3D and 4D structures in paraxial and nonparaxial nonlinear optics

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Introduction

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In the last decade theoretical nonlinear optics has developed a new branch through considering the formation of spatial patterns across the transverse profile of propagating laser beams [1-4]. The challenge of understanding these patterns and their evolution has meant that full numerical simulation of the governing nonlinear partial differential equations has, by necessity, formed the backbone of most investigations. However, reduction of the spatial complexity to competition between dominant nonlinear structures provides a means to understanding and quantifying the transient and asymptotic dynamics.

The work undertaken by this Investigation Group at the Study Centre, and proposed for future work, is an appropriate balance of computational and analytic studies. During the Study Centre, an application for resources on the Edinburgh Parallel Computing Centre Connection Machine was prepared, and a pump-priming allocation awarded, to undertake the initial investigations. Since the possible scope for the examination of 3D and 4D nonlinear structures is very large, it was necessary to keep considerations as simple, and as general, as possible. In particular nonlinear systems, robust structures, such as solitons, are simple exact solutions while in a great number of related systems their characteristics are manifest and can dominate much of the nonlinear evolution. The framework for this work is based on the 3D and 4D nonlinear Schrödinger equation (NLS). It is essential to understand how the fundamental structures of such a universal equation evolve, interact and stabilise before attempting to interpret their behaviour in systems involving, for example, higher order effects.

The paraxial 4D NLS, which in optics governs the evolution of the electric field envelope, u, of a beam propagating in the $+z$ direction through a Kerr medium, can be written as:

$$i\frac{\partial u}{\partial z} + \alpha\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \beta\frac{\partial^2 u}{\partial t^2} + \eta|u|^2 u \quad = \quad 0, \tag{1}$$

where α defines the scale of the transverse plane $(x - y)$, β is the dispersion parameter, t is time, and η parametrises the nonlinearity of the medium.

While diffraction fixes α to be positive, the following cases need to be considered:

Anomalous dispersion	$\Rightarrow \beta > 0,$
normal dispersion	$\Rightarrow \beta < 0,$
self-focusing media	$\Rightarrow \eta > 0,$
self-defocusing media	$\Rightarrow \eta < 0.$

Sufficient groundwork has been covered, regarding the 3D NLS, to extract meaningful information from computational explorations of the full 3D-space + 1D-time problem with the minimum of discretisation. A systematic exploration of the possible combinations of the above coefficients has revealed a number of cases which have not previously received attention. Solutions and instabilities for these novel configurations were discussed by the Investigation Group.

In parts A and B of this report we discuss each case for the paraxial NLS, giving background where work has previously been performed and proposing investigations where stable solutions are not known. In part C considerations of the nonparaxial NLS are made and an overview of parts A-C is presented. In part D analysis that was undertaken, and further application of these considerations, is outlined. Finally, in part E, considerations are generalised to include optical elements and cavities.

A) CW beams and transverse effects ($\beta = 0$).

i) <u>CW beams in self-focussing media ($\eta = 1$).</u>

In a period which now spans three decades, many attempts have been made to understand the physical processes involved in self-focusing [5-16]. A formulation of 2 dimensional self-trapped beams was presented in 1964 by Chiao et al [6]. It was proposed that a high intensity beam can modify the host medium and create a nonlinear waveguide through which the beam can propagate without experiencing diffractive spreading. It was shown in [6, 7], assuming the paraxial approximation and allowing for three space dimensions, that this self-trapping phenomenon was unstable, leading to collapse of the solution to a singularity. This collapse process can be halted by a number of other processes such as nonlinear absorption, saturation of the nonlinearity and material damage. The existence, and relative importance, of such mechanisms depends on the details of the nonlinear medium considered. It was not the intention of the Group to dwell on such features. Instead, in part C, results and proposed work, pertaining to a mechanism which is fundamental to the light itself, are discussed.

ii) <u>CW beams in self-defocussing media ($\eta = -1$).</u>

In the previous case, $\eta = 1$, the Kerr effect can lead to a runaway process which results in a singularly high beam intensity. For a self-defocusing medium such singular behaviour in the intensity does not occur. However, recent research has shown that, in this case, point singularities in the *phase* of the beam (optical vortices) can act as robust nonlinear structures which dominate and characterise the evolution of the system. Researchers [17-19] have examined whether any 3D manifestation of the 2D dark soliton solution [20] is possible. However, it has been found that dark soliton stripe (DSS) solutions, which are 1D phase singularities, break-up into rows of optical vortices when subject to perturbations and that these new nonlinear waves are more robust [21]. Other interesting results, concerning the 3D NLS, were the discovery of vortex gas solutions, stabilisation into regular crystals of *rotating* vortices and the motion of nonlinear vortices on diffracting beams [22].

B) Pulsed beams – the 4D problem ($\beta \neq 0$).

i) $\underline{\eta = 1, \beta = 1.}$

The full 4D problem has recently been considered for a particular set of parameters which describe the nonlinear interaction of light and matter (diffraction, *anomalous* dispersion and self-focusing nonlinearity on an equal phenomenological footings) [23]. In that work a space-time "light bullet" was proposed. The formal equivalence between temporal dispersion and spatial diffraction permits a solution which evolves symmetrically in space and time. In comparison with its 3D counterpart, this 4D solution collapses more rapidly to an *unphysical* singularity.

ii) $\underline{\eta = 1, \beta = -1.}$

It was reported at the Study Centre that, for *normal* dispersion, the 4D wavepacket can avoid catastrophic collapse through splitting into temporally resolved filaments whose individual power is insufficient to support the nonlinear mechanism of collapse [24].

iii) $\underline{\eta = -1, \beta = 1.}$

The generalisation of the results outlined in part A ii) to include temporal effects is expected to lead to completely new space-time structures. Perhaps the most closely related work to this section of the Report is that which dealt with the coupling of bright and dark solitons ("symbions") in the "1+2" NLS including dispersion [25]. However, in addition to the assumption of no transverse (lateral) instabilities, in that work, only one sign of group velocity dispersion was considered. The determination of the stability of the coupling between DSS's and dark temporal solitons and the formation, evolution, interactions and emsemble configurations of fully 4D vortices were discussed at the Centre and proposed as future work. The details of different initial value problems involving, for example noise perturbed CW backgrounds and topologically feasible soliton-vortex and soliton-soliton couplings were also discussed and outlined in presentation during the Reports period.

iv) $\eta = -1, \beta = -1.$

In this situation, the possibilies are analogous to those of part B iii) except that where a single 1D *dark* soliton is considered, in this case, it is replaced by consideration of a 1D *bright* soliton profile. Thus, aspects of symbions consisting of a DSS pattern (or vortex) coupled to a bright temporal soliton were made. For this type of dispersion, further effects may arise due to a susceptibility of temporally smooth regions of the field to a modulational instability.

C) Nonparaxial nonlinear optics.

The generalisation of equation (1) to include nonparaxial effects leads to the consideration of the following evolution equation:

$$\frac{\Theta^2}{4}\frac{\partial^2 u}{\partial z^2} + i\frac{\partial u}{\partial z} + \alpha\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \beta\frac{\partial^2 u}{\partial t^2} + \eta|u|^2 u \quad = \quad 0 \qquad (2)$$

where Θ is a parameter equivalent to the divergence angle for a Gaussian beam in linear propagation and naturally reflects the role of the transverse size of the beam and the tendency for light to travel off-axis in nonlinear propagation.

The central problem with the light bullet solution of [23] is the adoption of the paraxial approximation in its derivation. The paraxial wave description becomes invalid as the bullet focuses down to dimensions of the order of an optical wavelength. Recently, detailed considerations of the collapse process in the framework of nonparaxial theory have been made. As reported at the Study Centre, it has been discovered that a new class of symmetric solution exists in this regime [26]. In numerical simulations, an algorithm which allows self-focusing to be noncatastrophic is used to solve the nonlinear Helmholtz equation [27]. Analyses and simulations have shown that a *hierarchy* of nonparaxial modes can stabilise to *finite* size.

Further generalisation of these new results has been proposed within the framework of this investigation group. Simulation and further analysis of families of *nonparaxial light bullet* solutions and their interactions and applications is to be undertaken.

In very recent work, the numerical solution of the full vector Maxwell equations with a Kerr nonlinearity has been sought [28]. Preliminary investigations have attempted to realise solutions which are self-supporting and simulations have indicated the existence of light packets which can propagate in space with little change in their shape or spectral characteristics. The complexities of the full vector Maxwell model of nonlinear propagation can be overwhelming, both from a numerical and an analytical viewpoint. Consequently, results using the full fundamental set of equations can be hard to interpret. The two approaches, vector Maxwell and envelope equations, are complementary and a deeper understanding of both types of descriptions will be attained once results from the two approaches can be compared and contrasted.

Intermediate summary.

In addition to a computational exploration of the stability of new 4D structures, it is proposed that trial solutions, which can capture their evolutionary characteristics, will be sought and used in Lagrangian analysis to derive ordinary differential equations for the evolution of their characteristic parameters. To conclude parts A-C, we present the following table in which a brief outline of known, and expected, 3D and 4D structures is given. The cases for which results are already published, or for which participants of the investigation group have unpublished results, are highlighted. As can be seen in this table, there are still quite a number of cases to be considered.

β	η	Solutions	Paraxial	Nonparaxial
0	+1	Optical collapse and modes	•	•
0	-1	DSS break-up and vortices	•	
+1	+1	The optical bullet	•	
+1	-1	Vortices and symbions		
-1	+1	Arrest of collapse	•	
-1	-1	Vortices and symbions		

D) Related physical systems – periodic media and gap solitons.

The NLS is known to describe a very wide class of phenomena. In this part we report on work that was performed, during investigation periods, on a related physical system, for which the previous considerations of 3D and 4D structures may also apply. Analysis was undertaken to generalise the existing theory of gap solitons in periodic media to include, for the first time, transverse effects.

From Floquet-Bloch theory, the amplitude of a wave propagating in a periodic medium may experience exponential decay with distance if its frequency lies within one of the forbidden frequency bands (band gaps) which occur around the Bragg frequencies [29]. In a *nonlinear* periodic structure the effect of the incident light can be to partially close this gap. Thus, if the frequency of the light is sufficiently close to a band edge, then, in this case, transmission can be greatly enhanced. Further to this, it has been discovered that a transmission resonance can also occur for a frequency which is still *within* the band gap. Such a resonance has been linked to the formation of a "gap soliton", which may be a static envelope along the propagation axis [30]. In homogeneous media, solitons can arise due to the interplay of nonlinearity and host dispersion. In periodic media, the resulting band structure can provide this dispersion and underlying material dispersion is not required.

In [31] it was shown that stationary gap solitons are a particular case and that, more generally, propagating soliton solutions may also be found. In that work, the soliton envelope, $a(t, z')$, was shown to obey a "1+1" NLS.

$$i\frac{\partial a}{\partial t} + \frac{1}{2}\omega''_m \frac{\partial^2 a}{\partial z'^2} + \alpha_m |a|^2 a = 0 \tag{3}$$

where ω''_m and α_m are defined in terms of the particular Bloch function, ϕ_m, which is modulated. At the Study Centre, a more general formulism was derived. The starting point for this analysis was the nonlinear wave equation, for the electric field, E, which includes diffraction:

$$\frac{\partial^2 E}{\partial t^2} - c^2\left(\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2}\right) = -4\pi\chi(z)\frac{\partial^2 P}{\partial t^2} \tag{4}$$

where $\chi(z)$ is the periodic susceptibility and P is the polarisation wave. Following the multiple scales analysis and procedure outlined in [31], a more general form of equation (3) was derived. Within this context a future study of the transverse stability of gap solitons and possible higher dimensional structures, such as soliton-vortex symbions, was proposed.

E) Cavity effects and optical memory devices.

Investigation of the propagation, stability and interaction characteristics of fundamental nonlinear 3D and 4D light structures will determine their potential as elementary units (self-supporting "optical bits") in information technologies. It is also important to extend the above considerations, which deal exclusively with propagation, to include optical elements, such as mirrors, and thus the steering, manipulation and storage of such optical information. As a first step, we propose to study a nonparaxial single feedback mirror system and investigate its potential for the storage of optical information. Secondly, we propose a novel coupled-cavity configuration as an optical memory device and the generalisation of additive pulse modelocking to additive *beam* modelocking.

i) Nonparaxial single feedback mirror configuration.

The single feedback mirror configuration has been shown to give rise to spontaneous hexagonal optical patterns in the transverse plane [32-38]. The characteristic size of such patterns scales in proportion to the square root of the distance between the nonlinear medium and the feedback mirror. To exploit the possible information capacity associated with these transverse patterns, one would like to make this distance as small as possible. This direction of investigation leads to the consideration of a nonparaxial generalisation of published results and the analysis and simulation of a system with 4 dimensions. The 4D equations describe the propagating light and are coupled to evolution equations for the material variables [39, 40].

Separately, there is an interest in 2D soliton-like isolated states which correspond to single points of the hexagon pattern [41, 42]. Where such isolated states exist, they are bistable and so can be considered for use as pixels. In the nonparaxial regime these pixels would be of wavelength dimensions. It has been shown that when the nonlinear medium is a thin layer of two-level quantum systems (its longitudinal width being less than an optical wavelength) then the single feedback mirror configuration exhibits (plane-wave) optical bistability [43]. By analogy, it seems extremely likely that localised states will also exist. One can envisage implementation of such a system using a quantum-well layer, spacer and a dielectric stack mirror. Such a configuration offers the prospect of 1 bit per cubic wavelength storage capacity.

ii) Additive beam modelocking.

Additive pulse modelocking (APM) is a general method for short pulse production which is applicable to a wide range of lasers. The pulse shortening mechanism is similar to that of a fast saturable absorber and is based on the coherent addition of self-phase modulated pulses [44, 45]. The required self-phase modulation typically occurs due to the Kerr effect and, in the first demonstration of APM, was generated by coupling the main laser cavity to an external section which contained a length of optical fibre [46]. It was originally believed that this all-optical feedback scheme required solitonic behaviour, and thus a net negative group velocity dispersion, in the external section. However, modelling showed that this is not necessarily the case [47]. A further key observation, in terms of the proposed investigations here, is that this configuration can operate in *both* active and passive modelocking regimes.

The proposal of this group is to investigate the progressive generalisation of the principles underlying APM to higher dimensions and the inclusion of diffraction. Firstly, the equivalent scheme (of the same dimension) where dispersion is replaced directly with diffraction will be studied to demonstration what we have called *additive beam modelocking.* The context of these generalisations will not be to attempt to modelock the (global) transverse empty-cavity modes of the main cavity but to enable an arbitrary array of (local) nonlinear spatial filaments to be sustained within the main cavity. We are thus proposing an alternative architecture for optical memory devices.

It is envisaged that such coupled-cavities could be constructed as planar sandwich structures. Two distinct types of possible device were proposed, both types supplying a constant “read out” of the information stored. The first was a passive device, requiring constant CW illumination, for which the information arrays are coupled in along with the background illumination using a beam splitter. The second was active, without external illumination, and for which the optical information is written directly into the main cavity. Generally, attention will be directed towards the simplest configurations before more involved analysis and simulation, including, for example, pulsed beams and nonparaxial effects, are considered

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