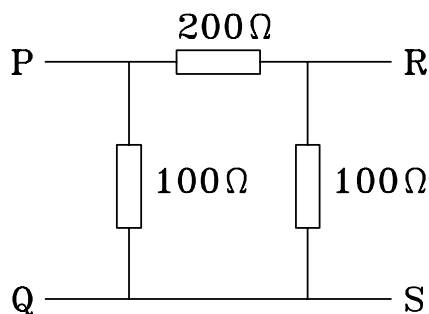
$$R_2 = R_1 A_1 L_2 / A_2 L_1$$

$$R_2 = 2.5 \times \pi \times (0.75 \times 10^{-3})^2 \times 5 / \pi \times (0.25 \times 10^{-3})^2 \times 20$$

$$R_2 = 1.4 \times 0.6^2 \times 10 / 0.4^2 \times 16$$

$$\mathbf{R_2 = 5.625 \Omega}$$

- c) Calculate the total effective resistance between P and Q in the network shown.



If a 12 V battery is now connected across PQ, calculate the voltage between points R and S.

(5 marks)

Looking from PQ the 200Ω resistor is in series with one of the 100Ω resistors and these are in parallel with the other 100Ω resistor. Therefore:

$$1/R_E = 1/100 + 1/300$$

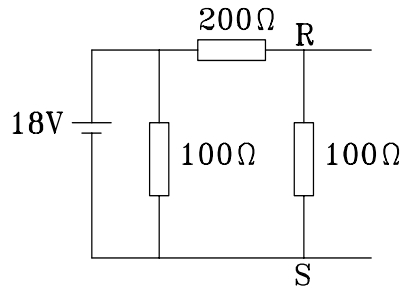
$$\mathbf{R_E = 75\Omega}$$

If a 12V battery is now connected across PQ then the circuit becomes a divider with 12V across the 200Ω and 100Ω resistors

$$V_{RS} = 12(100/300)$$

$$\mathbf{V_{RS} = 4V}$$

- d) A moving coil galvanometer is to be used to measure the current and voltage between points *R* and *S* in the following circuit. If the galvanometer has an internal resistance of $5\ \Omega$ and gives a full scale deflection when $5\ \text{mA}$ is applied across it, calculate the shunt resistance required to:
- enable the galvanometer to read the current
 - enable the galvanometer to measure the voltage



(15 marks)

The voltage drop across RS is:

$$(100/200+100)18 = V$$

$$V = 6 \text{ Volts}$$

The current passing between R and S is therefore $6/100 = 0.06\ \text{A}$

- a. Current through the coil = p.d. across the coil/resistance of the coil

$$5 \times 10^{-3} = \text{p.d.}/5$$

$$\text{p.d.} = 0.025\text{V}$$

The current through the shunt = Total current - current through the coil
 $= 60 \times 10^{-3} - 5 \times 10^{-3} = 0.035\text{A}$

Resistance of the shunt S = p.d. across S/current through S
 $= 0.025/0.035$

$$\mathbf{S = 0.714\Omega}$$

- b. Current through coil = p.d across galvanometer and shunt/the total resistance

$$5 \times 10^{-3} = 6/\text{total resistance}$$

$$\text{Total resistance} = 1200\Omega$$

The shunt is in series with the galvanometer, therefore:

Resistance of the shunt $S = 1200 - 5$

$$\mathbf{S = 1195\Omega}$$

3. Answer **ALL** parts of the question:

a) With relation to Kirchoff's law, give brief definitions of :

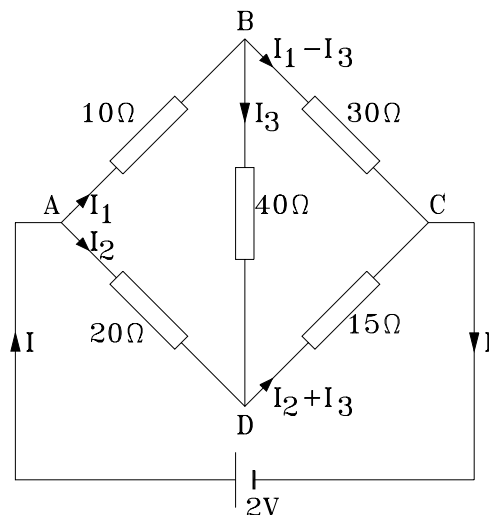
- i. a loop
- ii. the loop rule
- iii. a node
- iv. the node rule
- v. and a branch

(10 marks)

- **Loop:** All circuits can be divided into a number of closed loops
- **The loop rule** *The algebraic sum of the emf's in any closed loop is equal to the algebraic sum of the potential drops.* This rule states that the algebraic sum of the potential differences across all the circuit elements in a loop is zero.
- **Nodes:** A node is any point where 3 or more conductors come together
- **The node rule** *The total current flowing towards a node is equal to the total current flowing away from the node i.e. the algebraic sum of the currents flowing towards a node is zero.* The current is said to be positive if it flows into a node and negative if it flows away from a node.
- **Branch:** A path between two nodes which does not contain any other node.

b) Determine the value and confirm the direction of the current in BD using:

- i. Kirchoff's Law
- ii. Thevenin's theorem



Hint: When using Kirchoff's law, analyse loops ABC+battery, ABDA and BDCB

(30 marks)

i) By Kirchoff

Loop ABC+battery

$$\begin{aligned}2 - 10I_1 - 30(I_1 - I_3) &= 0 \\2 - 40I_1 + 30I_3 &= 0\end{aligned}\quad (a)$$

Loop ABDA

$$-10I_1 + 20I_2 - 40I_3 = 0 \quad (b)$$

Loop BDCB

$$\begin{aligned}-40I_3 - 15(I_2 + I_3) + 30(I_1 - I_3) &= 0 \\30I_1 - 15I_2 - 85I_3 &= 0\end{aligned}\quad (c)$$

Multiply (b) by 3 and (c) by 4 and add

$$\begin{aligned}-30I_1 + 60I_2 - 120I_3 &= 0 \\120I_1 - 60I_2 - 340I_3 &= 0 \\ \hline 90I_1 - 460I_3 &= 0\end{aligned}$$

$$I_1 = 5.111I_3$$

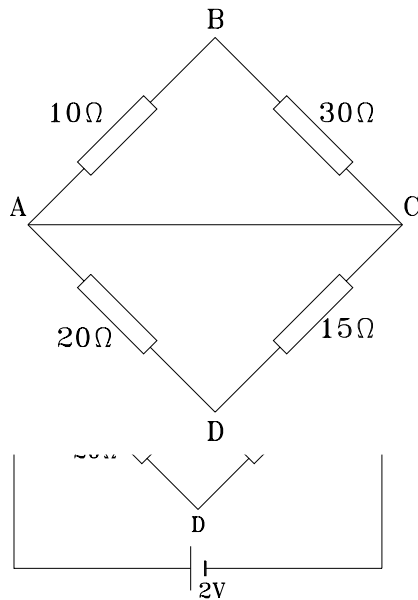
Substitute for I_1 in (a)

$$\begin{aligned}2 - 40(5.111I_3) + 30I_3 &= 0 \\ \mathbf{I_3 = 0.0114 A}\end{aligned}$$

Since the current is +ve then the direction shown on the circuit diagram is correct. i.e. from B to D

ii) By Thevenin

Since we require to find the current through the 40Ω resistor between BD the first step is to remove this resistor and find the potential difference between A - B and also between A - D so as to find the potential difference between B - D



The PD between A - B is

$$2 \times 10 / (10 + 30) = 0.5 \text{ V}$$

The PD between A - D is

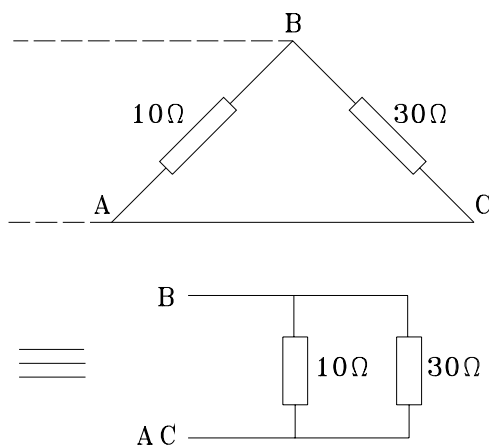
$$2 \times 20 / (20 + 15) = 1.143 \text{ V}$$

Therefore the PD between B - D is

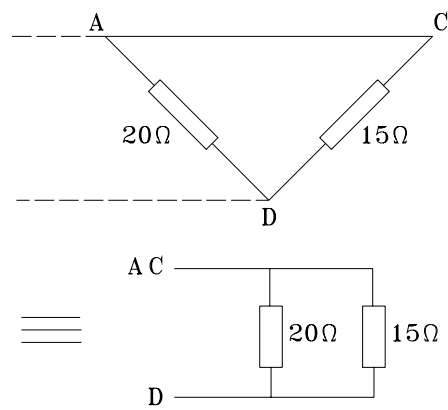
$$1.143 - 0.5 = 0.643 \text{ V}$$

The next step is to replace the battery with a resistance equal to the internal resistance of the battery - as this is negligible then junction A - C is short circuited.

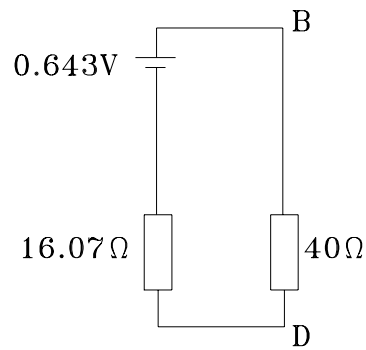
The equivalent resistance between AB is



$$= 10 \times 30 / (10 + 30) = 7.5 \Omega$$



$$= 20 \times 15 / (20 + 15) = 8.57\Omega$$



The total resistance between B and D is 16.07Ω

The current through BD = $0.643 / (16.07 + 40) = 0.0115\text{A}$

4 Answer **ALL** parts of the question:

a. Charles Coulomb observed that charged objects attracted or repelled one another and he therefore concluded that they must exert a force on each other. Describe his important observations regarding these forces and give the equation now known as Coulomb's law. Define all the terms. (10 marks)

- a) the force F depended on how far apart the charged objects were and falls inversely as the square of the distance r : $F \propto 1/r^2$
- b) the force depended on the amount of charge on each of the objects and is proportional to the product of the charges. $F \propto q_1q_2$

These statements can be summarised in Coulomb's Law, which shows the force F between two point charges q_1 and q_2 , separated by a distance r is:

$$F \propto q_1q_2/r^2$$

If we now introduce a proportionality constant k , then:

$$F = kq_1q_2/r^2$$

k is known as Coulomb's constant and is equal to:

$$k = 1/4\pi\epsilon_0$$

Therefore the force F in Newtons is:

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{r^2} \quad \text{Coulomb's Law}$$

ϵ_0 is the permittivity of free space = $8.854 \times 10^{-12} \text{ C}^2\text{N}^{-1}\text{m}^{-2}$

q_1 and q_2 are the amounts of charge in Coulombs

r is the distance between the charges in metres.

- b. A copper coin has a mass of 3.1g and contains an equal number of positive and negative charges. If each copper atom has a positive charge of $4.6 \times 10^{-18}\text{C}$ and the atomic weight of copper is 63.5 g/mol calculate:
- The total positive and negative charge contained within the coin.
 - The distance that the positive and negative charged have to separated to produce a force of 4.5 N.
 - The force of attraction if the positive and negative charges are 1 m apart.
 - Comment on the possibility of removing all the electrons from the coin.
- ϵ_0 the permittivity of free space = $8.854 \times 10^{-12} \text{ C}^2\text{N}^{-1}\text{m}^{-2}$

(10 marks)

- i. The total number of atoms contained by the coin = number of moles of copper
Avagadro's number.

$$= (3.1/63.5) \times 6.23 \times 10^{23} = 2.9 \times 10^{22} \text{ atoms}$$

Therefore the total positive charge = $2.9 \times 10^{22} \times 4.6 \times 10^{-18} = 133 \times 10^3 \text{ C}$ = total negative charge

- ii. Using Coulomb's law: $F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$

$$4.5 = \frac{1}{4\pi \times 8.854 \times 10^{-12}} \frac{133 \times 10^3 \times 133 \times 10^3}{r^2}$$

$$r^2 = 2 \times 10^{15} \times 133 \times 133$$

$$\mathbf{r = 5.9 \times 10^9 \text{ m}}$$

- iii. Again use Coulomb's law to give the force:

$$\mathbf{F = 1.6 \times 10^{20} \text{ N}}$$

- iv. The above two answers show that it would be impossible to remove all the electrons from the coin due to the huge forces of attraction.

- c. Two point charges of $+10^{-6}$ and -10^{-6} C are positioned horizontally and separated by 10 cm. Calculate the magnitude and direction of the electric field:

- i. at a point A mid-way between the charges
- ii. at a position B which is 5 cm vertically above the mid-point

ϵ_0 the permittivity of free space = $8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$

(20 marks)

- i. At the mid point between the charges the electric field due to the positive charge points away from the positive charge and that due to the negative charge points towards the negative charge. As the mid-point is being considered then the field, at this point, will be equal to twice that due to one charge i.e.

$$E = 2Q/4\pi\epsilon_0 r^2$$

$$E = 2 \times 10^{-6} / 4\pi \times 8.854 \times 10^{-12} \times 0.05^2$$

$$\mathbf{E = 7.2 \times 10^6 \text{ NC}^{-1}}$$

Direction will be towards the negative charge

- ii At position B, the field will again point away from the positive charge and towards the negative charge as shown in the diagram. Therefore the resultant field direction at B will be horizontal pointing away from the positive charge.

The total field at B $E_B = E_1 \cos 45^\circ + E_2 \cos 45^\circ = 2E_1 \cos 45^\circ$

$$E_1 = Q/4\pi\epsilon_0 r^2 \quad \text{where } r = \text{the distance between the charge and point B}$$

$$r^2 = 0.05^2 + 0.05^2 = 2 \times 0.05^2$$

$$E_B = (2 \times 10^{-6} / 4\pi \times 8.854 \times 10^{-12} \times 2 \times 0.05^2) \cos 45^\circ$$

$$\mathbf{E_B = 2.5 \times 10^6 \text{ NC}^{-1}}$$

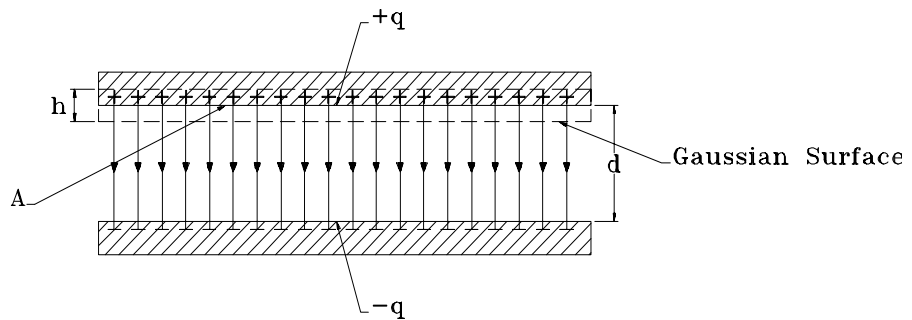
Direction is horizontal from B away from the positive charge

5. Answer **ALL** parts of the question:

a) Show, using the parallel plate and cylindrical capacitors as examples, that the capacitance only depends on the geometry of the capacitor.

(20 marks)

Parallel plate capacitor



This capacitor is formed from two plates with area A separated by a distance d . When the plates are connected to the terminals of a battery then a positive charge $+q$ will appear on one plate and a negative charge $-q$ will appear on the other. If we make the assumption that d is small compared to the area of the plates then it can be said that a uniform field E exists between the plates, with the lines of force being parallel and evenly spaced.

Using Gauss's law, it is possible to calculate the capacitance of the device. The Gaussian surface is of height h with an area equal to that of the plates (shown on the diagram). The only part of the Gaussian surface we are concerned with is that which lies between the plates as the surface which lies within the plate has zero field and the two side surfaces are at 90° to the field. For the surface between the plate E is constant and therefore the flux is equal to EA

Therefore: $\epsilon_0\phi = \epsilon_0EA = q$

The work needed to take a test charge q_0 from one plate to the other is equal to q_0V or the force (q_0E) multiplied by the distance (d). These expressions must be equal i.e.

$$q_0V = q_0Ed$$

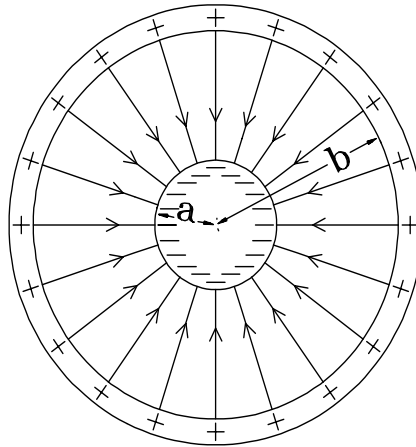
$$V = Ed$$

Substituting into the standard capacitor equation $C=q/V$ we get:

$$C = \frac{q}{V} = \frac{\epsilon_0EA}{Ed} = \frac{\epsilon_0A}{d} \qquad \text{Parallel plate capacitor}$$

A cylindrical capacitor

A cylinder capacitor is made up of two coaxial cylinders of radius a and b and of length l .



We assume that the length $l \gg b$ and construct a Gaussian surface which is a cylinder of radius r and length l , where $b > r > a$

Using Gauss's law:

$$\frac{q}{\epsilon_0} = \oint E \cdot dA$$

$$q = E\epsilon_0 2\pi r l$$

$$E = \frac{q}{2\pi r l \epsilon_0}$$

The potential difference between the plates is:

$$V = \int_a^b E dr = \int_a^b \frac{q}{2\pi \epsilon_0 l} \frac{dr}{r} = \frac{q}{2\pi \epsilon_0 l} \ln \frac{b}{a}$$

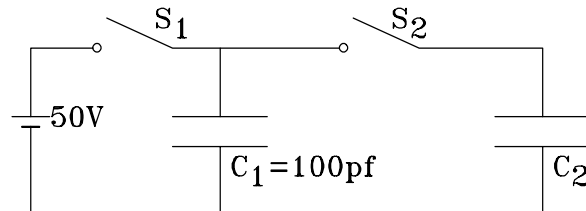
The capacitance is given by:

$$C = \frac{q}{V} = \frac{2\pi \epsilon_0 l}{\ln(b/a)}$$

The capacitance only depends on the geometry of the capacitor. If we look back at the equation for a parallel plate capacitor it can be seen that the capacitance is also only a function of the dimensions of the device.

- b) A 100 pF capacitor C_1 is charged to a potential difference of 50 volts by closing switch S_1 . S_1 is now opened and capacitor C_1 is connected to an uncharged capacitor C_2 by closing switch S_2 . If the measured potential difference drops to 35 volts, calculate:

- i. The capacitance of C_2
- ii. The stored energy before and after the switch S_2 is made. Explain why the energies calculated are not equal.



(20 marks)

- a) When switch S_2 is closed the charge q_0 on C_1 is shared between both the capacitors. Thus:
- $$q_0 = q_1 + q_2$$

This can be written:

$$\begin{aligned} C_1 V_0 &= C_1 V + C_2 V \\ C_2 &= C_1 (V_0 - V)/V \\ C_2 &= 100 (50 - 35)/35 \\ C_2 &= 42.86 \text{ pf} \end{aligned}$$

- b) The initial stored energy is:

$$\begin{aligned} U_0 &= \frac{1}{2} C_1 V_0^2 \\ U_0 &= \frac{1}{2} \times 100 \times 10^{-12} \times 50^2 \\ U_0 &= \mathbf{0.125 \times 10^{-6} \text{ J}} \end{aligned}$$

The final stored energy is:

$$\begin{aligned} U &= \frac{1}{2} C_1 V^2 + \frac{1}{2} C_2 V^2 \\ U &= (\frac{1}{2} \times 100 \times 10^{-12} \times 35^2) + (\frac{1}{2} \times 42.86 \times 10^{-12} \times 35^2) \\ U &= \mathbf{0.088 \times 10^{-6} \text{ J}} \end{aligned}$$

The final energy U is less than the original energy U_0 due to thermal energy loss in the connecting wires as the charge moves through them.