Tutorial 3 Solutions

1. The capacitance is the amount of charge (Q) that can be stored per unit of potential difference applied (V),

i.e. $\boxed{C = \frac{Q}{V}}$.

2. $C = 6 \times 10^{-4} \mu F = 6 \times 10^{-4} \times 10^{-6} F$

$V = 100V$

(a) Re-arrange the above formula to give:

$$Q = CV = 6 \times 10^{-4} \times 10^{-6} \times 100$$

$$= 6 \times 10^{-10} \times 10^{2}$$

i.e. $Q = 6 \times 10^{-8}$ C.

(b) $C = 6 \times 10^{-10} F$ (same capacitor)

$V = 50 V$

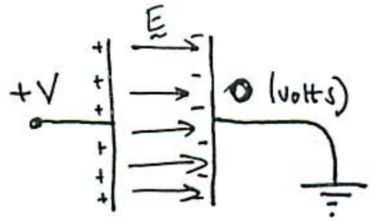
$$Q = CV = 6 \times 10^{-10} \times 50$$

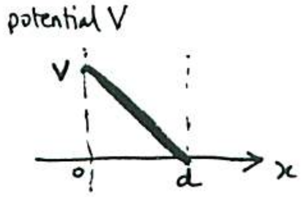
$$= 300 \times 10^{-10}$$

i.e. $Q = 3 \times 10^{-8}$ C.

Note: Since C is fixed, with half the potential difference we have a different circuit in which only half the charge is stored on the capacitor.

3. (a)





(b) d = plate separation

V = potential difference

Electric field magnitude,

$$\boxed{E = \frac{V}{d}}$$

(c) Capacitance, $\boxed{C = \frac{\varepsilon A}{d}}$, where A = plate area

d = plate separation distance

ε = permittivity (of medium between plates)

Close to a charged, conducting surface

$$\boxed{E = \frac{\sigma}{\varepsilon}}$$, where $\sigma = \frac{\text{charge}}{\text{area}}$ (surface charge density).

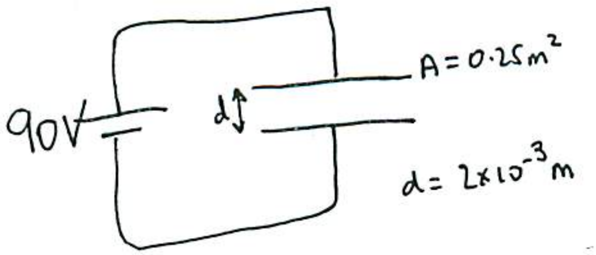
We also have that $E = \frac{V}{d} = \frac{\sigma}{\varepsilon}$

i.e. $\frac{V}{d} = \frac{Q}{\varepsilon A}$, where Q is the stored charge

, i.e. $\frac{V \varepsilon A}{d} = Q$

i.e. $\boxed{C = \frac{\varepsilon A}{d} = \frac{Q}{V}}$, by re-arranging.

4.



(a) In air, $\varepsilon \simeq \varepsilon_0$

i.e. $\varepsilon = 8.85 \times 10^{-12}\ Fm^{-1}$

$$C = \frac{\varepsilon A}{d} = \frac{8.85 \times 10^{-12} \times 0.25}{2 \times 10^{-3}}$$

$$\simeq 1.106 \times 10^{-9}$$

i.e. $C \simeq 1.1 \times 10^{-9}\ F$.

$$C = \frac{Q}{V} \qquad \therefore \quad Q = CV$$

$$= 1.1 \times 10^{-9} \times 90$$

i.e. $Q = 9.9 \times 10^{-8}\ C$.

b) Capacitor left charged (disconnected from battery) with $Q = 9.9 \times 10^{-8} C$.

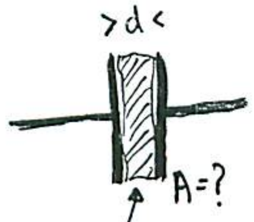
Twice distance between plates : $d \to 2d$

$$\Rightarrow \quad C_{new} = \frac{\varepsilon A}{2d} = \frac{1}{2} C_{old}$$

Since Q remains the same, $V_{new} = \dfrac{Q}{C_{new}} = \dfrac{2Q}{C_{old}}$

$$= 2V_{old} = 2 \times 90$$

$$= 180\ V.$$

5. 

$C = 1.0\mu F$

$\varepsilon_r = 2.6$ (relative permittivity)

$d = 0.05mm$

$: \; C = \dfrac{\varepsilon A}{d}$, where $\varepsilon = \varepsilon_r \varepsilon_0$

$\therefore \; A = \dfrac{Cd}{\varepsilon}$

i.e. $A = \dfrac{Cd}{\varepsilon_r \varepsilon_0}$

i.e. $A = \dfrac{1\times10^{-6} \times 0.05\times10^{-3}}{2.6\times 8.85\times10^{-12}}$

i.e. $A = \dfrac{5\times10^{-6-2-3}}{23.01\times10^{-12}} = 0.217\times10^{1}$

$\approx 2.2\ m^2.$

6. 

$C = 2.0\mu F$.

$d = 0.015\times10^{-3}m$

paper width, $w = 40\times10^{-3}mm$

Again, $C = \dfrac{\varepsilon A}{d}$, where $\varepsilon = \varepsilon_r\varepsilon_0$,

and area of paper = area of plates

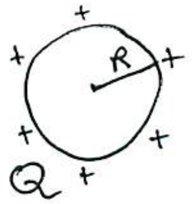
, $A = w(\text{width}) \times L(\text{length})$

$\therefore \; A = \dfrac{Cd}{\varepsilon_r\varepsilon_0}$

i.e. $wL = \dfrac{Cd}{\varepsilon_r\varepsilon_0}$

i.e. $L = \dfrac{\varepsilon d}{w\varepsilon_r\varepsilon_0} = \dfrac{(2.0\times10^{-6})(0.015)\times10^{-3}}{40\times10^{-3}\times2.5\times8.85\times10^{-12}} = 33.9m$,

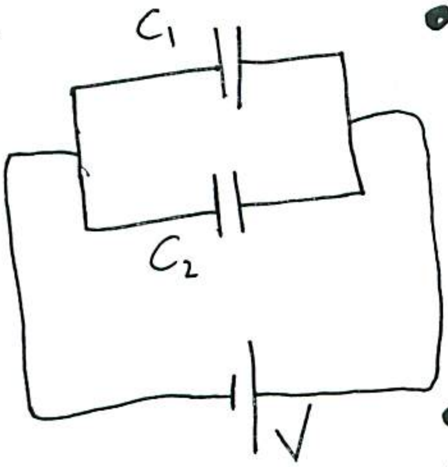
(kinda large! :-))

7. 

Outside, and on the surface, the potential is the same as if the charge Q is all at the point at the centre of the sphere.

$$\rightarrow \quad V = \frac{Q}{4\pi \varepsilon R}$$

The capacitance, C, is given in terms of the charge stored and potential on the surface of the sphere: $C = \frac{Q}{V} = 4\pi\varepsilon R.$

8. 

- In parallel, the potential difference V is across both capacitors, i.e. V is the same for both.

  $\rightarrow$ (i) is true.

- The amount of charge stored on each capacitor will depend on the individual capacitances

(recall that V is fixed for both and $C_1 = \frac{Q_1}{V}$, $C_2 = \frac{Q_2}{V}$)

$\rightarrow$ the charges are not necessarily the same

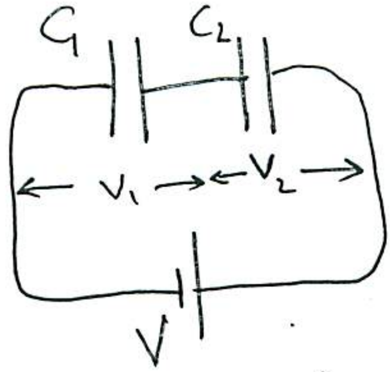
$\rightarrow$ (ii) is not always true.

Also, the total charge stored $Q_T = Q_1 + Q_2$.

- It follows from the fact that $Q_T = Q_1 + Q_2$, that the total capacitance $C_T = \frac{Q_1 + Q_2}{V} = \frac{Q_1}{V} + \frac{Q_2}{V}$

$$\text{i.e.} \quad C_T = C_1 + C_2$$

$\rightarrow$ (iii) is true: $\boxed{\rightarrow \text{THE ANSWER IS } \mathbf{C}.}$

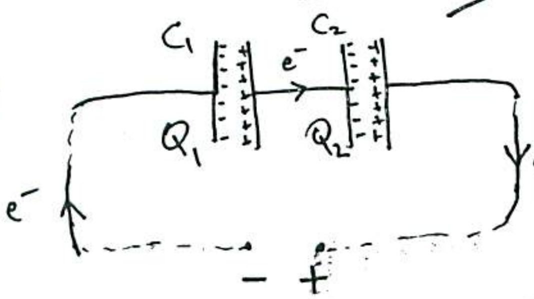
9. 

• One may envisage a gravitational potential analogy to see that the individual potentials (V_1 and V_2) add up to the potential V.... ... it is like climbing a ladder in two stages !

i.e. $V = V_1 + V_2$ (it is only when $C_1 = C_2$ that we would expect $V_1 = V_2$)

i.e. the potential difference across each capacitor is not always the same

$\Rightarrow$ (i) is not true.

• 

Electrons build up on the left plate of C_1. These repel electrons in the connecting wire towards C_2. Electrons are then repelled from the right plate of C_2 towards the battery.

The result is that there is a sequence of equal and opposite charges on the capacitor plates and $Q_1 = Q_2$.

$\Rightarrow$ (ii) is true.

• The potential difference across each capacitor is less that it would be if the other one was n't there. The amount of charged stored on each capacitor is proportional to the potential difference across it (e.g. $Q_1 = C_1 V_1$)

In fact (see lecture notes), total capacitance C_T is given by

$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C_T = \frac{C_1 C_2}{C_1 + C_2}$. For example, $C_T = C_1 \underbrace{\left(\frac{C_2}{C_1 + C_2}\right)}_{\text{always less than } 1}$

$\Rightarrow$ total capacitance is less that C_1 and less than C_2 $\Rightarrow C_T < C_1$.

$\Rightarrow$ (iii) is not true.

$\boxed{\rightarrow \text{THE ANSWER IS } \mathbf{E}.}$

10. $\boxed{C_1 = 8\mu F \ , \ C_2 = 12\mu F}$

(i) In series, equivalent/total capacitance C_T is given by

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\text{i.e.} \quad \frac{1}{C_T} = \frac{C_1 + C_2}{C_1 C_2}$$

$$\text{i.e.} \quad C_T = \frac{C_1 C_2}{C_1 + C_2}$$

Here, $C_T = \dfrac{8 \times 10^{-6} \times 12 \times \cancel{10^{-6}}}{(8+12) \times \cancel{10^{-6}}} = \dfrac{96}{20} \times 10^{-6} = 4 \cdot 8 \mu F.$

(ii) In parallel, equivalent/total capacitance $C_T = C_1 + C_2$

Here, $C_T = (8 \times 10^{-6}) + (12 \times 10^{-6})$

$= (8+12) \times 10^{-6} = 20 \mu F.$

11. $\boxed{C_1 = 0.4\mu F \ , \ C_2 = 0.6\mu F}$

(i) In series, $C_T = \dfrac{C_1 C_2}{C_1 + C_2} = \dfrac{(0.4 \times 0.6)}{(0.4 + 0.6)} \times 10^{-6} = \dfrac{0.24}{1} \times 10^{-6}$

$= 0.24 \mu F.$

In parallel, $C_T = C_1 + C_2 = (0.4 + 0.6) \times 10^{-6}$

$= 1 \mu F.$