TUTORIAL 4 SOLUTIONS

1. Energy stored, $E = \frac{1}{2}QV$, where Q = charge, V = p.d. when capacitor is <u>fully</u> charged.

<u>Alternatively</u>, we may use $C = \frac{Q}{V}$ to eliminate either Q or V and get a different expression for E.

(i) $Q = CV \rightarrow E = \frac{1}{2}(CV)V = \frac{1}{2}CV^2$,

(ii) $V = \frac{Q}{C} \rightarrow E = \frac{1}{2}Q\left(\frac{Q}{C}\right) = \frac{1}{2}\frac{Q^2}{C}$.

Here, we have $V = 12V$ and $Q = 10\mu C$.

Then, $E = \frac{1}{2}QV = \frac{1}{2} \times 10 \times 10^{-6} \times 12 = 6 \times 10^{-5}$

$= 60 \times 10^{-6}$ J

i.e. $E = 60\mu J$.

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2. $C = 10\mu F$, $Q = 20\mu C$

$$E = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$$

$$= \frac{1}{2}\frac{(20 \times 10^{-6})^2}{10 \times 10^{-6}}$$

$$= \frac{1}{2}\frac{(20)^2 \times 10^{-6} \times \cancel{10^{-6}}}{10 \times \cancel{10^{-6}}}$$

$$= \frac{1}{2}\frac{400 \times 10^{-6}}{10}$$

i.e. $E = 200 \times 10^{-7} = 20\mu J$.

3. $C = 100\mu F$, $V = 100V$

$$E = \frac{1}{2}QV = \frac{1}{2}CV^2$$

$$= \frac{1}{2} \times 100 \times 10^{-6} \times (100)^2$$

$$= 50 \times 10^{-6} \times 10^4$$

i.e. $E = 0.5 \text{ J}$.

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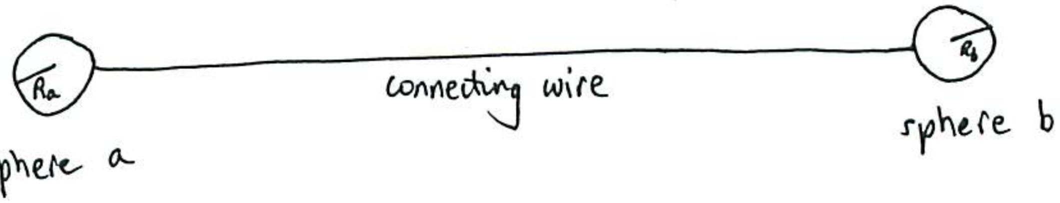
4. Capacitance $C = \frac{Q}{V}$ is the amount of charge stored per unit voltage.

Isolated (i.e. not grounded) conducting sphere has $C = 4\pi\varepsilon R$,

where, for air, $\varepsilon \approx \varepsilon_0$ $\rightarrow$ $C \approx 4\pi\varepsilon_0 R$, for sphere in air.



Charge will distribute itself until the potential of each sphere is the same.

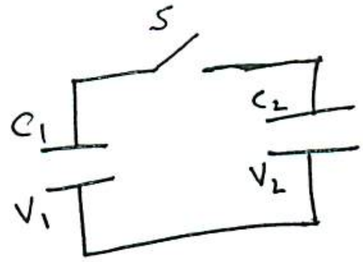
(a) $R_a = R_b \Rightarrow C_a = C_b \Rightarrow$ each sphere will have the <u>same</u> charge at the same potential

$\rightarrow$ $1\mu C$ will distribute evenly between the spheres ($0.5\mu C$ each)

(b) $C_a = \frac{Q_a}{V}$, $C_b = \frac{Q_b}{V}$ $\rightarrow$ $\frac{Q_a}{Q_b} = \frac{C_a}{C_b} = \frac{\cancel{4\pi\varepsilon_0} R_a}{\cancel{4\pi\varepsilon_0} R_b} = \frac{60}{20} = 3$, e.g. $Q_a = 3Q_b$ and $Q_a + Q_b = 1\mu C$

Simultaneous equations for Q_a and Q_b — Substitute $3Q_b + Q_b = 1\mu C$ i.e. $4Q_b = 1\mu C$

i.e. $Q_b = \frac{1}{4}\mu C$ and $Q_a = \frac{3}{4}\mu C$.



(a) $E = \frac{1}{2} C_1 V_1^2$

$= \frac{1}{2} \times 10 \times 10^{-6} \times (50)^2$

$= \frac{25}{2} \times 10^{-3} \quad = 1.25 \times 10^{-2}\ J.$

(b) The charge Q_1 on C_1 will be shared between C_1 and C_2 so that the potential difference across each capacitor, V_{new}, is the same.

Denoting the capacitance of the parallel combination as C_{new}, we will have that $C_{new} = \frac{\text{total charge}}{V_{new}} = \frac{Q_1}{V_{new}}$.

→ working out $Q_1 = C_1 V_1$ and C_{new} (from $C_{new} = C_1 + C_2$) will gives $V_{new} = Q_1 / C_{new}$.

Here, $Q_1 = C_1 V_1 = 10 \times 10^{-6} \times 50 = 5 \times 10^{-4}\ C.$

$C_{new} = C_1 + C_2 = (10 + 2.5)_{\mu F} = 12.5 \times 10^{-6}\ F.$

→ $V_{new} = \frac{5 \times 10^{-4}}{12.5 \times 10^{-6}} = 40\ V.$

(c) Total energy (after connection), $E = \frac{1}{2} C_1 V_{new}^2 + \frac{1}{2} C_2 V_{new}^2 = \frac{1}{2} C_{new} V_{new}^2$

(Can use either to calculate E)

e.g. $E = \frac{1}{2} C_{new} V_{new}^2 = \frac{1}{2} \times 12.5 \times 10^{-6} \times (40)^2$

$\therefore E = 100 \times 10^{-4} = 1.00 \times 10^{-2}\ J.$

Energy stored before connection was $1.25 \times 10^{-2}\ J$ → $0.25 \times 10^{-2}\ J$ has been converted to some other form of energy.

→ most of the 'lost energy' will be converted to heat in the circuit.

6. The 'relative permittivity' of a medium, ε_r, is defined through the equation:

$$\boxed{\varepsilon = \varepsilon_r \varepsilon_0}$$

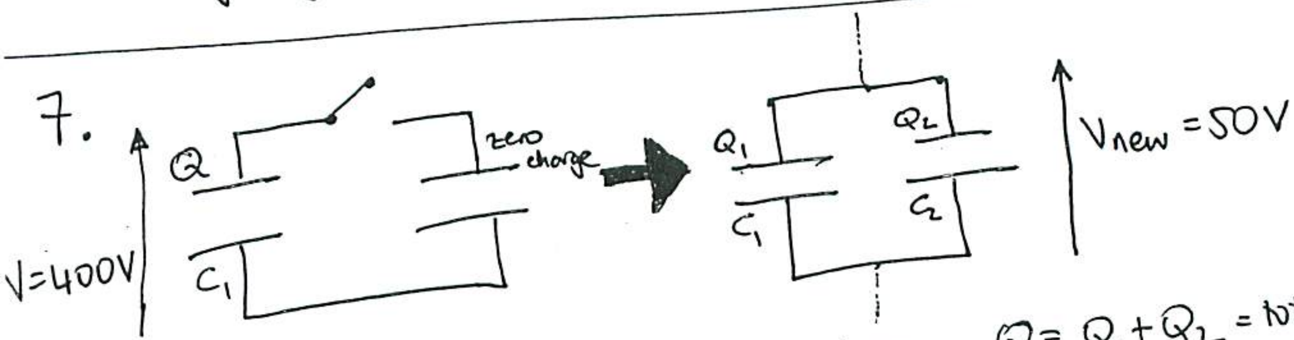
ε: medium permittivity

ε_r: relative permittivity

ε_0: permittivity of free space, i.e. a vacuum

So, we could say that $\boxed{\varepsilon_r = \dfrac{\varepsilon}{\varepsilon_0}}$. It tells us how much permittivity a medium has when compared to (expressed as a factor of multiplication) ε_0 i.e. that of a vacuum. By "permittivity", we can describe this as the degree to which a medium "permits" the storage of charge. Note that ε_r is also called the "dielectric constant."

7.



where $Q = Q_1 + Q_2$ = total charge stored

(a)

Parallel combination $\rightarrow$ $C_{new} = C_1 + C_2$

i.e. $C_{new} = \dfrac{\varepsilon_0 A}{d} + \dfrac{\varepsilon_r \varepsilon_0 A}{d}$ (ε_r ~~for~~ air $= 1$)

i.e. $C_{new} = \dfrac{\varepsilon_0 A}{d}(1 + \varepsilon_r)$ (note that 'identical dimensions' $\rightarrow$ $\dfrac{A}{d}$ same for both)

i.e. $C_{new} = C_1(1 + \varepsilon_r)$

<u>Also</u> $C_{new} = \dfrac{\text{total charge}}{V_{new}} = \dfrac{Q}{50}$ $\therefore Q = 50\,C_{new}$

$\therefore Q = 50\,C_1(1 + \varepsilon_r)$.

7. (b) $\varepsilon_r = ?$ where $Q = 50(1+\varepsilon_r)C_1$

Before connection $C_1 = \frac{Q}{V}$ i.e. $Q = C_1 V$

$\therefore \cancel{C_1} V = 50(1+\varepsilon_r)\cancel{C_1}$

$\therefore V = 50(1+\varepsilon_r)$, where $V = 400V$ (given)

$\therefore \frac{400}{50} = 1 + \varepsilon_r$

$\therefore 8 = 1 + \varepsilon_r \rightarrow \varepsilon_r = 7.$

(c) Energy before $E = \frac{1}{2}C_1V^2 = \frac{1}{2}C_1(400)^2$

$E = 80{,}000C_1 .$

Energy after $E = \frac{1}{2}C_1V_{new}^2 + \frac{1}{2}C_2V_{new}^2$

$= \frac{1}{2}C_1V_{new}^2 + \frac{1}{2}(C_1\varepsilon_r)V_{new}^2$, since $C_2 = \varepsilon_r C_1$

$= \frac{1}{2}C_1(50)^2 + \frac{1}{2}C_1 . 7 . (50)^2$

$E = C_1(10{,}000) .$

i.e. energy stored drops from $80{,}000C_1 \rightarrow 10{,}000C_1$

$\rightarrow$ usually most of the difference leads to heating of the circuit components.

8. Time constant, $\tau = RC$ (it is a characteristic time reflecting how long it takes to charge or discharge the capacitor)

Here, $\tau = 500 \times 10^{3} \times 2 \times 10^{-6}$

i.e. $\tau = 1000 \times 10^{-3} = 1\,s.$

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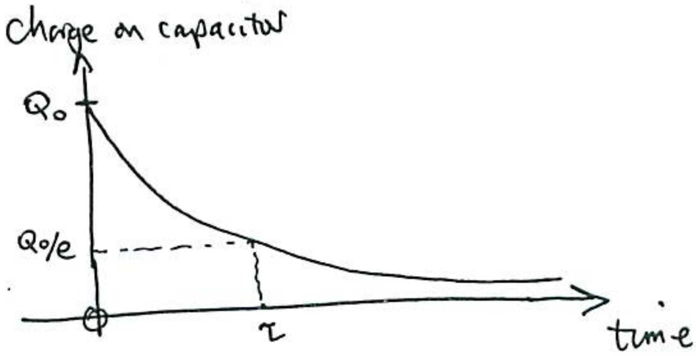
9. $\tau = RC$, $R = 50\,k\Omega = 50 \times 10^{3}\,\Omega$

$C = 20\,\mu F = 20 \times 10^{-6}\,F$

$\rightarrow \tau = 50 \times 10^{3} \times 20 \times 10^{-6} = 1000 \times 10^{-3} = 1\,s.$

10.



(in fact, after each time period of τ, the charge stored decreases by a factor of $\frac{1}{e}$)

$\tau = RC$

ie. $\tau = 500 \times 10^{3} \times 5 \times 10^{-6}$

ie. $\tau = 2500 \times 10^{-3}$

ie. $\tau = 2.5\,s.$