FY Electricity

TUTORIAL 1
SOLUTIONS

1. Strict definition : 1 coulomb is the charge that passes through a cross-section of a conductor per second when a current of 1A (one ampère) flows.

Alternatively, Electron charge, $1e = 1.6 \times 10^{-19} C$

$$\therefore \frac{1}{1.6 \times 10^{-19}} e = 1 C$$

$$\text{i.e. } 6.25 \times 10^{18} e = 1 C$$

i.e. 1 C is equivalent to the charge of 6.25×10^{18} electrons or protons.

2. E is the force exerted by the electrostatic field on a unit (positive) charge at that point. Equivalently, it is also the 'force per unit charge' at that point.

3. 

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2} .$$

If $Q_1 = Q$ and Q_2 is a unit (positive) test charge, i.e. $Q_2 = 1C$,

then $E = \frac{Q.1}{4\pi\varepsilon_0 r^2} = \frac{Q}{4\pi\varepsilon_0 R^2}$, where we have specified that $r = R$.

Equivalently, $E = \frac{F}{Q_2} = \frac{Q_1}{4\pi\varepsilon_0 R^2} = \frac{Q}{4\pi\varepsilon_0 R^2}$,

i.e. the force per unit charge of Q_2 due to the presence of $Q_1 = Q$.

4.

$Q_1 = 2\times10^{-2}\,\mu C$

$Q_2 = -5\times10^{-2}\,\mu C$

$r = 25\,cm$

$F = k\dfrac{Q_1Q_2}{r^2}$,

where

$k = \frac{1}{4\pi\varepsilon_0} = 9\times10^{9}\,mF^{-1}$.

But, we need all quantities in SI units.

$Q_1 = 2\times10^{-2}\,\mu C = 2\times10^{-2}\times10^{-6}\,C = 2\times10^{-8}\,C$

$Q_2 = 5\times10^{-2}\,\mu C = 5\times10^{-2}\times10^{-6}\,C = 5\times10^{-8}\,C$

$r = 25cm = 25\times10^{-2}\,m$

Note that Coulomb's Law only gives the magnitude (not direction) of the force and thus we consider only the magnitude (not the sign) of Q_1, Q_2

$$\rightarrow F = \frac{9\times10^{9}\times2\times10^{-8}\times5\times10^{-8}}{(25\times10^{-2})^2} = \frac{90\times10^{-7}}{25^2\times10^{-4}} = \frac{90}{25^2}\times10^{-3} \approx 1{\cdot}4\times10^{-4}\,N.$$

: UNLIKE CHARGES ATTRACT.

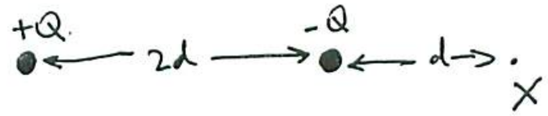
5. $F = \dfrac{Q_1Q_2}{4\pi\varepsilon r^2}$: $F \propto \frac{1}{r^2}$ (not $F \propto r^2$) → (i) FALSE

$F \propto Q_1Q_2$ → (ii) TRUE

$F \propto \frac{1}{\varepsilon}$ (not $F \propto \varepsilon$) → (iii) FALSE

∴ Only (ii) is true, and the answer is E.

6.



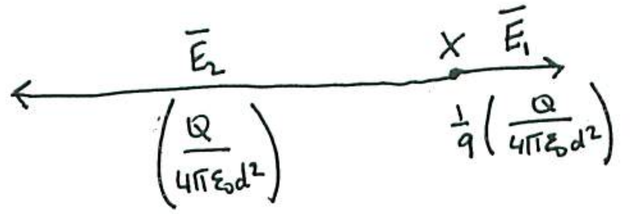
Consider a (positive) test charge at X.

E at X due to +Q : $E_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{(3d)^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{9d^2}$

(repulsive : to the right)

E at X due to -Q : $E_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{d^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{d^2}$

(attractive : to the left)



$$E\,(\text{total}) = \frac{Q}{4\pi\varepsilon_0 d^2} - \frac{1}{9}\left(\frac{Q}{4\pi\varepsilon_0 d^2}\right) = \left(1-\frac{1}{9}\right)\frac{Q}{4\pi\varepsilon_0 d^2} = \overset{2}{\frac{8}{9}}\cdot\frac{Q}{4\pi\varepsilon_0 d^2}$$

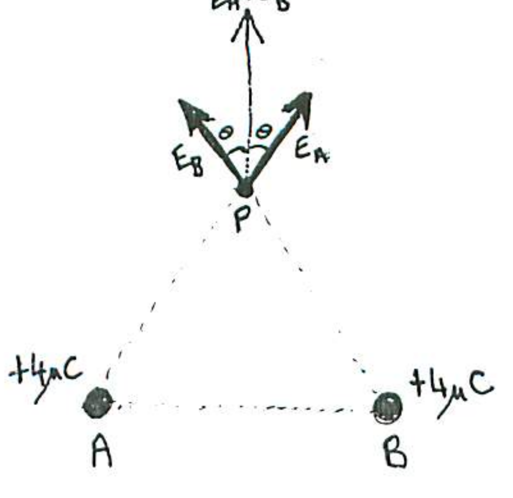
$$= \frac{2}{9}\cdot\frac{Q}{4\pi\varepsilon_0 d^2},$$

in magnitude

(direction is to left)

∴ Answer D is correct.

7. (i) both charges positive.



Due to +4μC at A : $Q = 4\mu C = 4\times10^{-6}\,C$
$r = 1m$

Magnitude, $E_A = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

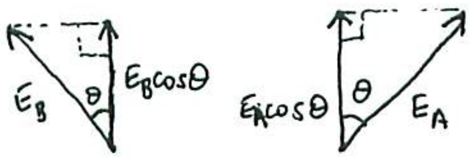
$$E_A = \frac{9\times10^9\times4\times10^{-6}}{1^2}$$

i.e. $E_A = 3{\cdot}6\times10^4\ NC^{-1}$. (or Vm^{-1})

Magnitude, $E_B = 3{\cdot}6\times10^4\,NC^{-1}$, as well.

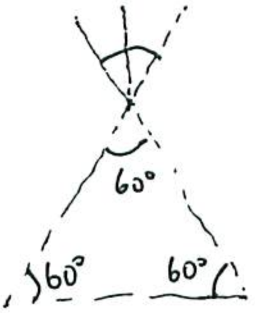
Total E at P results from the vector addition of E_A and E_B.

In general, we add the horizontal components and vector components, separately.



Vertical components add together
(horizontal components cancel out, here)

What is θ?



$$\theta = \frac{60^\circ}{2} = 30^\circ$$

∴ Resultant points upwards,

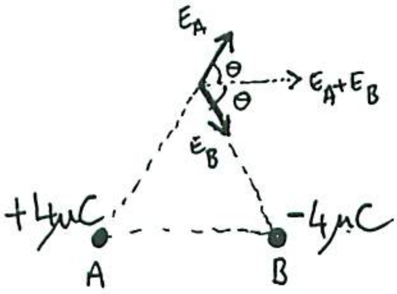
$$E_{TOTAL} = E_A\cos\theta + E_B\cos\theta$$

$$= 2 \times 3.6\times10^4 \times \cos\theta$$

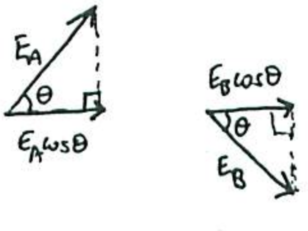
$$= 2 \times 3.6\times10^4 \times \cos 30^\circ$$

i.e. $E_{TOTAL} = 6.2 \times 10^5\ NC^{-1}$.

(ii) one charge positive, other negative

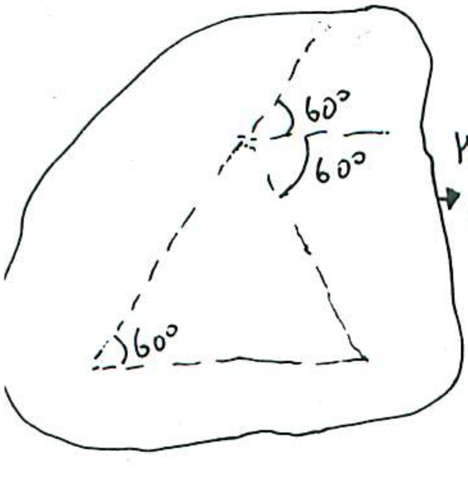


The magnitudes of E_A and E_B are the same as in part (i).



Horizontal components add,
vertical components cancel out.

∴ Resultant points to the right.



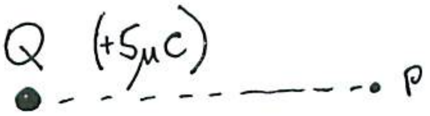
Here, θ=60°.

$$E_{TOTAL} = E_A\cos\theta + E_B\cos\theta \quad (\text{now, } \theta = 60^\circ)$$

$$= 2 \times 3.6\times10^4 \times \cos 60^\circ$$

i.e. $E_{TOTAL} = 3.6 \times 10^4\ NC^{-1}$.

8.



$r = 80\text{mm}$
$= 80 \times 10^{-3}\text{m}$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

$$= \frac{9\times10^9 \times 5\times10^{-6}}{(8\times10^{-2})^2}$$

$$= \frac{45\times10^3}{64\times10^{-4}}$$

$$= 7\times10^6\ \text{NC}^{-1}.$$

9. Corona Discharge.
It occurs in air when $E > 3\,\text{MVm}^{-1}$ (i.e $3\times10^6\ \text{Vm}^{-1}$).
For example, Van der Graaff generator.

10. A Faraday cage can be used to electrostatically shield sensitive equipment or humans.
Inside the cage, $E = 0$.

11. $E = \dfrac{\sigma}{\varepsilon_0}$,

σ = surface charge density $= \left(\dfrac{\text{charge}}{\text{area}}\right)$
ε_0 = permittivity of free space

<u>Note</u>: This same expression was derived in the lecture notes for close to : a charged, conducting sphere

AND a charged, conducting plate.

→ General result, for 'near a charged, conducting surface'.

12. Uniform $E = 0.4\,MVm^{-1} = 0.4 \times 10^{6}\,Vm^{-1}$

A proton has charge $Q = 1.6 \times 10^{-19}\,C$

(a) $E = \frac{F}{Q}$, i.e. Force, $F = EQ$

$= 0.4 \times 10^{6} \times 1.6 \times 10^{-19}$

i.e. $F = 6.4 \times 10^{-14}\,N$.

(b) Acceleration, $F = ma$ gives $a = \frac{F}{m}$

where proton mass $m = 1.67 \times 10^{-27}\,kg$

$\therefore\ a = \frac{6.4 \times 10^{-14}}{1.67 \times 10^{-27}} = 3.8 \times 10^{13}\,ms^{-2}$.

(c) Assume that each proton starts off at rest

→ distance travelled $s = \frac{1}{2}at^{2}$, where t = time elapsed.

We know the distance and acceleration, e.g. $s = 5m$,

→ solve for the time taken : $2s = at^{2}$

i.e. $\frac{2s}{a} = t^{2}$

$\therefore\ t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 5}{3.8 \times 10^{13}}} = 5.1 \times 10^{-7}\,s$.