Spatial soliton collisions at arbitrary angles

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The geometries of soliton collisions reveal symmetries that cannot be reproduced by previous descriptions based on the NLS equation. We present consistent results, based on the nonlinear Helmholtz equation, valid for arbitrary angles.

Helmholtz solitons

The propagation of a CW optical beam at an arbitrary angle in a focusing Kerr medium, can be accurately described by the NHE which, when re-cast as a NNLS [1], becomes

$$\kappa\frac{\partial^2 u}{\partial \zeta^2}+i\frac{\partial u}{\partial \zeta}+\frac{1}{2}\frac{\partial^2 u}{\partial \xi^2}+|u|^2u=0.$$

The exact Helmholtz bright soliton solution, describing the propagation an angle θ in unscaled coordinates, where $\tan^2\theta=2\kappa V^2$ [1].

$$u(\xi,\zeta)=\eta\,\mathrm{sech}\left[\frac{\eta(\xi+V\zeta)}{\sqrt{1+2\kappa V^2}}\right]\exp\left[i\frac{\sqrt{1+2\kappa\eta^2}}{2\kappa}\left(\frac{-2\kappa V\xi+\zeta}{\sqrt{1+2\kappa V^2}}\right)\right]\exp\left(\frac{-i\zeta}{2\kappa}\right)$$

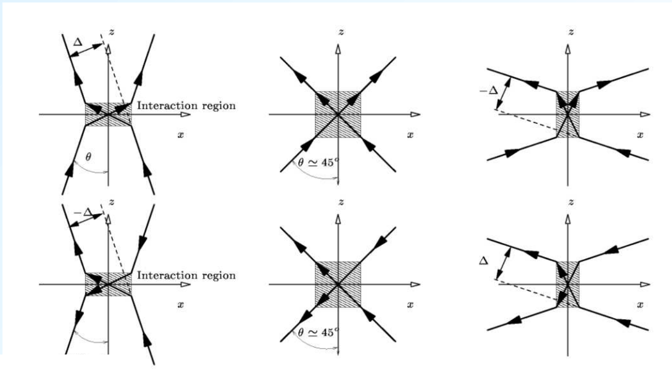
and verifications of Helmholtz solitons as robust attractors, were reported earlier [2,3].

Geometry of soliton collisions

We consider simultaneous propagation of two Kerr soliton beams, u_j and u_{3-j}, at angles θ and $-\theta$, respectively, to the z axis. The evolution of two soliton beams, for all but very small angles, is captured in the coupled system of equations

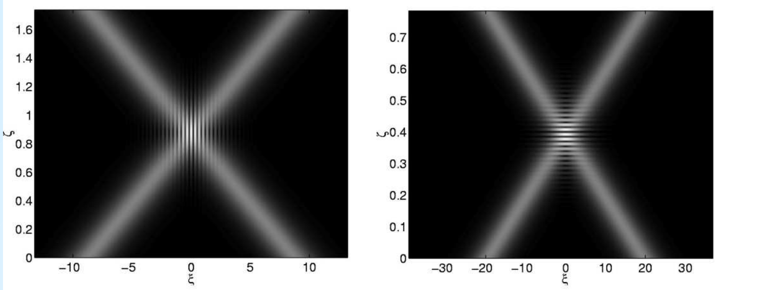
$$\kappa\frac{\partial^2 u_j}{\partial \zeta^2}+i\frac{\partial u_j}{\partial \zeta}+\frac{1}{2}\frac{\partial^2 u_j}{\partial \xi^2}+\left[|u_j|^2+(1+h)|u_{3-j}|^2\right]u_j=0$$

where j=1,2, and h has limits 0 and 1 for incoherent and coherent collisions, respectively. Helmholtz soliton collisions thus involve interactions governed by beam intensities, and result in trajectory phase shifts Δ in the beam paths, as defined in the figure. While the NLS predicts a monotonically decreasing value of Δ with growing θ [4], Δ should vanish at θ=45º for Helmholtz solitons, since no transverse force acts on either of the beams. Moreover, the value of Δ at 90º-θ should correspond to that of θ, except for a change in sign. Hence, rotation of each copropagation configuration by 90º permits one to obtain an equivalent description of corresponding counterpropagating solitons. In each case, soliton trajectories result in the same intensity traces.

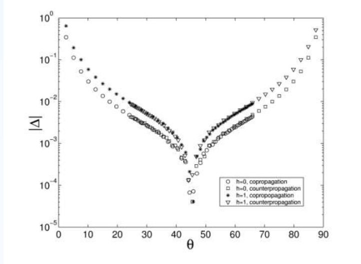


Numerical results

The true geometry of soliton collisions also suggests a possible computation scheme, suitable to model particularly large collision angles. Solutions of the NNLS and the numerical algorithms used [5] can describe both forward and backward propagating waves, permitting the use of boundary conditions for numerical integration that consist of both input and output solitons. The global solution can also be rotated by 90º to obtain results for equivalent propagation geometries. The figure shows the intensity patterns for configurations with θ=24.095º and 90º-θ=24.095º, i.e. for copropagating and counterpropagating cases, respectively. As expected on physical grounds, the orientation of the index grating changes from the transverse to the propagation direction when switching to counterpropagating beams.

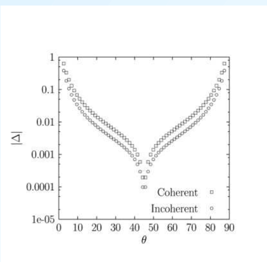


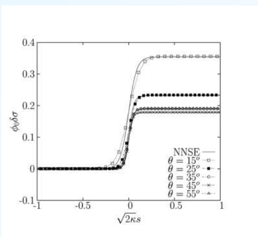
The figure shows Δ arising from coherent and incoherent collisions. Helmholtz results are in perfect agreement with the symmetry of the problem, and testify to the validity of the computations in the counterpropagation configurations



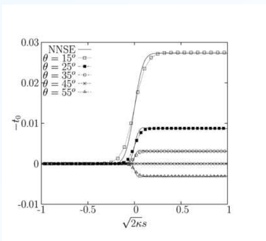
Quasi-particle theory

The figure below reports the behaviour of Δ as predicted by using an adiabatic perturbation approach (full details will be presented at CLEO: oral session IML 5/17). Also shown are the deviations (from unperturbed propagation) of soliton position and phase, as obtained from the analytical approach (points) and from full numerical integration of equations (solid lines).





Soliton phase



Soliton position

References

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