# Module M2.5 Working with vectors



Exit module

# 1 Opening items

## 1.1 Module introduction

When modelling the physical world mathematically we need to use quantities that can be measured. Such physical quantities mainly fall into two classes, *scalars* and *vectors*. The mathematics of scalars such as mass and temperature is simply the familiar mathematics of real numbers and functions, but the mathematics of vector quantities such as force and velocity, which have both a *magnitude* and a *direction*, is quite different, and is the subject of this module. Vectors are often introduced as geometric entities — more like arrows than numbers. Our main purpose in this module is to show you how the study of vectors may be transformed from geometry to algebra.

Section 2 of this module reviews the definition of scalar and vector quantities, the graphical (or geometric) representation of vectors, and the notation used in referring to vector quantities. (A more detailed introduction is given elsewhere in *FLAP*.) This section also reviews the basic operations of *scaling*, *vector addition* and *vector subtraction* in terms of the geometric representation. The section ends with a review of the *resolution* of vector quantities into *component vectors*, using the geometric representation.

Section 3 introduces the use of *unit vectors* to denote directions and, in particular, the use of $\boldsymbol{i}$, $\boldsymbol{j}$, and $\boldsymbol{k}$ to specify the directions of the $x$-, $y$-, and $z$-axes in a right-handed Cartesian coordinate system. Section 3 continues by showing how these Cartesian unit vectors may be used to represent any vector in three dimensions, and then how vectors represented in *Cartesian form* may be manipulated in some of the basic operations of vector algebra, such as scaling and vector addition.

Section 4 introduces an alternative algebraic representation of vector quantities in terms of *ordered triples* of numbers. Sections 3 and 4 each conclude with a specific example which illustrates how, in practice, vector operations may be carried out using the Cartesian form, or the ordered triple representations, respectively.

***Study comment*** Having read the introduction you may feel that you are already familiar with the material covered by this module and that you do not need to study it. If so, try the *Fast track questions* given in Subsection 1.2. If not, proceed directly to *Ready to study?* in Subsection 1.3.

## 1.2 Fast track questions

***Study comment*** Can you answer the following *Fast track questions*? If you answer the questions successfully you need only glance through the module before looking at the *Module summary* (Subsection 5.1) and the *Achievements* listed in Subsection 5.2. If you are sure that you can meet each of these achievements, try the *Exit test* in Subsection 5.3. If you have difficulty with only one or two of the questions you should follow the guidance given in the answers and read the relevant parts of the module. However, *if you have difficulty with more than two of the Exit questions you are strongly advised to study the whole module*.

### Question F1

Three forces $\boldsymbol{F}_1$, $\boldsymbol{F}_2$ and $\boldsymbol{F}_3$ acting on a particle are such as to keep the particle in equilibrium. $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ are given as $\boldsymbol{F}_1 = (-\boldsymbol{i} + 2\boldsymbol{j} + 4\boldsymbol{k})$ newtons and $\boldsymbol{F}_2 = (-\boldsymbol{i} - 5\boldsymbol{k})$ newtons.

(a) Find the vector $\boldsymbol{F}_3$.

(b) Suppose the magnitude of $\boldsymbol{F}_1$ is doubled, and the magnitude of $\boldsymbol{F}_3$ is tripled (without changing their directions), while $\boldsymbol{F}_2$ remains unchanged. Find the new resultant force acting on the particle, the magnitude of this resultant force, and the unit vector acting in the direction of this resultant force.

## Question F2

In the absence of an air current, the velocity of a particle is given by $\boldsymbol{v}_{\rm p} = (2, 2, -1)\,{\rm m\,s^{-1}}$. Find the resultant velocity of the particle, and the time taken for the particle to travel a distance of 5 m along its resultant path, when an air current of velocity $\boldsymbol{v}_{\rm c} = (-1, 1, 3)\,{\rm m\,s^{-1}}$ is present.

***Study comment***

Having seen the *Fast track questions* you may feel that it would be wiser to follow the normal route through the module and to proceed directly to *Ready to study?* in Subsection 1.3.

Alternatively, you may still be sufficiently comfortable with the material covered by the module to proceed directly to the *Closing items*.

## 1.3 Ready to study?

***Study comment*** Although the basic concepts of *scalar* and *vector* quantities are reviewed briefly in this module, it is assumed that you have some knowledge of these already. For example, you should be familiar with the definitions and methods of representation of *scalar* and *vector quantities*, and terms such as *zero vector*. You should understand the ideas of the *scaling of a vector quantity* by a scalar, the *addition* and *subtraction of vectors*, and the resolution of vectors into *component vectors*. Also, you should be familiar with the terms: *acceleration*, *displacement*, *electric charge*, *mass*, *momentum*, *temperature* and *velocity.* In addition, it is assumed that you have met *Newton's laws of motion*, and that you appreciate the conditions required for a particle to be in *equilibrium*. You should be familiar with basic mathematical concepts including the ideas of *Cartesian coordinate systems*, *Pythagoras's theorem* and the use of *trigonometric functions.* You should understand also the ways in which directions may be given with respect to points of the compass. If you are uncertain about any of these terms then you can review them by reference to the *Glossary* which will indicate also where in *FLAP* they are developed. The following *Ready to study questions* will allow you to establish whether you need to review some of the topics before embarking on this module.

### Question R1

A right-angled triangle ABC has sides AB and BC of length 12 cm and 5 cm, respectively. Angle $\mathrm{A\hat{B}C}$ is 90°. Determine the angles $\mathrm{B\hat{A}C}$ and $\mathrm{A\hat{C}B}$ and the length of the side AC.

### Question R2

A ladder 5 m long stands on level ground with its highest point against a wall at a point 4.5 m above the ground. Determine the angle of inclination of the ladder to the ground, and the distance of the foot of the ladder from the bottom of the wall.

### Question R3

Points A and B are given by their Cartesian coordinates in two dimensions as (1, 2) and (4, 3), respectively. Determine the lengths OA, OB and AB, and the angle $\mathrm{A\hat{O}B}$, where O is the origin of the coordinate system.

# 2 A brief review of scalars and vectors

## 2.1 What are scalars and vectors?

A **vector** is a mathematical object that has both *magnitude* and *direction*. Vectors may be represented diagramatically by *arrows* or *directed line segments*, as shown in Figure 1 where the magnitude and direction of a vector are indicated by the length and orientation of an arrow or line segment. These graphical representations of vectors are sometimes referred to as **geometric vectors**.

It is customary to use distinctive notation when referring to vectors in order to emphasize the importance of their direction as well as their magnitude. For example, the vector from A to B in Figure 1 can be denoted by $\overrightarrow{AB}$ and, similarly, the vector from C to D can be denoted $\overrightarrow{CD}$. Alternatively, and more commonly, we might denote the vector $\overrightarrow{AB}$ by $\boldsymbol{a}$, where the bold typeface warns us that $\boldsymbol{a}$ is a vector with direction as well as magnitude.

It should be noted that geometric vectors are completely specified by their magnitude and direction so $\overrightarrow{AB} = \overrightarrow{CD}$, even though they have been drawn at different points on the page.

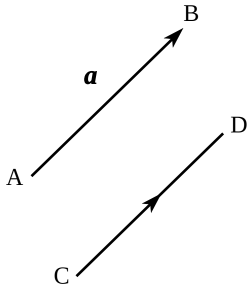



**Figure 1** Vector $\boldsymbol{a}$ or $\overrightarrow{AB}$ is represented by an arrow, and $\overrightarrow{CD}$ by a directed line segment. Notice that while the vectors $\overrightarrow{AB}$ and $\overrightarrow{BA}$ have the same magnitude, they do not have the same direction, and they are therefore distinct vectors.

The sum of two vectors is determined by the **triangle rule**, see Figure 2a, and the **vector sum** $\boldsymbol{a} + \boldsymbol{b}$ is often known as the **resultant vector**. The result of adding any number of vectors, such as $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$, and $\boldsymbol{d}$ in Figure 2b, is obtained by simply joining them ‘nose to tail’.

While ‘geometric’ vectors are valuable concepts, their practical use is limited by the difficulty of drawing such objects in three dimensions, and the major purpose of this module is to show you how a geometric construction like that shown in Figure 2b for the sum of four ‘geometric’ vectors can be replaced by a simple form of algebra.

## Scalar quantities

Many physical quantities can be specified completely by a single number together with an appropriate unit of measurement, for example length, mass, time and temperature. Such quantities are called **scalar quantities** or simply **scalars**, and we commonly denote them by symbols such as $L$, $m$, $t$ and $T$.

☞

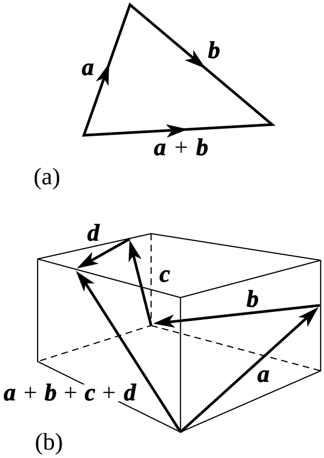



**Figure 2** (a) The sum of two vectors obtained from the triangle rule. (b) The sum of four vectors (in three dimensions).

## Vector quantities

There are many physical quantities, for example *displacement*, *velocity*, *acceleration* and *force*, which cannot be completely defined by their magnitudes (in the appropriate units); each of them is only really specified if we are also told the direction in which they act. The essence of such quantities is that they have both magnitude *and* direction, where the **magnitude** is a non-negative scalar quantity which gives the ‘size’ of the quantity.

> **Vector quantities** are physical quantities that have both magnitude *and* direction (and which can be adequately represented by geometric vectors).

## Notation

In this module (and generally in *FLAP*) we will follow the standard convention of using bold type to indicate vectors, but in handwritten work it is usual to indicate a vector, $\boldsymbol{a}$, by a wavy underline, $\underset{\sim}{a}$, ☞ while a geometric vector or displacement vector from A to B may be written $\overrightarrow{AB}$.

In *FLAP* we usually denote the magnitude of a vector $\boldsymbol{a}$ by $|\,\boldsymbol{a}\,|$, while in your written work you should use $|\,\underset{\sim}{a}\,|$. (It is important that you remember to include the underline to denote a vector, and the vertical lines for a magnitude, in your written work.) ☞

You will also encounter an alternative notation, used by many authors, in which the magnitude of the vector $\boldsymbol{a}$ is denoted by an unemboldened $a$.

✦ Which of the following are vector quantities?

(a) Your weight, (b) the pressure at a certain depth in the sea, (c) a charging rhinoceros.

✦ Is it generally true that $|\,\boldsymbol{a} + \boldsymbol{b}\,| = |\,\boldsymbol{a}\,| + |\,\boldsymbol{b}\,|$?

## 2.2 Scaling a vector, unit vectors and the zero vector

In order to **scale** a vector $\boldsymbol{a}$ by a factor 2 say, we simply double its length, and denote the result by $2\boldsymbol{a}$, which is compatible with the triangle rule of addition because (as we would expect) we then have $\boldsymbol{a} + \boldsymbol{a} = 2\boldsymbol{a}$.

The scaling factor may be positive or negative, so that, for example, if $\boldsymbol{s}$ is a displacement of 10 km north, then $3\boldsymbol{s}$ is a displacement of 30 km north; $-2\boldsymbol{s}$ is a displacement of 20 km south, and dividing by 2, we have $\frac{1}{2}\boldsymbol{s}$, or $\boldsymbol{s}/2$, a displacement of 5 km north.

## Unit vectors

If $\boldsymbol{s}$ is a displacement of 10 km north, then dividing $\boldsymbol{s}$ by 10 km produces a vector of magnitude 1 in the northerly direction.

In general, if we divide an arbitrary vector $\boldsymbol{a}$ by its magnitude we obtain a (dimensionless) vector in the same direction as $\boldsymbol{a}$ and of magnitude 1 (a dimensionless number). Such a vector is known as a **unit vector** in the direction of the vector $\boldsymbol{a}$, and is denoted by $\hat{\boldsymbol{a}}$, so that

$$\hat{\boldsymbol{a}} = \frac{\boldsymbol{a}}{|\boldsymbol{a}|} \tag{1}$$

❖ If $\hat{\boldsymbol{n}}$ is a unit vector pointing north, and $\hat{\boldsymbol{w}}$ is a unit vector pointing west, what is the magnitude of $\hat{\boldsymbol{n}} + \hat{\boldsymbol{w}}$?

### Question T1

If $\boldsymbol{F}$ represents a force directed vertically downwards, find a unit vector directed vertically upwards. ❏

## The zero vector

We are intent on establishing an algebra of vectors, so clearly one of our requirements is a **zero vector**, $\mathbf{0}$, which is defined so that

$$\boldsymbol{a} + (-1)\boldsymbol{a} = \mathbf{0} \qquad (2)$$

for any vector $\boldsymbol{a}$. The zero vector has magnitude zero, and its direction is undefined (since it is irrelevant). You may have noticed that we have written zero as a vector, since the sum of two vectors must of course be a vector, and you should use a wavy underline with every occurrence of the zero vector in your written work.

## 2.3 Addition and subtraction of vectors

The result of adding two vector quantities, say $\boldsymbol{a}$ and $\boldsymbol{b}$, is determined by the triangle rule of Figure 2a, or equivalently by the **parallelogram rule** illustrated in Figure 3a and is of the same physical type as the original vectors. (The choice of rule is merely a matter of taste.)

We follow the practice of ordinary algebra and usually replace $(-1)\boldsymbol{a}$ by $-\boldsymbol{a}$. The vector $-\boldsymbol{a}$ has the same magnitude as the vector $\boldsymbol{a}$, but it acts in the opposite direction (it is *antiparallel* to $\boldsymbol{a}$), for example, in Figure 3b $\overrightarrow{JN} = -\overrightarrow{NJ}$. One further point, which we take for granted in ordinary algebra, is that the order in which we add vectors is immaterial, so

$$\boldsymbol{a} + \boldsymbol{b} = \boldsymbol{b} + \boldsymbol{a} \tag{3}$$

and

$$\boldsymbol{a} + (\boldsymbol{b} + \boldsymbol{c}) = (\boldsymbol{a} + \boldsymbol{b}) + \boldsymbol{c} \tag{4}$$

*Note* A mathematician would recognize Equations 3 and 4 as the *commutative* and *associative* properties of *vector addition*.

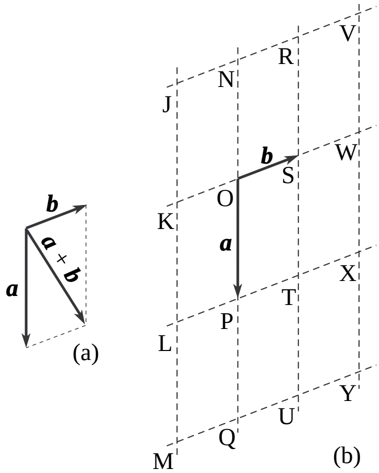



**Figure 3** (a) The parallelogram rule of addition. (b) An array of points defining many different vectors.

Vector subtraction is achieved by scaling a vector by –1 and then using vector addition as before. For example, in Figure 3b $\overrightarrow{OS} = \boldsymbol{b}$ and therefore $\overrightarrow{SO} = -\boldsymbol{b}$, then $\overrightarrow{SP} = \overrightarrow{OP} + \overrightarrow{SO} = \boldsymbol{a} + (-\boldsymbol{b}) = \boldsymbol{a} - \boldsymbol{b}$

✥ With reference to Figure 3:

(a) Express the vector $\overrightarrow{JW}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

(b) What is the final position of an object which is placed at R and then displaced by the vector $3\boldsymbol{a} - 2\boldsymbol{b}$?

(c) Write down two vectors equal to $2\boldsymbol{a} + 2\boldsymbol{b}$.

Remember that the position in which a geometric vector is drawn is not relevant, and the vectors $\overrightarrow{NX}$ and $\overrightarrow{KU}$ in Figure 3 are equal, in spite of the fact that they are defined by distinct line segments (and the vectors $\overrightarrow{JT}$ and $\overrightarrow{OY}$ would do equally well). Notice also $2\boldsymbol{a} + 2\boldsymbol{b} = 2(\boldsymbol{a} + \boldsymbol{b})$, which is a particular case of a more general rule, which states that for any scalar $k$

$$k\boldsymbol{a} + k\boldsymbol{b} = k(\boldsymbol{a} + \boldsymbol{b}) \qquad (5)$$

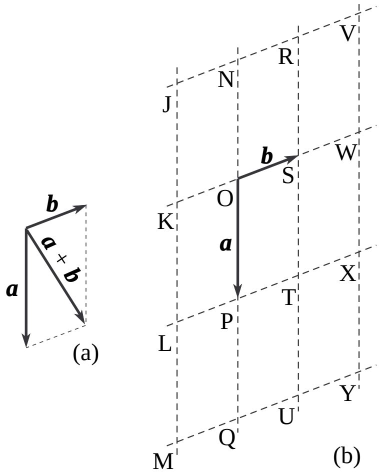



**Figure 3** (a) The parallelogram rule of addition. (b) An array of points defining many different vectors.

**Question T2**

With reference to Figure 3:

(a) Express the vectors $\overrightarrow{KU}$, $\overrightarrow{WL}$ and $\overrightarrow{YK}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

(b) Write down two vectors equal to $2\boldsymbol{a} - 3\boldsymbol{b}$. ❏

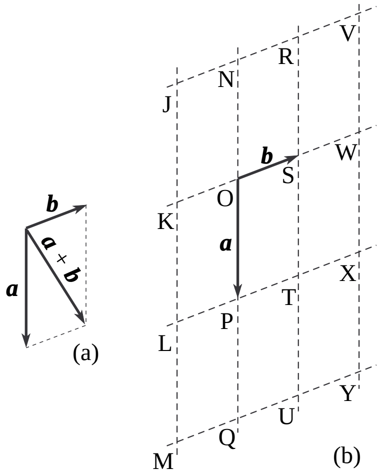
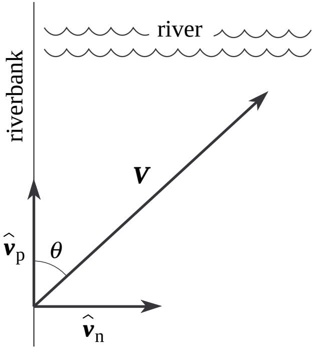



**Figure 3** (a) The parallelogram rule of addition. (b) An array of points defining many different vectors.

## 2.4 Resolution of vectors, and component vectors

Often when dealing with a vector quantity, we need to know how much the vector contributes in a given direction. For example, consider the two-dimensional situation in which a boat moves with velocity $\boldsymbol{V}$ across a river at an angle $\theta$ to the bank, as shown in Figure 4a. To find how long the boat takes to cross the river we need to find the effective velocity normal to (i.e. perpendicular to) the river bank. This can be obtained by treating the vector $\boldsymbol{V}$ as the vector sum of two orthogonal ☞ **component vectors** normal to the bank and parallel to it. ☞




(a)

**Figure 4** (a) Velocity $\boldsymbol{V}$ at an angle $\theta$ to the river bank, with unit vectors $\hat{\boldsymbol{v}}_{\rm n}$ and $\hat{\boldsymbol{v}}_{\rm p}$.

If $\hat{\boldsymbol{v}}_{\rm n}$ and $\hat{\boldsymbol{v}}_{\rm p}$ are unit vectors normal to, and parallel to the bank, respectively, then $\boldsymbol{V}$ can be written as the sum of two component vectors

so that $\quad \boldsymbol{V} = |\,\boldsymbol{V}\,|\sin\theta\,\hat{\boldsymbol{v}}_{\rm n} + |\,\boldsymbol{V}\,|\cos\theta\,\hat{\boldsymbol{v}}_{\rm p}$

The component vectors of $\boldsymbol{V}$ are in this case (see Figure 4b): $|\,\boldsymbol{V}\,|\sin\theta\;\hat{\boldsymbol{v}}_{\rm n}$ in a direction normal to the bank, and of magnitude $|\,\boldsymbol{V}\,|\sin\theta$, and, $|\,\boldsymbol{V}\,|\cos\theta\;\hat{\boldsymbol{v}}_{\rm p}$ in a direction parallel to the bank, and of magnitude $|\,\boldsymbol{V}\,|\cos\theta$.

If the width of the river is $d$, we can now see that the time taken for the boat to cross is $d/(|\,\boldsymbol{V}\,|\sin\theta)$.

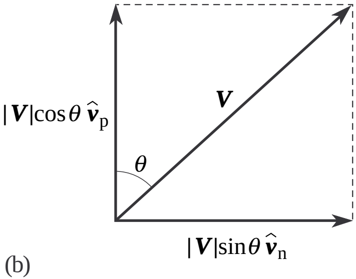



**Figure 4** (b) Velocity $\boldsymbol{V}$ resolved into component velocities $|\,V\,|\sin\theta\;\hat{\boldsymbol{v}}_{\rm n}$ and $|\,V\,|\cos\theta\;\hat{\boldsymbol{v}}_{\rm p}$ normal to, and parallel to, the river bank.

This process of **resolution** (i.e. splitting) into *component vectors* can be thought of as the reverse of vector addition; instead of adding two (or more ☞) vectors to produce a single resultant vector, we are replacing a single vector by the sum of two orthogonal *component vectors*. Note that when these component vectors are added together we obtain our original vector.

✦ The string of a simple pendulum exerts a force $\boldsymbol{F}$ of magnitude 15 N on the pendulum bob, at an angle of 15° to the (upward) vertical. Resolve this force into horizontal and vertical component vectors.

**Question T3**

A block of wood lies on a rough plane which is inclined at an angle of 25° to the horizontal. It is prevented from sliding down the plane by a static frictional force of magnitude 20 N acting up the plane. Find the horizontal and vertical component vectors of that frictional force. ❏

# 3 Vectors in a Cartesian coordinate system

In the previous section you saw how it was possible to represent the velocity of a boat in terms of the unit vectors $\hat{\boldsymbol{v}}_{\rm n}$ and $\hat{\boldsymbol{v}}_{\rm p}$ normal to, and parallel to, the bank of a river. This same idea can be extended to three dimensions, and provides a very convenient means of specifying three-dimensional vectors.

## 3.1 Cartesian unit vectors

The use of unit vectors is of particular importance in specifying the positive directions of the axes in a Cartesian coordinate system. In two dimensions the **Cartesian unit vectors** $\boldsymbol{i}$ and $\boldsymbol{j}$ are used to specify the directions of the *x*- and *y*-axes as shown in Figure 5.

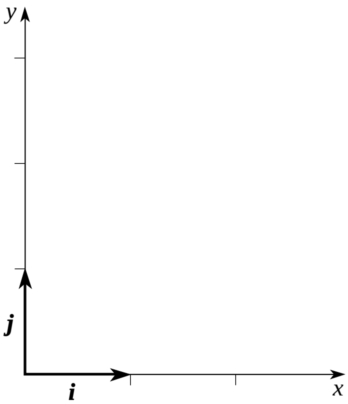



**Figure 5** Unit vectors $\boldsymbol{i}$ and $\boldsymbol{j}$ are directed along the *x*- and *y*-axes.

In three dimensions a third Cartesian unit vector $\boldsymbol{k}$ is added to specify the direction of the *z*-axis. However, there are two possible directions, opposite to one another, in which the *z*-axis can be directed so that it is normal to both the *x*- and *y*-axes. To remove any ambiguity about the choice of direction of the *z*-axis a *right-handed* Cartesian coordinate system is almost invariably used. There are various ways of describing this system, but one simple method only is included here.

If the thumb and first two fingers of your *right* hand are arranged approximately mutually perpendicular as in Figure 6, then, if the first and second finger point along the $x$- and $y$-axes, respectively, the thumb points along the $z$-axis in a right-handed system.

✥ Which of the following is a right-handed coordinate system:

(a) $\boldsymbol{i}$ points west, $\boldsymbol{j}$ points north and $\boldsymbol{k}$ points vertically upward,

(b) $\boldsymbol{i}$ is vertically downward, $\boldsymbol{j}$ points south and $\boldsymbol{k}$ points west?

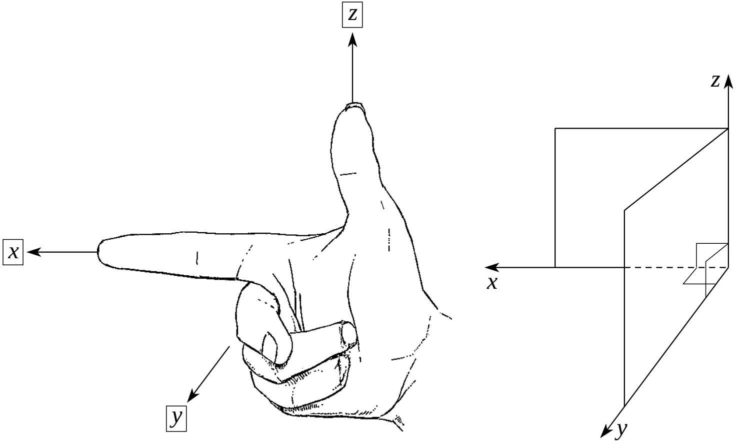



**Figure 6** Directions of the axes in a right-handed Cartesian coordinate system.

Now let us consider how these Cartesian unit vectors may be used to specify the Cartesian component vectors of any vector $\boldsymbol{a}$ in the $(x, y)$ plane, i.e. an arbitrary vector in two dimensions. The vector $\boldsymbol{a}$ can be resolved into two orthogonal component vectors parallel to the $x$- and $y$-axes, respectively, as shown in Figure 7. If $a_x$ and $a_y$ are the coordinates of the point A, then the component vectors will have magnitudes $a_x$ and $a_y$. Hence the component vectors will be $a_x\boldsymbol{i}$ and $a_y\,\boldsymbol{j}$. From vector addition, $\boldsymbol{a}$ can be expressed as:

$$\boldsymbol{a} = a_x\boldsymbol{i} + a_y\,\boldsymbol{j} \tag{6}$$

The important point to notice about Equation 6 is that, once we have specified the directions of $\boldsymbol{i}$ and $\boldsymbol{j}$ the vector $\boldsymbol{a}$ is completely determined (both in magnitude and direction) by the pair of scalar quantities $a_x$ and $a_y$.

The values $a_x$ and $a_y$ are known as the **Cartesian scalar components** of $\boldsymbol{a}$ (or simply the components of $\boldsymbol{a}$), $a_x\,\boldsymbol{i}$ and $a_y\,\boldsymbol{j}$ are the **Cartesian component vectors** of $\boldsymbol{a}$ (or simply the component vectors of $\boldsymbol{a}$).

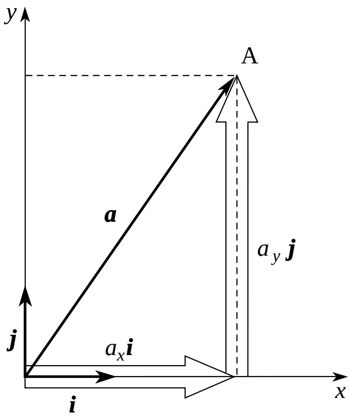



**Figure 7** The component vectors of vector $a$ in terms of $\boldsymbol{i}$ and $\boldsymbol{j}$.

We obtain the full benefit from this method of representing vectors when the idea is extended to three dimensions. In Figure 8 we show how a vector $\boldsymbol{a}$ in three dimensions can be resolved into three orthogonal component vectors parallel to the $x$-, $y$- and $z$-axes. If $a_x$, $a_y$ and $a_z$ are the coordinates of the point A, then

$$\boldsymbol{a} = a_x\,\boldsymbol{i} + a_y\,\boldsymbol{j} + a_z\,\boldsymbol{k} \tag{7}$$

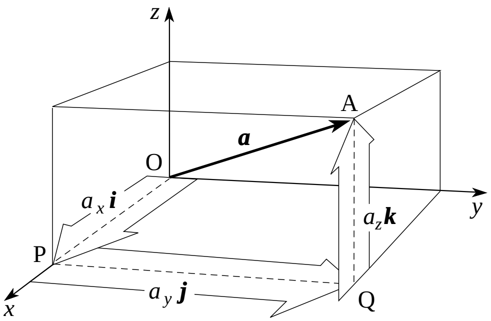



**Figure 8** The Cartesian component vectors of a vector $a$ in terms of $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$.

In this case the Cartesian component vectors of $\boldsymbol{a}$ are

$a_x\,\boldsymbol{i}$, $a_y\boldsymbol{j}$ and $a_z\,\boldsymbol{k}$

while the Cartesian scalar components of $\boldsymbol{a}$ are

$a_x$, $a_y$ and $a_z$ and when we write

$$\boldsymbol{a} = a_x\,\boldsymbol{i} + a_y\boldsymbol{j} + a_z\,\boldsymbol{k}$$

we say that $\boldsymbol{a}$ is expressed in **Cartesian form**.

The suffix notation used in the above definitions is quite convenient, and it is natural to express any given vector, $\boldsymbol{r}$ say, in the same form. Thus $r_x\,\boldsymbol{i}$ is the natural choice of notation for the vector component ☞ in the $x$-direction of the vector $\boldsymbol{r}$, while $r_y\,\boldsymbol{j}$ and $r_z\,\boldsymbol{k}$ denote its $y$- and $z$-component vectors.

Most of us have great difficulty visualizing objects, and their relative distances from each other, in three dimensions; vector methods largely overcome these problems.

✧ A portable rectangular cabin is 3 m wide, 5 m long and 2.5 m high, and it is placed on a horizontal foundation with its long axis pointing north/south. An origin is chosen in the bottom south-west corner of the cabin, with $\boldsymbol{i}$ pointing east, $\boldsymbol{j}$ pointing north, and $\boldsymbol{k}$ pointing vertically upward.

What vector displacement $\boldsymbol{r}$ is required to move an object from the origin to a point at the centre of the ceiling?

Once we have found the vector displacement $\boldsymbol{r}$ from the origin to the centre of the ceiling, then it is possible to find displacements *from this new point*. For example, the vector displacement of the opposite corner of the floor of the cabin from the origin is $\boldsymbol{a} = 3\boldsymbol{i} + 5\boldsymbol{j}$, and therefore the displacement of the opposite corner of the cabin from the centre of the ceiling is ☞

$$-\boldsymbol{r} + \boldsymbol{a} = -(1.5\boldsymbol{i} + 2.5\boldsymbol{j} + 2.5\boldsymbol{k})\,\mathrm{m} + (3\boldsymbol{i} + 5\boldsymbol{j})\,\mathrm{m} = (1.5\boldsymbol{i} + 2.5\boldsymbol{j} - 2.5\boldsymbol{k})\,\mathrm{m}$$

A portable rectangular cabin is 3 m wide, 5 m long and 2.5 m high, and it is placed on a horizontal foundation with its long axis pointing north/south. An origin is chosen in the bottom south-west corner of the cabin, with $\boldsymbol{i}$ pointing east, $\boldsymbol{j}$ pointing north, and $\boldsymbol{k}$ pointing vertically upward.

**Question T4**

Is the suggested coordinate system for the cabin described above right-handed? What vector displacement $\boldsymbol{s}$ will move an object from the centre of the ceiling to the mid-point of the northern wall? ❏

## 3.2 The magnitude and combination of vectors in Cartesian form

We can regard vectors in two dimensions as merely a special case of vectors in three dimensions; their $z$-component just happens to be zero. So hereafter our results apply equally well to vectors in two or three dimensions.

It is convenient to be able to determine the magnitude of a vector in terms of its Cartesian components, and this may be easily done if we apply Pythagoras's theorem to vector $\boldsymbol{a}$ in Figure 8. First we consider the $(x, y)$ plane only. Triangle OPQ is a right-angled triangle and so:

$$\mathrm{OQ}^2 = \mathrm{OP}^2 + \mathrm{PQ}^2$$

But $\mathrm{OP} = a_x$ and $\mathrm{PQ} = a_y$, and so:

$$\mathrm{OQ}^2 = a_x^2 + a_y^2$$

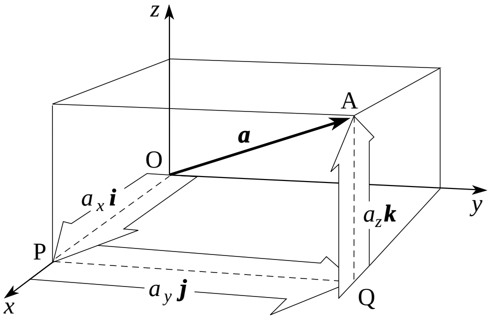



**Figure 8** The Cartesian component vectors of a vector $a$ in terms of $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$.

Now we consider the right-angled triangle OQA:

$$\mathrm{OA}^2 = \mathrm{OQ}^2 + \mathrm{QA}^2$$

But $\quad \mathrm{OQ}^2 = a_x^2 + a_y^2 \quad$ and $\quad \mathrm{QA}^2 = a_z^2$

and so: $\quad \mathrm{OA}^2 = a_x^2 + a_y^2 + a_z^2$

But $\quad \mathrm{OA} = |\,\boldsymbol{a}\,|$

therefore:

$$|\,\boldsymbol{a}\,| = (a_x^2 + a_y^2 + a_z^2)^{1/2} \qquad (8)$$

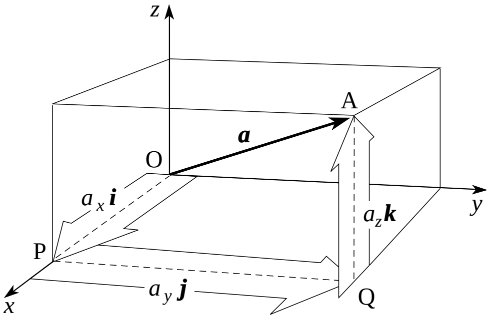



**Figure 8** The Cartesian component vectors of a vector $a$ in terms of $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$.

✥

Given that $\boldsymbol{a} = 2\boldsymbol{i} + 3\boldsymbol{j} + 4\boldsymbol{k}$ calculate $|\,\boldsymbol{a}\,|$.

**Question T5**

In Question T4 you were asked to determine a vector displacement $\boldsymbol{s}$.

$$\boldsymbol{s} = = (2.5\boldsymbol{j} - 1.25\boldsymbol{k})\,\text{m}$$

Now determine the magnitude of that displacement. ❑

As we saw in Subsection 2.2, the scaling of a vector is simply multiplication of the vector by a scalar quantity, and this operation is very straightforward when a vector is expressed in Cartesian form. For example let $\alpha$ ☞ be a scalar quantity and let $\boldsymbol{a} = a_x\,\boldsymbol{i} + a_y\,\boldsymbol{j} + a_z\,\boldsymbol{k}$, then:

$$\boxed{\alpha\,\boldsymbol{a} = \alpha\,a_x\,\boldsymbol{i} + \alpha\,a_y\,\boldsymbol{j} + \alpha\,a_z\,\boldsymbol{k} \qquad (9)}$$

The sum of two vectors in Cartesian form is equally easy.

Let $\boldsymbol{a} = a_x\,\boldsymbol{i} + a_y\,\boldsymbol{j} + a_z\,\boldsymbol{k}$ and $\boldsymbol{b} = b_x\,\boldsymbol{i} + b_y\,\boldsymbol{j} + b_z\,\boldsymbol{k}$, then:

$$\boldsymbol{a} + \boldsymbol{b} = (a_x + b_x)\boldsymbol{i} + (a_y + b_y)\boldsymbol{j} + (a_z + b_z)\boldsymbol{k} \qquad (10)$$

and similarly for subtraction

$$\boldsymbol{a} - \boldsymbol{b} = (a_x - b_x)\boldsymbol{i} + (a_y - b_y)\boldsymbol{j} + (a_z - b_z)\boldsymbol{k} \qquad (11)$$

❖ At 3a.m. a hedgehog of mass 1.5 kg starts crossing the M1 from west to east at a speed of 2.0 km h$^{-1}$. What is the momentum ☞ of the hedgehog?

❖ At 3.05a.m. a truck of mass 15 000 kg and speed 50 km h$^{-1}$ passes the same spot travelling north. What is the momentum of the truck?

**Question T6**

What is the magnitude of the combined momentum of the truck and the hedgehog at 3.06a.m., (and what was the name of the hedgehog)? ❑

Having considered the operations of scaling, addition and subtraction separately, we are now in a position to combine these processes when using the Cartesian component approach.

Suppose, for example, that $\boldsymbol{a} = 2\boldsymbol{i} + 3\boldsymbol{j} + 4\boldsymbol{k}$ and $\boldsymbol{b} = 3\boldsymbol{i} - \boldsymbol{j} + 4\boldsymbol{k}$ then

$$4\boldsymbol{a} - 3\boldsymbol{b} = 4(2\boldsymbol{i} + 3\boldsymbol{j} + 4\boldsymbol{k}) - 3(3\boldsymbol{i} - \boldsymbol{j} + 4\boldsymbol{k}) = -\boldsymbol{i} + 15\boldsymbol{j} + 4\boldsymbol{k}$$

and more generally, for scalars $\alpha$ and $\beta$,

$$\boxed{\alpha\,\boldsymbol{a} + \beta\,\boldsymbol{b} = (\alpha\,a_x + \beta\,b_x)\boldsymbol{i} + (\alpha\,a_y + \beta\,b_y)\boldsymbol{j} + (\alpha\,a_z + \beta\,b_z)\boldsymbol{k}} \quad (12)$$

❖ Is there any physical restriction on $\alpha$, $\beta$, $\boldsymbol{a}$ and $\boldsymbol{b}$ in order that Equation 12 makes sense?

## 3.3 An application of vectors in Cartesian form

To complete Section 3 let us consider how the concepts covered in this section may be applied in a specific example.

✥ A particle travels at velocity $\boldsymbol{v}_1$ for 3 s and then velocity $\boldsymbol{v}_2$ for 2 s, where $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ are given by: $\boldsymbol{v}_1 = (2\boldsymbol{i} - 3\boldsymbol{j} + 6\boldsymbol{k})\,\mathrm{m\,s^{-1}}$ and $\boldsymbol{v}_2 = (4\boldsymbol{i} + 12\boldsymbol{j} - 3\boldsymbol{k})\,\mathrm{m\,s^{-1}}$. Determine:

(a) the magnitudes of the velocities $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$;

(b) the displacements $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$ over the first and second intervals;

(c) the distances associated with these displacements;

(d) the total displacement, $\boldsymbol{s}_{\mathrm{t}}$;

(e) the magnitude of the total displacement; and

(f) the unit vector $\boldsymbol{\hat{s}}_{\mathrm{t}}$ in the direction of $\boldsymbol{s}_{\mathrm{t}}$.

## Question T7

Particle A, of mass 2 kg, travelling with velocity $\boldsymbol{v}_{\rm A} = (\boldsymbol{i} - 2\boldsymbol{j} + 4\boldsymbol{k})\,{\rm m\,s^{-1}}$ collides with particle B, of mass 3 kg, travelling with velocity $\boldsymbol{v}_{\rm B} = (3\boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k})\,{\rm m\,s^{-1}}$. The particles stick together on collision and then move as one combined body with momentum the same as the total momentum of A and B before the collision. Determine:

(a) the momenta ☞ of A and B before the collision;

(b) the total (resultant) momentum of A and B before the collision;

(c) the magnitudes of the momenta of A and B before the collision;

(d) and the velocity of the combined body after the collision. ❏

# 4 Vectors as ordered triples

## 4.1 Ordered triples and ordered pairs

A vector, $\boldsymbol{a} = 3\boldsymbol{i} - \boldsymbol{j} + 4\boldsymbol{k}$ say, in Cartesian form, is often abbreviated to three numbers in brackets, (3, −1, 4), in which the order of the numbers is of crucial importance, for otherwise we would not know which number referred to which component. Such a collection of numbers is called an **ordered triple**.

In general we write

$$\boldsymbol{a} = a_x\,\boldsymbol{i} + a_y\,\boldsymbol{j} + a_z\,\boldsymbol{k} = (a_x, a_y, a_z) \tag{13}$$

An ordered triple, such as (3, −1, 4), can mean the coordinates of a point in three dimensions or the vector $3\boldsymbol{i} - \boldsymbol{j} + 4\boldsymbol{k}$, but the context will make it clear which is intended. (In fact this dual meaning is an advantage, as you will see in Subsection 4.4.)

❖ Express the vector $2\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k}$ as an ordered triple.

In the case of two-dimensional vectors, we refer to **s** rather than ordered triples, for example, $\boldsymbol{v} = 2\boldsymbol{i} + 5\boldsymbol{j} = (2, 5)$. Therefore (2, 5) is the ordered pair which represents the vector $\boldsymbol{v}$.

## 4.2 Manipulation of ordered triples

The extension of the calculations of the last section, for vectors in Cartesian form, to vectors represented as ordered triples is very easy, but we will state the results 'for the record'.

If $\boldsymbol{a} = (a_x, a_y, a_z)$ and $\boldsymbol{b} = (b_x, b_y, b_z)$ then

$$|\boldsymbol{a}| = (a_x^2 + a_y^2 + a_z^2)^{1/2} \quad \text{(Eqn 8)}$$

$$\alpha\,\boldsymbol{a} = \alpha\,a_x\,\boldsymbol{i} + \alpha\,a_y\,\boldsymbol{j} + \alpha\,a_z\,\boldsymbol{k} \quad \text{(Eqn 9)}$$

$$\boldsymbol{a} + \boldsymbol{b} = (a_x + b_x)\boldsymbol{i} + (a_y + b_y)\boldsymbol{j} + (a_z + b_z)\boldsymbol{k} \quad \text{(Eqn 10)}$$

$$\alpha\,\boldsymbol{a} + \beta\,\boldsymbol{b} = (\alpha\,a_x + \beta\,b_x,\ \alpha\,a_y + \beta\,b_y,\ \alpha\,a_z + \beta\,b_z) \quad \text{(Eqn 12)}$$

### Question T8

Find the magnitude of the vector represented by the ordered triple (5, –2, –l). ❏

❖ Express $2(2\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k})$ as an ordered triple and find the ordered triple which represents the sum of $(\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k})$ and $(3\boldsymbol{i} - \boldsymbol{j} + 4\boldsymbol{k})$.

### Question T9

Find the ordered triple that represents the resultant of the vectors (2, –1, –2) and (3, –2, 7), and find also the ordered triple which represents the unit vector in the direction opposite to that of the resultant. ❏

## 4.3 An application of ordered triples

The following exercises are intended to illustrate how vectors may be manipulated using the ordered triple notation.

❖ Forces $\boldsymbol{F}_1 = (2, -1, 4)$ N and $\boldsymbol{F}_2 = (1, 3, 1)$ N act simultaneously on a particle. Determine:

(a) the magnitudes of $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$;

(b) the resultant of $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$;

(c) the magnitude of this resultant;

(d) the vector required to cancel the combined effect of $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$;

(e) the vectors $2\boldsymbol{F}_1$, $5\boldsymbol{F}_2$ and $2\boldsymbol{F}_1 + 5\boldsymbol{F}_2$.

### Question T10

A particle travels with velocity $\boldsymbol{v}_1 = (2, 1, -2)\,\mathrm{m\,s^{-1}}$ for 5 s,then with velocity $\boldsymbol{v}_2 = (-1, 2, 3)\,\mathrm{m\,s^{-1}}$ for 2 s, and finally with velocity $\boldsymbol{v}_3 = (-2, -1, 2)\,\mathrm{m\,s^{-1}}$ for 3 s. Find the total displacement of the particle over the 10 s, the magnitude of this total displacement, and the unit vector in the direction of the total displacement. ❏

## 4.4 Position vectors

There are two main applications of vectors in physics. The first concerns the mathematical modelling (or representation, if you prefer) of physical quantities such as force, displacement and velocity. The second is equally important, and concerns the location of objects in three dimensions, and is essentially a form of geometry. Generally we do not associate a vector with a particular point in space (although later you may encounter applications where it is desirable to do just that), and vectors such as $\overrightarrow{JN}$ and $\overrightarrow{UY}$ in Figure 3 are defined to be the same vector. On the other hand, the vector $\boldsymbol{a}$ in Figure 8 specifies the position of the point A provided that we know that the end of the vector is fixed at the origin. Vectors that are used in this way to determine the positions of points are often known as **position vectors**.

✦ Two points A and B are specified by the position vectors

$$\overrightarrow{OA} = (1, -3, 5)\,\text{cm} \quad \text{and} \quad \overrightarrow{OB} = (-2, 2, 4)\,\text{cm}$$

Find the distance between the points A and B, and the position vector of the mid-point M of AB.

Is a position vector $\boldsymbol{r}$ identical to a displacement vector $\boldsymbol{r}$?

Notice that in the above exercise we were quite happy to use exactly the same notation for the coordinates of M and for the position vector $\overrightarrow{OM}$. The context makes it absolutely clear which is intended, and, in any case, there is very little difference between saying that a point P is determined by the coordinates (1, 2, 3) and that P is determined by the position vector (1, 2, 3).

### Question T11

A particle is moving with velocity $\boldsymbol{v} = (1, 1, 2)\,\mathrm{m\,s^{-1}}$ and at time $t = 0$ it is at the point P with position vector $\overrightarrow{OP} = (2, 3, -4)$ m. What is the position vector $\overrightarrow{OQ}$ of the particle at time $t = 5\,\mathrm{s}$? ❏

# 5 Closing items

## 5.1 Module summary

1 *Scalar quantities* can be specified completely by a single number together with an appropriate unit of measurement.

2 *Vector quantities* can be specified by a magnitude and a direction. Geometric vectors are represented by arrows or *directed line segments* and vector quantities are often represented pictorially by geometric vectors. In print, vectors are denoted by bold typeface, and in handwritten material, by a wavy underline.

3 The *magnitude* of a vector is a non-negative scalar that represents the 'length' or 'size' of that vector. The magnitude of $\boldsymbol{a}$ is denoted by $|\,\boldsymbol{a}\,|$ (or sometimes by $a$) in print, and by $|\,\underset{\sim}{a}\,|$ in handwritten material.

4 Any vector $\boldsymbol{a}$ may be multiplied by a scalar $\alpha$ to produce a scaled vector $\alpha\,\boldsymbol{a}$ which points in the same direction as $\boldsymbol{a}$ if $\alpha > 0$ and in the opposite direction if $\alpha < 0$. The magnitude of $\alpha\,\boldsymbol{a}$ is $|\,\alpha\,\boldsymbol{a}\,| = |\,\alpha\,|\,|\,\boldsymbol{a}\,|$.

5 If any non-zero vector $\boldsymbol{a}$ is divided by $|\,\boldsymbol{a}\,|$, a vector of unit magnitude is obtained which points in the same direction as $\boldsymbol{a}$. Such a vector is called a *unit vector* and is denoted by $\boldsymbol{\hat{a}}$.

6 Vectors may be added geometrically using either the *triangle* or *parallelogram rule* (see Figures 2 and 3).

7 A vector may be *resolved* into *component vectors* along appropriately chosen directions. Given a vector $\boldsymbol{a}$, its orthogonal component vectors parallel and normal to a direction inclined at an angle $\theta$ to $\boldsymbol{a}$ are of magnitude

$$|\,\boldsymbol{a}_{\rm p}\,| = |\,\boldsymbol{a}\,|\cos\theta \quad \text{and} \quad |\,\boldsymbol{a}_{\rm n}\,| = |\,\boldsymbol{a}\,|\sin\theta$$

8 The *Cartesian unit vectors* in the directions of the Cartesian axes $x$, $y$ and $z$ are denoted by $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$.

9 A vector $\boldsymbol{a}$ can be expressed in Cartesian form as

$$\boldsymbol{a} = a_x\boldsymbol{i} + a_y\boldsymbol{j} + a_z\boldsymbol{k} \qquad \text{(Eqn 7)}$$

The scalars $a_x$, $a_y$ and $a_z$ are called the Cartesian scalar components of $\boldsymbol{a}$, whereas the vectors $a_x\boldsymbol{i}$, $a_y\boldsymbol{j}$ and $a_z\boldsymbol{k}$ are called the Cartesian component vectors of $\boldsymbol{a}$. ☞

10 The *magnitude* of the vector $\boldsymbol{a}$ is given by

$$|\,\boldsymbol{a}\,| = (a_x^2 + a_y^2 + a_z^2)^{1/2} \qquad \text{(Eqn 8)}$$

11 The operations of *scaling* and *vector addition* take the following Cartesian algebraic forms (for any scalar $\alpha$ and vectors $\boldsymbol{a}$ and $\boldsymbol{b}$):

$$\alpha\,\boldsymbol{a} = \alpha\,a_x\,\boldsymbol{i} + \alpha\,a_y\,\boldsymbol{j} + \alpha\,a_z\,\boldsymbol{k}$$

and $$\boldsymbol{a} + \boldsymbol{b} = (a_x + b_x)\boldsymbol{i} + (a_y + b_y)\boldsymbol{j} + (a_z + b_z)\boldsymbol{k} \qquad \text{(Eqn 10)}$$

12 A vector represented by $\boldsymbol{a} = a_x\,\boldsymbol{i} + a_y\,\boldsymbol{j} + a_z\,\boldsymbol{k}$ can also be represented by the abbreviated notation of an *ordered triple*:

$$\boldsymbol{a} = (a_x, a_y, a_z)$$

13 The operations of scaling and vector addition take the following abbreviated forms (for any scalar $\alpha$ and vectors $\boldsymbol{a}$ and $\boldsymbol{b}$):

$$\alpha\,\boldsymbol{a} = (\alpha\,a_x, \alpha\,a_y, \alpha\,a_z)$$

and $$\boldsymbol{a} + \boldsymbol{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$$

14 Vectors may be used to determine the position of points relative to a chosen origin, and they are then known as *position vectors*.

## 5.2 Achievements

Having completed this module, you should be able to:

Al Define the terms that are emboldened and flagged in the margins of the module.

A2 Identify quantities as being scalars or vectors, given the definitions of the quantities.

A3 Recognize and use the notations $(\boldsymbol{a}, \underset{\sim}{a}, |\,\boldsymbol{a}\,|, |\,\underset{\sim}{a}\,|)$ to represent vectors and their magnitudes.

A4 Carry out and represent graphically the operations of scaling, addition and subtraction of vectors, and of resolving a vector into orthogonal component vectors.

A5 Determine a unit vector in the same direction as a given vector.

A6 Use Cartesian unit vectors and Cartesian scalar components to represent a given vector in Cartesian form.

A7 Evaluate the magnitude of any vector in terms of its Cartesian components.

A8 Scale, add and subtract vectors in Cartesian form.

A9 Use Cartesian scalar components to represent a given vector as an ordered triple (or an ordered pair in two dimensions).

A10 Scale, add and subtract vectors using the ordered triple notation for vectors.

A11 Use a position vector to specify the location of a point.

***Study comment*** You may now wish to take the *Exit test* for this module which tests these Achievements. If you prefer to study the module further before taking this test then return to the *Module contents* to review some of the topics.

## 5.3 Exit test

***Study comment*** Having completed this module, you should be able to answer the following questions, each of which tests one or more of the *Achievements.*

**Question E1**

(*A2*) The electric field at any point in space can be defined as the electrical force experienced by a positive charge at that point, divided by the magnitude of the charge. Given that charge is a scalar quantity, decide whether electric field is a scalar or a vector quantity, and justify your decision.

**Question E2**

(*A4*) On Treasure Island, Captain Flint decides to bury his treasure at the point at which he arrives after making a displacement of 20 m west and then a displacement of 10 m north, starting from a distinctive rock which he uses as his reference point. Illustrate these displacements graphically, and find the magnitude and direction of the resultant displacement. Find also the component of the resultant displacement in the north-west direction.

### Question E3

(*A3*, *A5*, *A6*, *A7* and *A8*)  Vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are given by

$$\boldsymbol{a} = -\boldsymbol{i} + 5\boldsymbol{j} - 2\boldsymbol{k} \quad \text{and} \quad \boldsymbol{b} = 3\boldsymbol{i} - 2\boldsymbol{j} - 4\boldsymbol{k}$$

Find $\boldsymbol{a} + \boldsymbol{b}$, $|\boldsymbol{a} + \boldsymbol{b}|$ and the unit vector in the direction of $\boldsymbol{a} + \boldsymbol{b}$.

### Question E4

(*A3*, *A6*, *A7* and *A8*)  Given the forces: $\boldsymbol{F}_1 = (\boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k})\,\mathrm{N}$, and $\boldsymbol{F}_2 = (3\boldsymbol{i} + \boldsymbol{j} - 4\boldsymbol{k})\,\mathrm{N}$, find the force given by $3\boldsymbol{F}_1 + 2\boldsymbol{F}_2$, the magnitude of this force and its Cartesian vector component along the *z*-axis.

## Question E5

(*A3*, *A9* and *A10*) Given the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ in Cartesian form as:

$$\boldsymbol{a} = -\boldsymbol{i} + 4\boldsymbol{j} - 2\boldsymbol{k} \quad \text{and} \quad \boldsymbol{b} = 2\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$$

express the vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{a} + \boldsymbol{b}$ in terms of ordered triples. What physical restriction must apply to vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ for the resultant to have any sensible meaning?

## Question E6

(*A9* and *A10*) Vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are given by: $\boldsymbol{a} = (2, l, -l)$ and $\boldsymbol{b} = (1, -3, 2)$. Find, in ordered triple notation, the vectors $2\boldsymbol{a} + 5\boldsymbol{b}$, $3\boldsymbol{a} - 2\boldsymbol{b}$, and the vector $\boldsymbol{c}$ such that $\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c} = (0, 0, 0)$.

## Question E7

(*A9* and *A10*) Given the forces $\boldsymbol{F}_1 = (2, 0, 2)\,\mathrm{N}$, $\boldsymbol{F}_2 = (\mathrm{l}, -2, 3)\,\mathrm{N}$ and $\boldsymbol{F}_3 = (-3, \mathrm{l}, 2)\,\mathrm{N}$, determine:

(a) the ordered triple representing $\boldsymbol{F}_1 + \boldsymbol{F}_2 + \boldsymbol{F}_3$;

(b) the magnitude of $\boldsymbol{F}_1 + \boldsymbol{F}_2 + \boldsymbol{F}_3$;

(c) the ordered triple representing the unit vector in the direction of $\boldsymbol{F}_1 + \boldsymbol{F}_2 + \boldsymbol{F}_3$;

(d) the component vector in the $z$-direction of the force which would completely counteract the force $\boldsymbol{F}_1 + \boldsymbol{F}_2 + \boldsymbol{F}_3$;

(e) the scalar component of $\boldsymbol{F}_1 + \boldsymbol{F}_2 - \boldsymbol{F}_3$ in the $y$-direction.

## Question E8

(*A11*) A pyramid has a square base of length 100 m and its height is 20 m. Cartesian coordinates are chosen with one corner of the base as the origin, with $\boldsymbol{i}$ and $\boldsymbol{j}$ in the direction of the adjacent edges of the base, and with $\boldsymbol{k}$ vertically upward. Find the position vector of the vertex of the pyramid.

***Study comment*** This is the final *Exit test* question. When you have completed the *Exit test* go back to Subsection 1.2 and try the *Fast track questions* if you have not already done so.

If you have completed **both** the *Fast track questions* and the *Exit test*, then you have finished the module and may leave it here.