FY Electricity

TUTORIAL 2

SOLUTIONS

1. V is the work done required to bring a 1C charge to this point from ∞. Equivalently, it is the electrostatic potential energy per unit charge at this point.

   E is the electrostatic force that acts on a 1C charge at this point. Equivalently, it is the electrostatic force per unit charge at this point.

2. (i) V is a scalar (given by just a number) since work done and potential energy are scalars. The SI unit is volts (V). This is equivalent to Joules/coulombs, recalling that it is energy per unit charge.

   (ii) E is a vector (it has both a number associated, which gives the magnitude/size of the force per unit charge, <u>and</u> a direction, which gives the direction of this force). A unit could be $\frac{N}{C}$, recalling that $E = \frac{F}{Q} \rightarrow \frac{\text{newtons}}{\text{coulombs}}$, or $\frac{V}{m}$, since $E = -\frac{dV}{dx} \rightarrow \frac{\text{volts}}{\text{metres}}$.

3. Electric potential due to a point charge is $V = \frac{Q}{4\pi\varepsilon_0 r}$,

   where Q = charge (in C) causing the electric field and associated potential,

   r = distance (in m) from Q

   ε_0 = permittivity of free space (close to that for air)

$$V = \frac{Q}{4\pi\varepsilon_0 r} \approx 9\times10^9 \times \frac{Q}{r} = 9\times10^9 \times \frac{5\times10^{-6}}{80\times10^{-3}} = \frac{45}{80}\times10^{9-6+3}$$

$$= \frac{45}{80}\times10^6 \simeq 0.56\times10^6 = 5.6\times10^5\ \text{V}.$$

4. The potential at X will be the sum of the potentials due to +Q and -Q. In each case, we are dealing with V due to a point charge.

<u>Due to +Q</u> : $r = 2d + d = 3d$, charge $= +Q$

$$\therefore V_{+Q} = \frac{+Q}{4\pi\varepsilon_0(3d)} = +\frac{Q}{12\pi\varepsilon_0 d}.$$

<u>Due to -Q</u> : $r = d$, charge $= -Q$

$$\therefore V_{-Q} = \frac{-Q}{4\pi\varepsilon_0 d}.$$

The total potential at X $= V_{+Q} + V_{-Q}$

$$= \frac{Q}{12\pi\varepsilon_0 d} - \frac{Q}{4\pi\varepsilon_0 d}$$

$$= \frac{Q}{4\pi\varepsilon_0 d}\cdot\left(\frac{1}{3} - 1\right)$$

$$= \frac{Q}{4\pi\varepsilon_0 d}\cdot\left(\frac{1}{3} - \frac{3}{3}\right)$$

$$= \frac{Q}{4\pi\varepsilon_0 d}\cdot\left(\frac{-2}{3}\right)$$

$$= \frac{-2Q}{12\pi\varepsilon_0 d}$$

$$= -\frac{Q}{6\pi\varepsilon_0 d}.$$

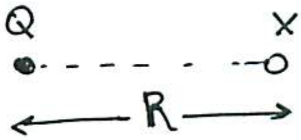
$\therefore$ The answer is B.

5. Coulomb's Law : $F = \dfrac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$

Note: We could have ε_0 or ε here.

Consider the electric field due to $Q = Q_1$ and the force per unit charge that results a distance R away from Q

$$E = \frac{F}{Q_2} = \frac{Q}{4\pi\varepsilon_0 R^2}$$



The potential at X is the sum of all the 'work dones' in taking a unit charge from ∞ to R (in small steps)

$$\rightarrow \quad V = -\int_{\infty}^{R} E\,dr = -\int_{\infty}^{R} \frac{Q}{4\pi\varepsilon_0 r^2}\cdot dr = -\frac{Q}{4\pi\varepsilon_0}\int_{\infty}^{R}\frac{1}{r^2}\,dr$$

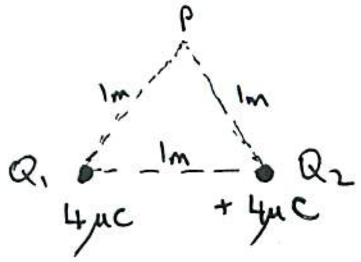
$$= -\frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{\infty}^{R} = -\frac{Q}{4\pi\varepsilon_0}\left\{-\frac{1}{R} - \left(-\frac{1}{\infty}\right)\right\}$$

tends to zero

$$\therefore \quad V = +\frac{Q}{4\pi\varepsilon_0 R}.$$

6. In each case, the contributions to the electric potential add up.

(i)



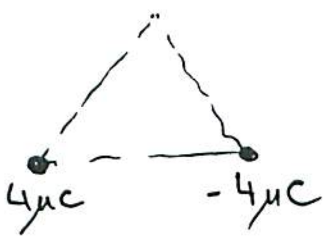
$$V = \frac{Q_1}{4\pi\varepsilon_0 r} + \frac{Q_2}{4\pi\varepsilon_0 r} = \frac{4\times10^{-6}}{4\pi\varepsilon_0 \cdot 1} + \frac{4\times10^{-6}}{4\pi\varepsilon_0 \cdot 1}$$

$$= \frac{8\times10^{-6}}{4\pi\varepsilon_0} \approx 9\times10^{9}\times 8\times10^{-6}$$

$$= 72\times10^{9-6} = 72\times10^{3}$$

$$= 7.2\times10^{4}\,V.$$

(ii)

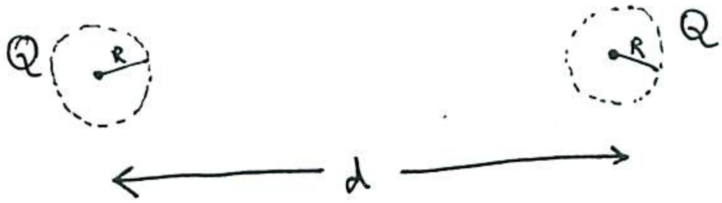


Now, $V = 0$ since ...

$$V = \frac{4\times10^{-6}}{4\pi\varepsilon_0 \cdot 1} + \frac{(-4\times10^{-6})}{4\pi\varepsilon_0 \cdot 1}$$

$$= \frac{4\times10^{-6}}{4\pi\varepsilon_0} - \frac{4\times10^{-6}}{4\pi\varepsilon_0} = 0.$$

7. The charged spheres appear like point charges (with total charge at the centre of each sphere) in terms of the resulting force, electric field and electric potential <u>outside</u> each sphere.



The force is given by Coulomb's Law

i.e. $F = \frac{Q.Q}{4\pi\varepsilon_0 d^2} = \frac{Q^2}{4\pi\varepsilon_0 d^2}$ (spheres are identical and will thus carry the same excess charge Q)

We are given the potential of each sphere, that results from the charge Q that lies on the surface (radius R) of each sphere :

$V = \frac{Q}{4\pi\varepsilon_0 R}$. This tells us the charge on each sphere which, in turn, allows us to work out the force.

$$\therefore \; Q = 4\pi\varepsilon_0 R . V$$

$$\approx \frac{1}{9\times10^9} \times (5\times10^{-3}) \times 100 \;, \quad \text{here} \quad \frac{1}{4\pi\varepsilon_0} \approx 9\times10^9$$

$$\rightarrow 4\pi\varepsilon_0 \approx \frac{1}{9\times10^9}$$

$$= \frac{5}{9} \times 10^{-9-3+2}$$

and $R = \frac{\text{diameter}}{2} = \frac{10\text{mm}}{2}$

$$= \frac{5}{9} \times 10^{-10}$$

i.e. $Q \approx 0.56\times10^{-10}\,\text{C}$

$$\therefore \; F = \frac{Q^2}{4\pi\varepsilon_0 d^2} \approx \frac{9\times10^9 \times (0.56\times10^{-10})^2}{(100\times10^{-3})^2} = \frac{9\times(0.56)^2\times10^9\times10^{-20}}{10^{-2}}$$

$$= 9\times(0.56)^2\times10^{-9} \approx 2.8\times10^{-9}\,\underline{\text{N}}.$$

8. The potential difference V_{YX} is equal to the work done to move a unit charge (+1C) from X to Y.
(This is also equal to the electrostatic potential energy gained by the unit charge.)

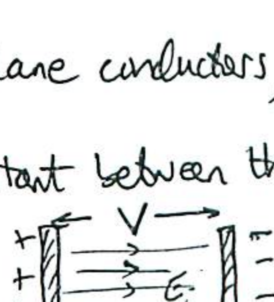
9. The potential difference (25 keV) is the work done / energy gained for a unit charge.
For a charge Q, work done/energy gained is $W = QV$
For an electron, $Q = e$ and $W = eV$ (here V = pot. diff. and not volts!!)

Energy gained (in eV units) $= 25\,\text{keV} = 25\times10^{3}\,\text{eV}$

Energy gained (in J) $= 25\times10^{3}\times1.6\times10^{-19}\,\text{C.V}$
$= 25\times10^{3}\times1.6\times10^{-19}\,\text{J}$
$= 25\times1.6\times10^{-16}\,\text{J}$
$= 40\times10^{-16}\,\text{J}$
$= 4\times10^{-15}\,\text{J}.$

10. $E = -\frac{dV}{dx}$, where $\frac{dV}{dx}$ is the potential gradient.

This is generally true. For the case of two parallel plane conductors, the E-field and potential gradient are assumed constant between the plates. Then, $\frac{dV}{dx} = \frac{\text{potential difference}}{\text{plate separation}} = \frac{V}{d}$.



10. (continued) $E = \frac{V}{d} = \frac{400}{20 \times 10^{-3}} = 20 \times 10^{3} = 2 \times 10^{4}\ Vm^{-1}$.

$Q = 8 \times 10^{-19}\ C$ (+5 electronic charges)

$E = \frac{F}{Q} \rightarrow F = EQ$

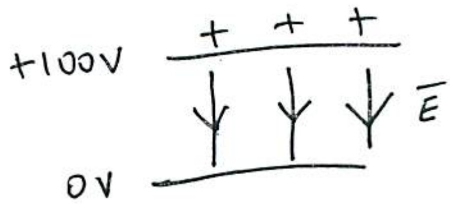
$= 2 \times 10^{4} \times 8 \times 10^{-19}$

$= 16 \times 10^{-15}$

$\therefore F = 1.6 \times 10^{-14}\ N$.

11. $E = \frac{V}{d} = \frac{100V}{20 \times 10^{-3} m} = \frac{1}{2} \times 10^{2} \times 10^{2} = 5 \times 10^{3}\ V/m$

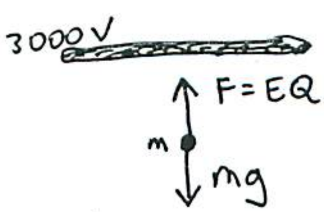
$F = EQ = 5000Q$ (in newtons)



Assuming Q is a positive charge,

The force is downwards, i.e. same direction a $\bar{E}$ vector.

$\rightarrow$ Answer is E.

12. 

Gravitational force balances force due to E.

$\therefore mg = EQ$, where $E = \frac{V}{d}$.

$\therefore mg = \frac{V}{d} Q$

i.e. $Q = \frac{mgd}{V}$

12. (continued)

$$Q = \frac{mgd}{V}$$

$$= \frac{9.75\times10^{-15}\times 10\times(100\times10^{-3})}{3\times10^{3}}$$

$$= \frac{9.75}{3}\times 10^{-15+1+2-3-3}$$

$$= \frac{9.75}{3}\times 10^{-18} = 3.25\times10^{-18}\ C.$$

1 electron has 1.6×10^{-19} C i.e. $1e = 1.6\times10^{-19}$ C

i.e. $\frac{1}{1.6\times10^{-19}}e = 1$ C

i.e. $\frac{3.25\times10^{-18}}{1.6\times10^{-19}}e = 3.25\times10^{-18}$ C

i.e. $\frac{3.25}{1.6}\times10e = 3.25\times10^{-18}$ C

ie $\approx 20e = 3.25\times10^{-18}$ C.

Note Top plate has higher potential, as shown in diagram. This gives the electrostatic force upwards on the negative charge

The answer is 20 electrons.

13. $\boxed{E = \frac{\sigma}{\varepsilon_0}}$: σ = surface charge density $\left(= \frac{\text{charge}}{\text{area}}\right)$

Note For fixed charge Q, the smaller the sphere radius then the higher the charge density σ and the higher E is close to the surface.

As radius R decreases to $R = R_{min}$ and gives $E = 3\,MVm^{-1}$ then a spark may result.

13 (continued)

On the surface of a sphere of radius R, we have

$$E = \frac{Q}{4\pi\varepsilon_0 R^2}$$

$$V = \frac{Q}{4\pi\varepsilon_0 R}$$

Then, $\boxed{E = \frac{V}{R}}$ on surface of charged sphere.

A spark can happen when

$$E = 3\ \mathrm{MVm^{-1}} = 3\times10^6\ \mathrm{Vm^{-1}} = \frac{V}{R_{min}} = \frac{0.6\times10^6}{R_{min}}$$

$$\therefore \quad 3\times10^6 = \frac{0.6\times10^6}{R_{min}}$$

i.e. $$R_{min} = \frac{0.6\times10^6}{3\times10^6}$$

$$= \frac{0.6}{3}$$

$$\therefore \quad R_{min} = 0.2\ \mathrm{m}.$$