OSA
November 2000 Vol. 11 No. 11

opn
Optics & Photonics News

Pyramid-Shaped Pixels for Organic Light-Emitting Displays

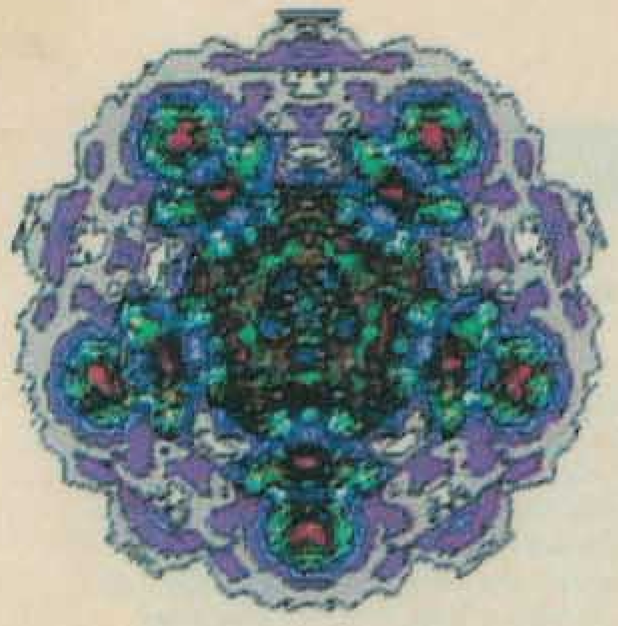
Lasers, Kaleidoscopes, and Fractals

Toward Very Large-Scale Integrated Photonics

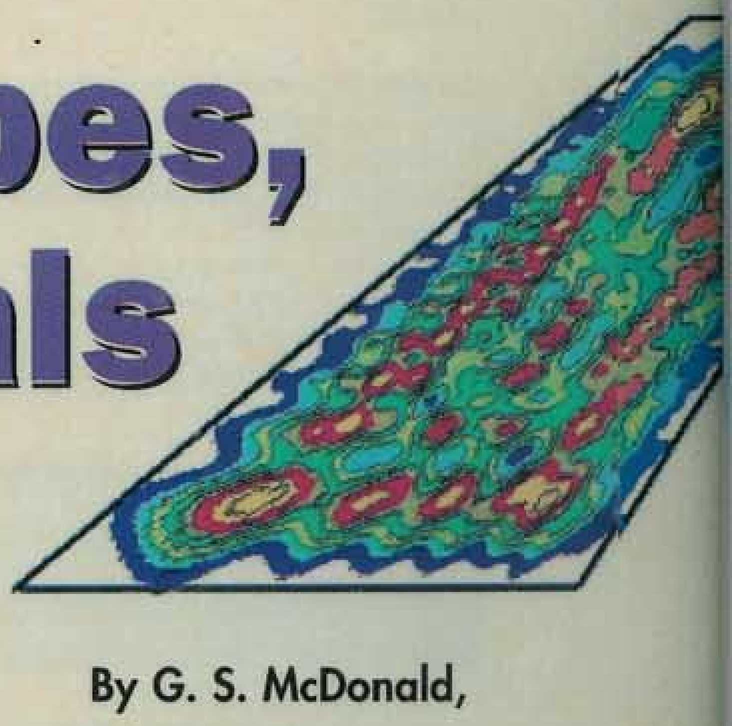
Recent Progress in Binocular Design

Diffractive Optics Beauty Contest

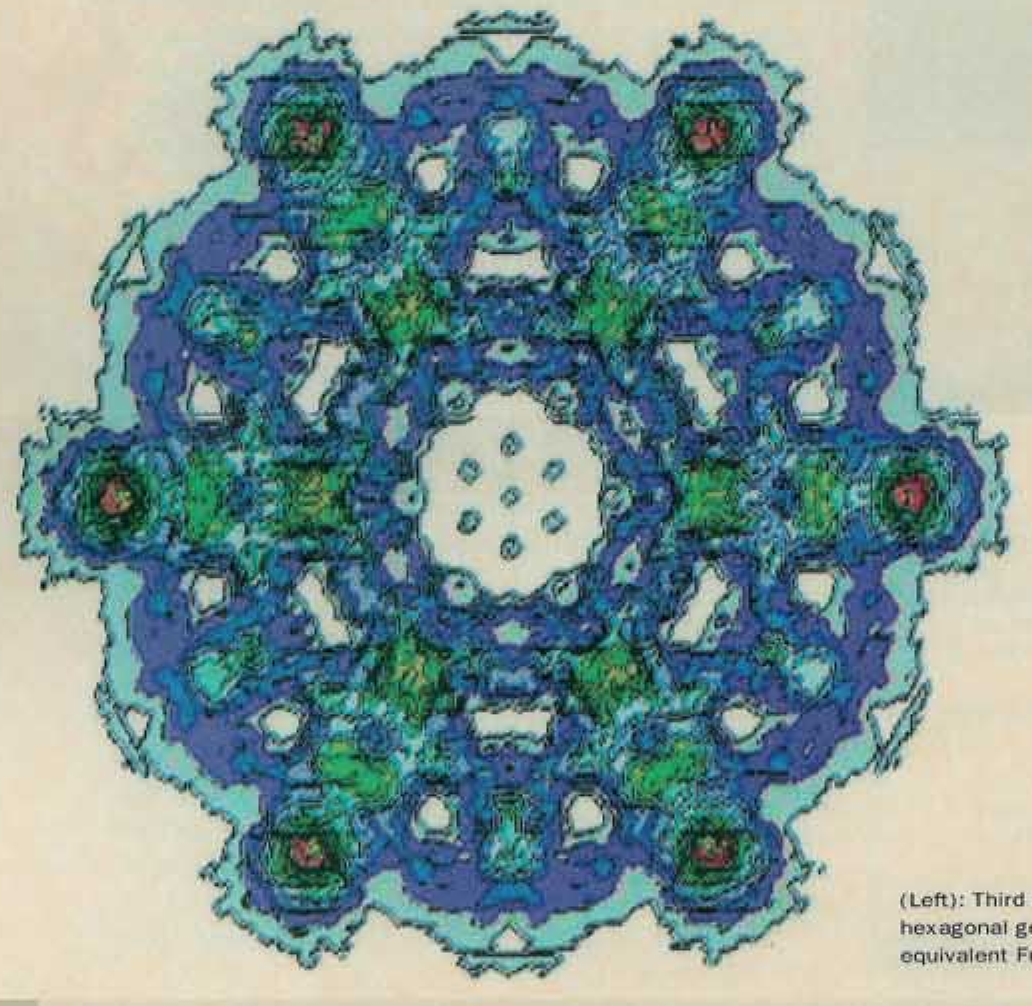
Silicon Eye Using MEMs Micro-Mirrors



Lasers, Kaleidoscopes, and Fractals



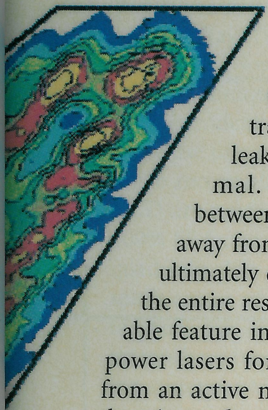
By G. S. McDonald, G. H. C. New, and J. P. Woerdman



(Left): Third member of the mode family for a cavity with hexagonal geometry, a magnification of 1.3, and an equivalent Fresnel number of 3.5.

1047-6938/00/11/0022/02-$0015.00

Glance at the mode patterns of an unstable cavity laser and the immediate impression is of their beauty and complexity. Perform a detailed mathematical examination of their structure and one discovers that the patterns possess fractal character.[1-4] We have christened the unstable cavity system as a kaleidoscope laser,[5] after the device invented by Sir David Brewster in 1816. Just as the patterns in a toy kaleidoscope vary endlessly as the tube is shaken, so the modes of the kaleidoscope laser change with the shape of the defining aperture and the dimensions of the cavity (represented by the Fresnel number). Although experiments have been concentrated on relatively low Fresnel number systems, mode profiles have been generated computationally for much higher values and for numerous aperture shapes including triangles, squares, pentagons, hexagons, octagons, circles, and rhombuses.[6] Typical modes are shown in the pictures on the left.



Most lasers are based on stable optical cavities in which the mode energy is confined to the region near the cavity axis; light rays that circulate in the cavity are then trapped forever between the mirrors, and leakage around the mirror edges is minimal. In contrast, the rays that bounce between the mirrors of an unstable cavity run away from the axis toward the mirror edges and ultimately escape. The associated cavity modes fill the entire resonator volume, which is a highly desirable feature in some contexts, for example, in high-power lasers for which maximum energy extraction from an active medium is required. Energy spillage at the mirror edges is an inherent feature of unstable cavity lasers, so it is not surprising that the shape of the mirror (or whatever component defines the aperture of the system) is critical. Indeed, the mode profiles are formed from the repeated diffraction of the field that circulates in the cavity at the aperture boundary.

Typical mode patterns of stable cavities can be found in just about any textbook on laser physics. A fascinating semi-analytical technique for calculating the mode profiles of unstable cavities was developed by Southwell in the 1980s.[7] In his so-called virtual source (VS) method, an unstable resonator is unfolded to create a corridor of virtual images of the defining aperture. A plane wave is launched into the far end of the corridor, and the mode is formed from the superposition of the patterns diffracted at the individual apertures—the virtual sources that give the technique its name. By applying a self-consistency condition, Southwell was able to obtain the mode eigenvalues that control the way in which diffraction from the different apertures combines to form the modes. The whole operation takes a second or two on a PC.

The chief limitation of the Southwell method is that it basically works only in one transverse coordinate; two-dimensional (2-D) modes can be found in just a couple of special cases—for square geometry in which 2-D modes are the product of two one-dimensional (1-D) modes, and for circular geometry in which the single transverse coordinate is the radius of the circle. We have undertaken the first investigation of 2-D modes of unstable cavity lasers in the general case. Although we have made encouraging progress in extending the VS method to two transverse coordinates, the best way of generating the 2-D modes is by brute force numerical calculation. Finding the lowest-loss mode is quite easy for almost any cavity geometry because one only needs to propagate an arbitrary initial pattern repeatedly around the cavity and wait for all the higher-loss modes to die away. This is the well-known Fox-Li method that has a history dating back to the 1960s. We have gone beyond this, however, by including a filtering operation that allows us to select different higher-order modes at will, a technique closely analogous to the insertion of a Fabry-Perot etalon into a real laser. The hexagonal mode shown is in fact the third member of the mode family.

The key question is: what is the origin of the fractal structure of the modes? In particular, how is it that a simple linear optical system that involves merely repeated diffraction leads to such an exotic characteristic? Although the complete answer to these questions awaits more detailed study, some general pointers have already been identified. First, the degree of instability in an unstable resonator is governed by its magnification factor, which determines the rate at which rays run away from the axis. This suggests a picture in which a mode is formed by the repeated superposition of similar images of different sizes, and it is then a short step to self-similarity and fractals. Although this picture does not tell the whole story, its conclusion is correct. Visual inspection of the 2-D images does indeed reveal the expected multiple scales and, when a box-counting algorithm is applied to the highly structured 1-D modes, fractional dimensions in the 1.2-1.6 range have been duly recorded.[1]

Recently, we developed a more sophisticated mathematical explanation of the fractal nature of the modes. It is well known that fractals can be characterized by their spatial power spectrum. Surprisingly, we determined that Fresnel diffraction patterns provide just the right building blocks, so that the superposition of many such patterns yields a profile with the appropriate spectral properties for fractal structure.

References

1. G. P. Karman and J. P. Woerdman, "Fractal structure of eigenmodes of unstable cavity lasers," Opt. Lett. **23**, 1909 (1998).
2. G. P. Karman, G. S. McDonald, G. H. C. New, and J. P. Woerdman, "Fractal modes in unstable resonators," Nature (London) **402**, 138 (1999).
3. "Fractals discovered in laser modes", Opto & Laser Europe (October 1999).
4. "Lasers that look like child's play," Newsline (EPSRC), Issue 15 (January, 2000).
5. G. S. McDonald, G. P. Karman, G. H. C. New, and J. P. Woerdman, "Kaleidoscope laser," J. Opt. Soc. Am. B **17**, 524 (2000).
6. G. P. Karman, G. S. McDonald, G. H. C. New, and J. P. Woerdman, "Excess noise dependence on intracavity aperture shape," Appl. Opt. **38**, 6874 (1999).
7. W. H. Southwell, "Unstable resonator mode derivation using virtual source theory," J. Opt. Soc. Am. A **3,** 1885 (1986).

G. S. McDonald is with the Joule Physics Laboratory, University of Salford, Salford, UK. He can be reached by e-mail at g.s.mcdonald@salford.ac.uk. G. H. C. New is with the Department of Physics, Imperial College, London, UK. He can be reached by e-mail at g.new@ic.ac.uk. J. P. Woerdman is with the Hyugens Laboratory, University of Leiden, Leiden, The Netherlands. He can reached by e-mail at go@rulhm1.leidenuniv.nl.