# Families of Two-Colour Helmholtz Spatial Solitons

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Multi-colour spatial solitons comprise localized optical components at distinct temporal frequencies [1]. The components (which may be bright-like and dark-like) tend to overlap in space, thereby allowing the interplay between linear spreading (diffraction) and nonlinear effects (self- and mutual-focusing) to result in an electromagnetic structure with a stationary intensity pattern. Two-colour spatial solitons for a Kerr-type medium were proposed by De La Fuenete and Barthelemy [2] within the context of an intuitive nonlinear Schrödinger model. Subsequent experiments, using continuous-wave (CW) laser light at red and green wavelengths, demonstrated that such mutually-trapped light beams could be generated in $CS_2$ waveguides [3]. This opened up the possibility of multi-colour photonic device applications and architectures [4].

Here, we introduce a novel Helmholtz model for two-colour CW optical fields whose temporal frequency separation is similarly large. A key advantage of our approach is that it allows one full access to multi-component geometries involving propagation at arbitrary angles and orientations with respect to the reference direction [5] – such considerations are central to off-axis configurations involving, for instance, beam multiplexing [6] and interface [7] scenarios. In contrast, classic paraxial models [2,3] capture angles (in the laboratory frame) that are negligibly, or near-negligibly, small [4]. The two-colour modulational instability problem can be solved in a range of physically relevant regimes. Bright-bright and bright-dark solitons are also reported, each of which having *co-propagation* and *counter-propagation* solution classes that are connected by geometrical transformation. Extensive computations [8] have confirmed the validity of analyses (see Fig. 1).

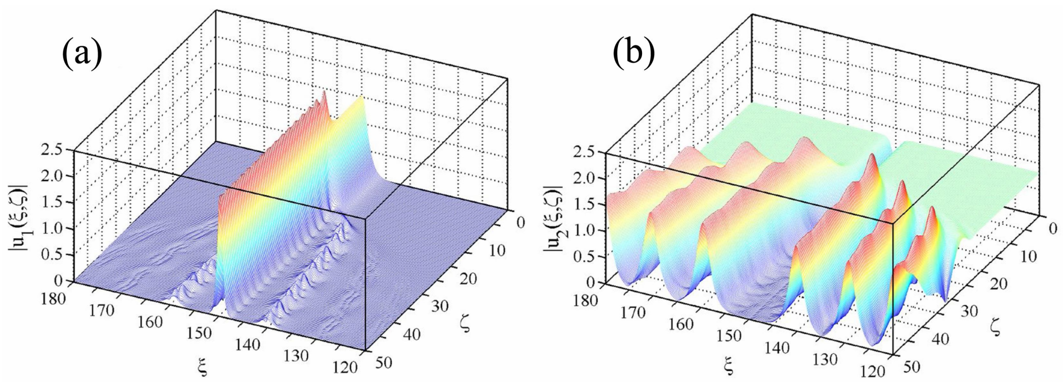


**Fig. 1** Inherent instability of the exact analytical bright-dark Helmholtz soliton – bright component shown in (a), dark component in (b). Modulational instability develops initially on the finite-amplitude plane-wave background of the dark component in $u_2$, leading to filamentation [the dominant spatial frequency in part (b) is predicted by linear analysis]. This instability then feeds through the system, via nonlinear coupling, to destabilize the bright component $u_1$.

## References

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