1. Ohm's law : $V = IR$ i.e. current I through a conductor is proportional to the applied voltage (potential difference) V, R is the resistance.

The ohm : $R = \frac{V}{I}$ gives $1\,\Omega \equiv \frac{1V}{1A}$, in units.

Therefore, One ohm (1Ω) is the resistance of a conductor that permits the flow of a current of 1A when a p.d. of 1V is applied.

2. Current $=\frac{\text{charge}}{\text{time}}$ i.e. $I = \frac{Q}{t}$ and $1A = \frac{1C}{1s}$

→ 1 ampere = 1 coulomb per second ⊛

potential difference (voltage) $= \frac{\text{work done (energy)}}{\text{charge}}$, i.e. $V = \frac{E}{Q}$

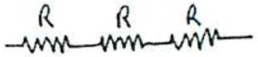
(when bringing a charge Q up to voltage V)

→ 1 volt = 1 joule per coulomb ⊛

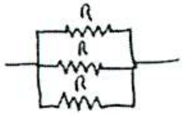
3. Resistance R of a conductor (expressed in ohms) is the ratio V/I,

where V is the potential difference across the conductor, and I is the current flowing through it.

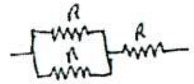
4. 3 resistors of 8Ω each.

(i) 

$R_T = R + R + R = 8+8+8 = 24\Omega$.

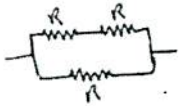
(ii) 

$\frac{1}{R_T} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$, $\therefore R_T = \frac{8}{3}\Omega$.

(iii) 

Two in parallel give $\frac{1}{R_p} = \frac{1}{R} + \frac{1}{R} \rightarrow R_p = \frac{8}{2}\Omega$. (4Ω)

Plus one in series: $R_T = R + \frac{8}{2} = 8 + \frac{8}{2} = 12\Omega$.

(iv) 

Two in series $R_s = R + R = 8+8 = 16\Omega$

Plus one in parallel: $\frac{1}{R_T} = \frac{1}{R_s} + \frac{1}{R} = \frac{1}{16} + \frac{1}{8} = \frac{8+16}{16\times 8}$

$\rightarrow R_T = \frac{16\times 8}{8+16} = \frac{16}{3}\Omega$.

---

5. 3 resistors of 12Ω. Similar to above.

(i) $R_T = 3\times 12 = 36\Omega$, (ii) $\frac{1}{R_T} = \frac{3}{R} = \frac{3}{12} \rightarrow R_T = 4\Omega$,

(iii) Two in $\parallel^{el}$: $\frac{1}{R_p} = \frac{2}{R} = \frac{2}{12} \rightarrow R_p = 6\Omega$

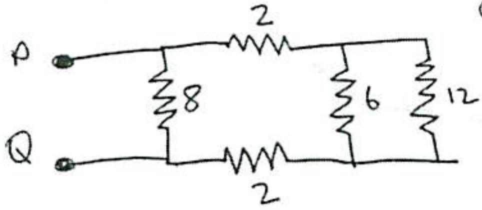
Plus one in series: $R_T = R + R_p = 12 + 6 = 18\Omega$.

(iv) Two in series: $R_s = 2R = 24\Omega$.

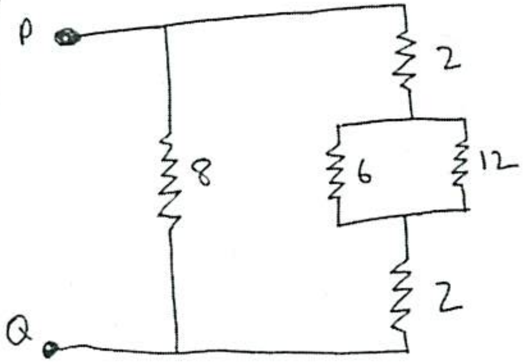
Plus one in parallel: $\frac{1}{R_T} = \frac{1}{R} + \frac{1}{R_s} = \frac{1}{12} + \frac{1}{24} = \frac{3}{24}$

$\rightarrow R_T = 8\Omega$.

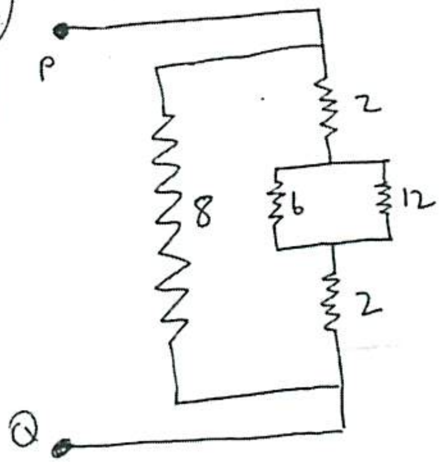
6. It may help to redraw the <u>same</u> circuit so that it looks more familiar ....

i.e. 

OR



OR



So, 6Ω in parallel with 12Ω gives R_p, where

$\frac{1}{R_p} = \frac{1}{6} + \frac{1}{12} = \frac{12+6}{6\times 12} = \frac{3}{12}$

$\rightarrow R_p = 4\Omega$.

Then, R_p in series with the two 2Ω resistors gives

$R_s = 4\Omega + 2\Omega + 2\Omega$

i.e. $R_s = 8\Omega$.

Then, 8Ω in parallel with R_s gives R_T, where

$\frac{1}{R_T} = \frac{1}{8} + \frac{1}{R_s}$

i.e. $\frac{1}{R_T} = \frac{1}{8} + \frac{1}{8} = \frac{2}{8}$

i.e. $R_T = 4\Omega$.

7. The structure of the resistor circuit here is the same as in the previous question. (4)

Here, 3Ω in parallel with 6Ω gives $R_p = \frac{3\times6}{3+6} = \frac{18}{9} = 2\Omega$.

R_p in series with two 1Ω gives $R_s = 2+1+1 = 4\Omega$.

R_s in parallel with 4Ω gives $R_T = \frac{4\times R_s}{4+R_s} = \frac{4\times4}{4+4}$

i.e. $R_T = \frac{16}{8} = 2\Omega$.

8. Resistivity ρ is defined as a material constant (at a particular temperature) and is given as the constant of proportionality in the equation:

$$\boxed{R = \rho\frac{L}{A}}$$

, where R = resistance, L = length, A = cross-sectional area } of the material

when $R = 10\Omega$, $L = 5m$, $A = 0.2m^2$, $\rho = ?$

Re-arrange the above equation to find ρ . . .

$$\rho = \frac{RA}{L} = \frac{10\times0.2}{5} = 0.4\ \Omega m.$$

Note dimensions: $[\rho] = \frac{[R][A]}{[L]} = \frac{\Omega m^2}{m} = \Omega m$.

9. (5)

$$\boxed{R = \frac{\rho L}{A}}$$

$R = 4\Omega$
$L = 3m$
$A = 0.3mm^2 = 0.3\times(10^{-3}m)^2$
$= 0.3\times10^{-6}m^2$

$$\rho = \frac{AR}{L} = \frac{0.3\times10^{-6}\times4}{3} = 4\times10^{-7}\ \Omega m.$$

10.

$$\boxed{R = \frac{\rho L}{A}}$$

(i) if $L \to 2L$ and diameter d doubled,

If cross-section is either square or circular

$d \to 2d \Rightarrow A \to 4A$

since A depends on $d^2 \to (2d)^2 = 4d^2$

$$\therefore R \to \frac{\rho.(2L)}{(4A)} = \frac{1}{2}\frac{\rho L}{A} = \frac{1}{2}R_{previous}$$

i.e. resistance is halved.

(ii) when $L \to L/2$ and $d \to d/2$ $(A \to A/4)$

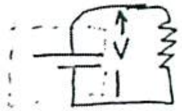
$$R \to \frac{\rho.(L/2)}{(A/4)} = \left(\frac{4}{A}\right).\rho.\left(\frac{L}{2}\right) = \frac{2\rho L}{A} = 2R_{previous}$$

$\therefore$ resistance is doubled.

11. The electromotive force (or emf) of a battery is the output voltage when no current is drawn, i.e. in open-circuit ..

.. the emf is usually represented by the symbol E.

The potential difference, V, is a more general term that also applied to when current is drawn, i.e. in closed-circuit ...

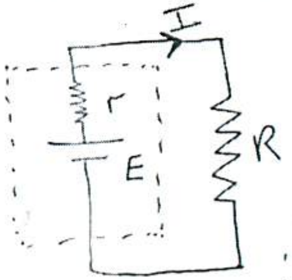


- When no current is drawn, $E = V$
- When current is drawn, $E > V$.

- The difference in magnitude is because the battery itself presents some resistance to current flow (its "internal resistance", r).

  And, some of the battery's emf is dropped across its internal resistance, leaving less potential difference across the battery terminals (when a current flows).

- One can derive an equation relating E and V by considering the battery to be an ideal battery (with output voltage E) in series with a resistor of resistance r (representing the internal resistance)..



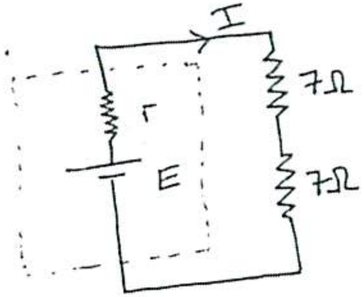
The total resistance in the circuit is $r+R$.

Ohm's Law gives $E = I(R+r)$

i.e. $E = \underbrace{IR}_{\text{actual output p.d. } V \text{ at terminals}} + \underbrace{Ir}_{\text{voltage dropped across internal resistance}}$

$\therefore \boxed{E = V + Ir}$

12.



where emf $E = 1.5\,V$

internal resistance $r = 1\,\Omega$

Total circuit resistance, $R_T = r + 7 + 7$

i.e. $R_T = 1 + 7 + 7$

i.e. $R_T = 15\,\Omega$.

- Ohm's Law : $E = IR_T$

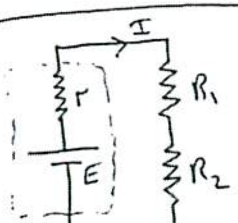
  gives $I = E/R_T = \frac{1.5}{15} = 0.1\,A$.

  (same current in each resistor)

- Voltage across each resistor is also given by Ohm's law

$$V_{7\Omega} = IR_{7\Omega} = I \times 7$$
$$= 0.1 \times 7$$
$$\text{i.e. } V_{7\Omega} = 0.7\,V.$$

13.



where $E = 6\,V$ (emf)

$r = 1\,\Omega$ (internal resistance)

$R_1 = 4\,\Omega$

$R_2 = 5\,\Omega$. Total circuit resistance,

$R_T = r + R_1 + R_2 = 1 + 4 + 5 = 10\,\Omega$

(i) Current $I = \frac{E}{R_T} = \frac{6}{10} = 0.6\,A$.

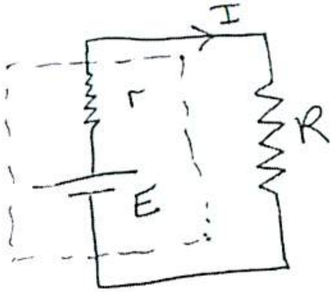
(ii) p.d. across $5\,\Omega$ resistor $V_{R_2} = IR_2 = 0.6 \times 5 = 3\,V$.

(iii) actual potential difference at battery terminals : $V = I(R_1 + R_2)$

i.e. $V = 0.6 \times (4+5) = 5.4\,V$.

We could alternatively used the fact that $V = E - Ir$

14.



$E = 5\,V$ (emf)

$I = 0.4\,A$

$R = 12\,\Omega$

(i) $r = ?$

Ohm's law: $E = I(r+R)$ ✓ ✓ ? ✓

→ Solve to find r. $\frac{E}{I} = r + R$

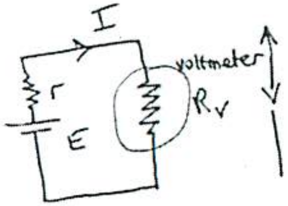
$\therefore \frac{E}{I} - R = r$

ie. $r = \frac{E}{I} - R = \frac{5}{0.4} - 12 = \frac{10}{4}\times 5 - 12 = \frac{50}{4} - 12$

$= 12.5 - 12$

$\therefore r = 0.5\,\Omega.$

(ii)



where $E = 5V$, $r = 0.5\,\Omega$.

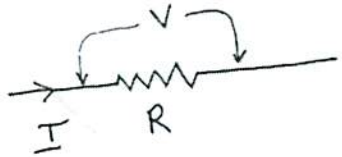
Ohm's Law: $E = I(r + R_V) = Ir + \underbrace{IR_V}_{\text{p.d. across voltmeter, } V = 4.9V}$

$\therefore E = Ir + V$

ie. $5 = Ir + 4.9 \rightarrow Ir = 0.1 \rightarrow I = \frac{0.1}{r} = \frac{0.1}{0.5} = 0.2\,A.$

$\therefore V = IR_V$ and $R_V = V/I = \frac{4.9}{0.2} = 24.5\,\Omega.$

15.

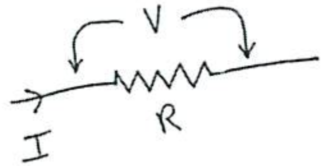


$\boxed{\text{Power dissipated},\ P = IV}$

Ohm's Law gives $V = IR$, (i) $P = IV = I(IR) = I^2R.$

(ii) $P = I.V = \left(\frac{V}{R}\right)V = \frac{V^2}{R}$, again using Ohm's law.

16.

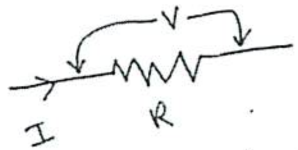


where $V = 7V$

$I = 3A$

Power dissipated, $P = IV = 3 \times 7 = 21\,W.$

17.



where $R = 5\,\Omega$

$I = 1A$

Power dissipated, $P = I^2R = 1^2 \times 5 = 5\,W.$

Power $= \frac{\text{energy}}{\text{time}, t}$, where time $= 3\,\text{min} = 3\times 60$ sec $= 180$s.

$\therefore$ energy $= P.t = 5.180 = 900\,J.$

18. $P = 60W = \frac{\text{energy}}{\text{time}} \rightarrow$ energy $= 60 \times 1 \times 60 = 3600\,J.$

$\left.\begin{array}{l} V = 240V \\ P = 60W \end{array}\right\} \Rightarrow R = ?$ $\therefore P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P} = \frac{(240)^2}{60} = 960\,\Omega.$

19.

$P = 2kW = 2\times10^3 W$

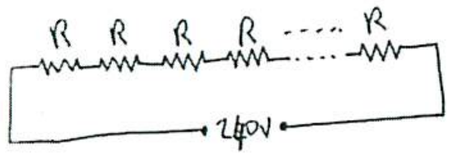
$V = 240V$

$R = ?$

$P = \frac{V^2}{R}$ $\quad \therefore R = \frac{V^2}{P}$

i.e. $R = \frac{(240)^2}{2\times10^3} = 288\Omega.$

20.



Twenty lamps, each with resistance R. Total power consumed/dissipated $P = 24W$

(i) Total resistance, $R_T = R+R+R+\dots+R = 20R$

Total power, $P = \frac{V^2}{R_T}$

$\therefore R_T = \frac{V^2}{P} = \frac{(240)^2}{24} = 2400\Omega$

$\therefore 20R = 2400$

i.e. $R = \frac{2400}{20} = 120\Omega.$

(ii) 19 lamps $\rightarrow R_T = 19R = 19\times120 = 2280\Omega.$

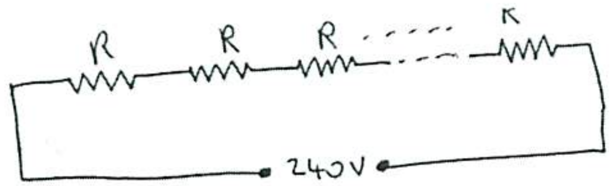
Then, $P = \frac{V^2}{R_T} = \frac{(240)^2}{2280} = 25.3\ W,$ note that this is higher than 24W and there are less lamps!

(iii) low voltage test : $V = 0.1V,\ I = 10mA$

$\Rightarrow R = \frac{V}{I} = \frac{0.1}{10\times10^{-3}} = 10\Omega,$ very much less than 120Ω under higher voltage conditions.

Much higher voltage $\rightarrow$ much hotter lamp filament $\rightarrow$ much higher resistance.

21.



Twenty four lamps, each with resistance R

Total power consumed/dissipated, $P = 24W$

(i) Total resistance, $R_T = 24R$. $P = \frac{V^2}{R_T}$ gives $R_T = \frac{V^2}{P}$

$\therefore R_T = \frac{(240)^2}{24} = 2400\Omega = 24R$. $\quad \therefore R = \frac{2400}{24} = 100\Omega.$

• Ohm's law : $I = \frac{V}{R_T} = \frac{240}{(24\times100)} = 0.1A.$

(ii) 23 lamps : $R_T = 23\times100 = 2300\Omega$: $P = \frac{V^2}{R_T} = \frac{(240)^2}{2300} \approx 25W$

(i.e. higher with less lamps)

(iii) Low voltage test : $\left.\begin{array}{l} V = 1V \\ I = 0.1A \end{array}\right\} \rightarrow R = \frac{V}{I}$

i.e. $R = \frac{1}{0.1} = 10\Omega < 100\Omega$

much higher voltage $\left(V = 1V \rightarrow V = \frac{240}{24} = 10V\right)$

gives a much hotter filament and this results in a much higher resistance.