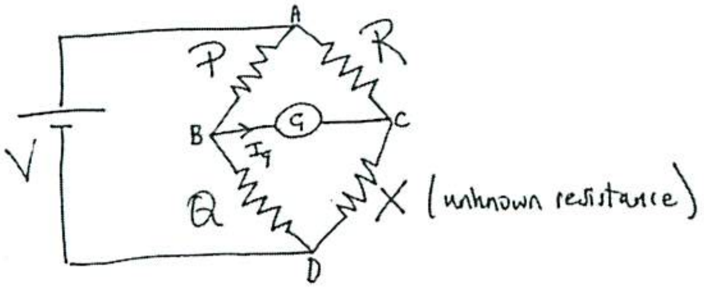
①

TUTORIAL 6 SOLUTIONS

1. (a)



'At balance' : $I_g = 0$ i.e. points B and C have the same potential

Considering the circuit as a 'double potential divider', $I_g = 0$ then implies that the following potential differences are equal:

$$\left.\begin{aligned} V_{AB} &= V_{AC} \\ V_{BD} &= V_{CD} \end{aligned}\right\} \text{(i)}$$

Which, in terms of currents (using Ohm's Law), gives:

$$\left.\begin{aligned} I_1 P &= I_2 R \\ I_1 Q &= I_2 X \end{aligned}\right\} \text{(ii)}$$

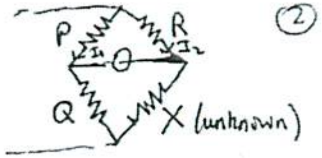
[Since $I_1 = I_{AB} = I_{BD}$ and $I_2 = I_{AC} = I_{CD}$, noting that $I_g = 0$ and Kirchhoff's current law]

Dividing equations (ii) gives the balance condition : $\boxed{\dfrac{P}{Q} = \dfrac{R}{X}}$

Then, to work out the unknown resistance, X, re-arrange to get: $\boxed{X = \dfrac{QR}{P}}$

②

1. (b) $\left(\begin{matrix}\text{lamp} \\ \text{resistance}\end{matrix}\right)$ $X = \dfrac{QR}{P}$



(i) $\left.\begin{aligned} P &= 1000\Omega \\ Q &= 0.5\Omega \\ R &= 1000\Omega \end{aligned}\right\} \Rightarrow X = \dfrac{0.5 \times 1000}{1000} = 0.5\Omega.$

(ii) $\left.\begin{aligned} P &= 1000\Omega \\ Q &= 1000\Omega \\ R &= 5\Omega \end{aligned}\right\} \Rightarrow X = \dfrac{1000 \times 5}{1000} = 5\Omega.$

In case (i) I_1 and I_2 are balanced and small (both currents pass through $(1000 + 0.5)\Omega$

In case (ii) $\left.\begin{aligned} R + X &= (5+5) = 10\Omega \\ P + Q &= (1000+1000) = 2000\Omega \end{aligned}\right\}$ current through R and X (the bulb) is much larger here

→ The higher current through the bulb (X) in the second case is consistent with the bulb glowing brightly.

Note also that, in the second case, the resistance of X is also much higher ($0.5\Omega \to 5\Omega$). This is consistent with a much hotter (i.e. brightly glowing) bulb, since resistance increases with temperature increasing.

1. (c) (i) $P = 100\Omega$, $Q = 1\Omega$, $R = 100\Omega$ $\Big\} \rightarrow X = \frac{QR}{P} = \frac{1 \times 100}{100} = 1\Omega$

(ii) $P = 100\Omega$, $Q = 100\Omega$, $R = 10\Omega$ $\Big\} \rightarrow X = \frac{QR}{P} = \frac{100 \times 10}{100} = 10\Omega$.

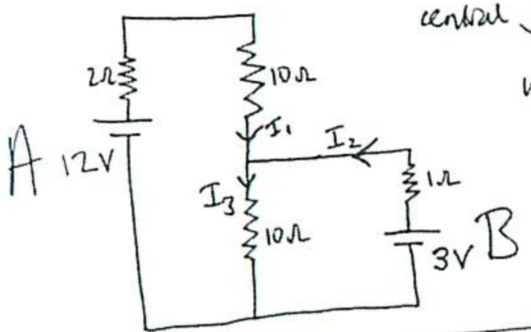
---

2. Kirchhoff's Laws : (i) Algebraic sum of the currents INTO a junction is zero. (current law)

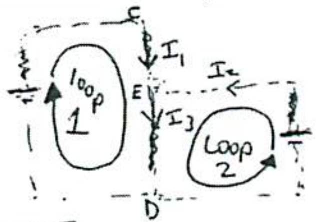
(ii) Algebraic sum of the potential differences around any closed loop is zero (voltage law)

---

3. Draw a circuit diagram that includes the internal resistances and ideal batteries (that only have an emf and no intrinsic resistance); also label the currents with respect to the central junction.



We will consider two loops :



Kirchhoff's current law : $\boxed{I_{ED} = I_3 = I_1 + I_2}$ (this will allow us to eliminate I_3 from the two equations we get from applying Kirchhoff's voltage law.....

(I_1 will be the current through battery A, I_2 will be the current through battery B)

 3. CONTINUED.

Note that potential difference relating to the section ED, and giving rise to I_3, is

$$V_{ED} = I_3 10 = 10I_1 + 10I_2, \quad = 10(I_1 + I_2).$$

Loop 1 $\quad 12 = 2I_1 + 10I_1 + 10(I_1 + I_2)$

Loop 2 $\quad 3 = 1.I_2 + 10(I_1 + I_2)$

2 equations in 2 unknowns (I_1 and I_2)

$$\Rightarrow \boxed{\begin{aligned} 12 &= 22 I_1 + 10 I_2 \quad &(i) \\ 3 &= 10 I_1 + 11 I_2 \quad &(ii) \end{aligned}}$$

$11 \times (i) \rightarrow \quad 132 = 242 I_1 + 110 I_2 \quad (i)$

$10 \times (ii) \rightarrow \quad 30 = 100 I_1 + 110 I_2 \quad (ii)$

subtract : $\quad 102 = 142 I_1 \longrightarrow I_1 = \frac{102}{142} \approx 1.39A.$

Then, $3 = 10I_1 + 11I_2$ gives $I_2 = \frac{3 - 10I_1}{11} = -0.99A.$

$\therefore$ I_2 actually goes in the other direction (and will be charging up battery B, rather than draining battery B).

Current through ED is $I_3 = I_1 + I_2 = 1.39 + (-0.99)$

$= 0.40A.$

3. (c) Power dissipated in resistor between E and D,

$P = I^2R$, where $I = I_3 = 0.4A$, $R = 10\Omega$

$\rightarrow P = (0.4)^2 \times 10 = 1.6\,W$.

Energy dissipated in resistor between C and E,

Energy = Power x time, where Power $= I_1^2 R$ and $I_1 = 1.39A$, $R = 10\Omega$

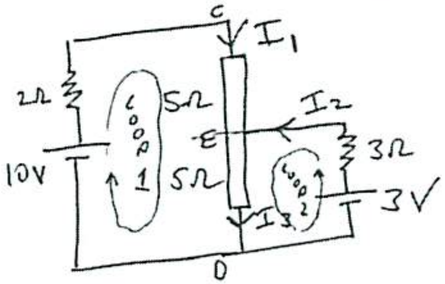
i.e. Power $= (1.39)^2 \times 10 \approx 19.3\,W$.

Time $= 5\,min = 5 \times 60\,s = 300\,s$

$\therefore$ Energy $= 19.3 \times 300 \approx 5{,}796\,J \approx 5.8\,kJ$.

4. Circuit with internal resistances, emf's and currents through the batteries...

(a)



Kirchhoff's current law: $I_3 = I_1 + I_2$

(will use this in the equations from the voltage law)

$V_{ED} = I_3 \cdot 5 = (I_1 + I_2)5$.

Loop 1 $\quad 10 = 2I_1 + 5I_1 + 5(I_1 + I_2) \rightarrow 10 = 12I_1 + 5I_2 \quad$ (i)

Loop 2 $\quad 3 = 3I_2 + 5(I_1 + I_2) \rightarrow 3 = 5I_1 + 8I_2 \quad$ (ii)

solve for I_1 and I_2

$8 \times$(i) $\rightarrow 80 = 96I_1 + 40I_2$

$5 \times$(ii) $\rightarrow 15 = 25I_1 + 40I_2$

subtract $\quad 65 = 71I_1 \rightarrow I_1 = \frac{65}{71} \approx 0.92A$.

4. (a) CONTINUED

$I_1 = 0.92A$, e.g. $3 = 5I_1 + 8I_2$ then gives...

$I_2 = \frac{3 - 5I_1}{8} = -0.2A$.

i.e. I_2 goes clockwise round loop 2 (and charges battery B).

- Current through ED $= I_3 = I_1 + I_2$
  $= 0.92 + (-0.2)$
  $= 0.72A$.

(b) Power dissipated between CE: $P = I_1 R^2$, where $I_1 = 0.92A$, $R = 5\Omega$.

i.e. $P = (0.92)^2 \times 5 = 4.23\,W$.

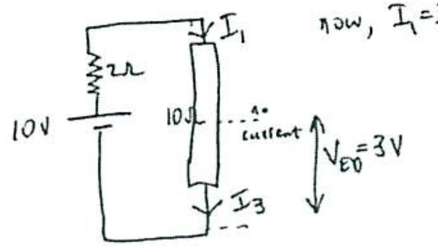
Energy dissipated in 3 minutes: Energy = Power x time, time = 3 min = 180 s,

$\therefore$ Energy $= 4.23 \times 180 = 761\,J$.

(c) No current through battery B $\Rightarrow I_2 = 0 \Rightarrow$ no p.d. dropped across 3Ω internal resistance,

and battery emf (3V) matches V_{ED}

We can then consider

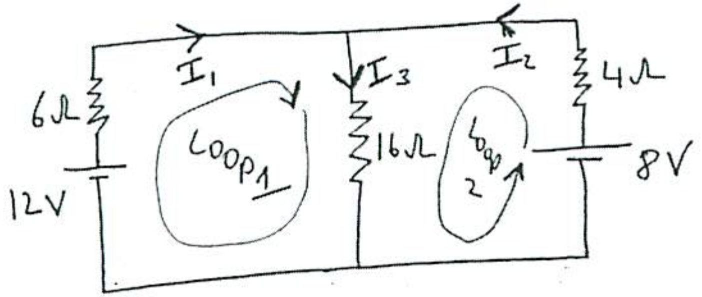


now, $I_1 = I_3 = I$, say $\quad I = \frac{10V}{(2+10)\Omega}$ (ohm's law)

i.e. $I \approx 0.83\,A$.

and $R_{ED} = \frac{V_{ED}}{I} = \frac{3}{0.83} \approx 3.6\Omega$.

5. (i) Draw circuit, label currents ("coming from" the batteries), and apply Kirchhoff's current and voltage laws.



Current law $\quad I_3 = I_1 + I_2 \quad$ (will use to eliminate I_3 from the equations we get from the voltage law)

e.g. Voltage dropped across $R_L = 16\Omega$, $\quad V_L = I_3 16$

i.e. $V_L = (I_1 + I_2)16$.

Voltage law

Loop 1 : $\quad 12 = 6I_1 + 16(I_1+I_2) \rightarrow 12 = 22I_1 + 16I_2 \quad$ (i)

Loop 2 : $\quad 8 = 4I_2 + 16(I_1+I_2) \rightarrow 8 = 16I_1 + 20I_2 \quad$ (ii)

(divide both equations by 2) $\rightarrow$ $\left.\begin{array}{ll} 6 = 11I_1 + 8I_2 & \text{(i)} \\ 4 = 8I_1 + 10I_2 & \text{(ii)} \end{array}\right\}$ SOLVE FOR I_1 and I_2

$10\times$(i) $\rightarrow \quad 60 = 110I_1 + 80I_2$

$8\times$(ii) $\rightarrow \quad 32 = 64I_1 + 80I_2$

subtract $\quad 28 = 46I_1 \quad \rightarrow \quad I_1 = \frac{28}{46} \approx 0.6$ A.

(current supplied by 12V battery)

5. CONTINUED $\quad I_1 = 0.6$ A $\quad$, e.g. $4 = 8I_1 + 10I_2$

$$\rightarrow \quad I_2 = \frac{4 - 8I_1}{10} \approx -0.08 \text{ A.}$$

i.e. actually flowing clockwise in loop 2 and charging (rather than draining the 8V battery).

Current in 16Ω (load) resistor $= I_3 = I_1 + I_2$

$= 0.6 - 0.08$

i.e. $I_3 = 0.52$ A.

Here, I_1 and I_3 flow in direction that we picked at the beginning but I_2 flows in the other direction.

(ii) Power dissipation $= I^2R$. $\quad$ (r_1) $\quad P = I_1^2 r_1 = (0.6)^2 6$

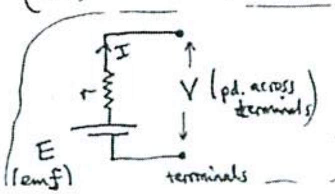
$= 2.16$ W.

(r_2) $\quad P = I_2^2 r_2 = (-0.08)^2 4$

$= 0.026$ W.

(R_L) $\quad P = I_3^2 R_L = (0.52)^2 16 = 4.33$ W.

(iii) in each case we have



Kirchhoff's voltage law

$\rightarrow E = Ir + V$

$\rightarrow V = E - Ir$

$V_1 = E_1 - I_1r_1 = 12 - (0.6)6 = 8.4$ V.

$V_2 = E_2 - I_2r_2 = 8 - (-0.08).4$

$= 8 + 0.08\times4$

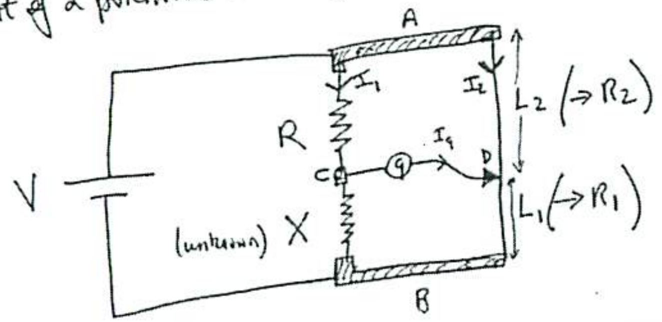
$= 8.32$ V.

(9)

5. (iii) continued : battery 1 is drained and some of the emf E_1 is dropped across internal resistance r_1 → reduced voltage at battery terminals.

battery 2 has current flowing into its positive terminal and is being charged. The p.d. across the terminals is the sum of the p.d. across r_2 and the Emf and is larger than the battery emf alone.

6. Metre bridge gets its name from the 1m length of wire that forms part of a potential divider ($L = L_1 + L_2 = 1m$) ...



Balance condition arises when $I_g = 0$ (as a result of the same potential at points C and D)

This implies that : $V_{AC} = V_{AD}$ and $V_{CB} = V_{DB}$ ⇒ $I_1 R = I_2 R_2$, $I_1 X = I_2 R_1$ → divide equations

BALANCE CONDITION (equation)

$$\boxed{\frac{R}{X} = \frac{R_2}{R_1} = \frac{L_2}{L_1}}$$

since $R = \rho \frac{L}{A}$ i.e. $R \propto L$.

(10)

7. $R = R_0 (1 + \alpha T)$

R: Resistance at temperature T

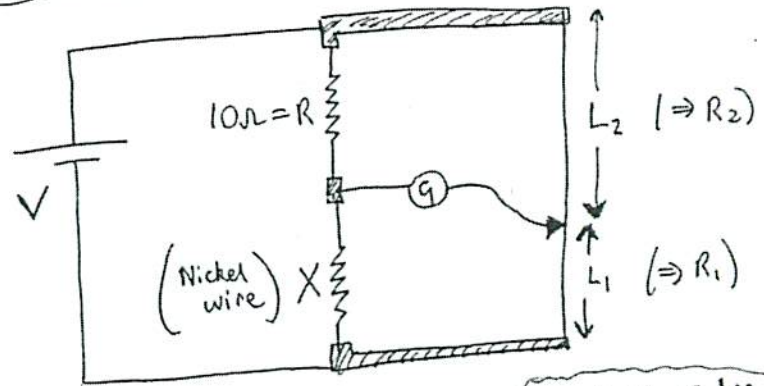
R_0: Resistance at temperature of 0°C

α: Temperature coefficient of Resistance

T: Temperature T in °C

$$\boxed{\alpha = \frac{\text{increase in resistance per °C}}{\text{resistance at 0°C}}}$$

8.



$L = L_1 + L_2 = 1m$

Balance condition (zero current reading in galvanometer) $\boxed{\frac{R}{X} = \frac{L_2}{L_1}}$ (see solution to question 6.)

(a) $R = 10\Omega$. $T = 0°C \to L_1 = 50cm$ ($L_2 = 100 - 50 = 50cm$)
$T = 100°C \to L_1 = 60cm$ ($L_2 = 100 - 60 = 40cm$)

$T = 0°C$ $X_0 = \frac{RL_1}{L_2} = \frac{10 \times 50}{50} = 10\Omega$.

$T = 100°C$ $X_{100} = \frac{RL_1}{L_2} = \frac{10 \times 60}{40} = 15\Omega$.

8. CONTINUED $\quad R = R_0(1+\alpha T)$

$$X_{100} = X_0(1+\alpha T)$$

i.e. $15 = 10(1+\alpha.100)$

i.e. $1.5 - 1 = \alpha.100$

i.e. $\alpha = 5\times10^{-3}\ {}^\circ C^{-1}$.

Require T when $X_{54} = \frac{RL_1}{L_2}$ (and $L_1 = 54\,cm$, $L_2 = 100-54 = 46\,cm$)

$$X_{54} = \frac{10\times 54}{46}$$

i.e. $X_{54} \simeq 11.74\,\Omega$.

$\therefore\ X_{54} = X_0(1+\alpha T)$, where we need to find T.

i.e. $11.74 = 10(1+5\times10^{-3}.T)$

Re-arranging gives $T \simeq 34.8^\circ C$.

(b) $R = \frac{\rho L}{A}$, where $R \equiv X_{54} = 11.74\,\Omega$
$L = 300\,cm = 3\,m$
$A = 5\times10^{-4}\,cm^2 = 5\times10^{-4}\times(10^{-2}m)^2$
i.e. $A = 5\times10^{-4}\times10^{-4}m^2 = 5\times10^{-8}m^2$

$$\rho = \frac{RA}{L} = \frac{11.74\times5\times10^{-8}}{3} \simeq 1.96\times10^{-7}\,\Omega m.$$

(c) Since $X \simeq 10\,\Omega$, balance point near middle of $L \rightarrow$ highest accuracy (e.g. allows neglecting resistance connecting strips of the bridge).

9. Notation used is as in the previous solution.

$R = 10\,\Omega$. $\quad T=0^\circ C \rightarrow L_1 = 40\,cm$, $L_2 = 100-40 = 60\,cm$
$T=100^\circ C \rightarrow L_1 = 50\,cm$, $L_2 = 100-50 = 50\,cm$

$T=0^\circ C$: $X_0 = \frac{RL_1}{L_2} = \frac{10\times40}{60} \simeq 6.67\,\Omega$

$T=100^\circ C$: $X_{100} = \frac{RL_1}{L_2} = \frac{10\times50}{50} \simeq 10\,\Omega$.

This can give us the temperature coefficient of resistance

$X_{100} = X_0(1+\alpha T) \rightarrow \alpha = \frac{1}{X_0}\left(\frac{X_{100}-X_0}{T}\right)$

i.e. $\alpha = \frac{1}{6.67}\left(\frac{10-6.67}{100}\right) \simeq 5\times10^{-3}\ {}^\circ C^{-1}$.

(a) T when $L_1 = 44\,cm$? $\quad L_2 = 100-44 = 56\,cm$

$$X = \frac{RL_1}{L_2} = \frac{10.44}{56} \simeq 7.86\,\Omega$$

then $X = X_0(1+\alpha T)$ where $7.86 = 6.67(1+5\times10^{-3}.T)$
$\rightarrow T \simeq 34.5^\circ C$.

(b) $R = \frac{\rho L}{A}$, where $R = X = 7.86\,\Omega$
$L = 150\,cm = 1.5\,m$
$A = 2.5\times10^{-4}\,cm^2 = 2.5\times10^{-4}\times(10^{-2}m)^2$
i.e. $A = 2.5\times10^{-4}\times10^{-4}m^2 = 2.5\times10^{-8}m^2$.

$$\therefore\ \rho = \frac{RA}{L} = \frac{7.86\times2.5\times10^{-8}}{1.5} \simeq 1.3\times10^{-7}\,\Omega m.$$