# Module P3.1 Introducing fields

# 1 Opening items

## 1.1 Module introduction

We are all familiar with the effect of gravitational forces. They help to keep us ‘down to Earth’! On the surface of the Earth the strongest gravitational influence is that of the Earth itself, but gravitational forces are thought to be universal — they exist between all masses, wherever they may be. This is the basic message of the famous *law of universal gravitation* first propounded by Isaac Newton. When dealing with such a universal force it is useful to introduce the idea of a *gravitational field*. The gravitational force acts only where there is a body for it to act upon, but the gravitational field can exist at all points within a region of space. The gravitational field tells us the force that would act on a body if one were present and it characterizes the distribution of masses that cause such a force. If we know how the gravitational field varies from place to place, we can predict the behaviour of a body placed in the field.

Fields are of fundamental importance in physics. They arise naturally whenever we deal with quantities that are defined simultaneously at many different points or throughout an entire region. In this module we will begin by drawing the distinction between two quite separate kinds of field, *scalar fields* and *vector fields*. Gravitational fields and the closely similar *electrostatic fields*, used to describe electrical forces, are both examples of vector fields. It is these fields, and their associated forces, that occupy us throughout Subsections 2.2 to 2.5. In particular, we see how the fields are defined, how they are represented mathematically and pictorially using vectors and *field lines*, and we investigate the particular fields that correspond to some very simple distributions of mass and charge.

In the second part of the module (Section 3) we examine the work done on a particle when it is displaced from one point to another within a gravitational or electrostatic field. We learn how to calculate the *potential energy* that such a particle possesses by virtue of its position within a field, and we see how the gravitational (or electrostatic) force on a particle is related to the rate at which its gravitational (or electrostatic) potential energy changes with position. Finally we introduce *gravitational* and *electric potential*, which are scalar fields, and show how to represent fields by *equipotential surfaces* or *contours*.

***Study comment*** Having read the introduction you may feel that you are already familiar with the material covered by this module and that you do not need to study it. If so, try the *Fast track questions* given in Subsection 1.2. If not, proceed directly to *Ready to study?* in Subsection 1.3.

## 1.2 Fast track questions

***Study comment*** Can you answer the following *Fast track questions*?. If you answer the questions successfully you need only glance through the module before looking at the *Module summary* (Subsection 4.1) and the *Achievements* listed in Subsection 4.2. If you are sure that you can meet each of these achievements, try the *Exit test* in Subsection 4.3. If you have difficulty with only one or two of the questions you should follow the guidance given in the answers and read the relevant parts of the module. However, *if you have difficulty with more than two of the Exit questions you are strongly advised to study the whole module*.

### Question F1

The hydrogen atom may be crudely modelled as an electron orbiting a proton in a circular orbit of radius $5 \times 10^{-11}$ m. Treating the proton and the electron as positive and negative point charges, respectively, estimate the ratio of the electrostatic force to the gravitational force between the two particles.

(Mass of electron $m_e = 9.11 \times 10^{-31}$ kg, mass of proton $m_p = 1.67 \times 10^{-27}$ kg, the magnitude of the charge on the electron and proton $e = 1.60 \times 10^{-19}$ C. Universal gravitational constant $G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$. Electrostatic constant $1/(4\pi\varepsilon_0) = 8.98 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$.)

### Question F2

Figure 1 shows the field lines in a region containing two objects both of which are charged and have mass.

(a) With no more information than this, can you tell whether the field shown is the gravitational field between the two objects or the electric field?

(b) Given the additional information that the two objects carry charges of opposite sign, which type of field is shown?

(c) Which object has the greater mass?

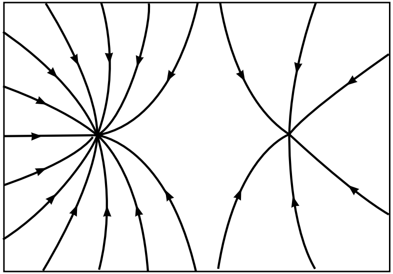


**Figure 1** See Question F2.

### Question F3

Calculate the change in gravitational potential energy when a satellite of mass $m_s = 4.00 \times 10^3$ kg is put into an orbit of radius $R_s = 4.20 \times 10^7$ m about the Earth. Treat the Earth as a point mass situated at its centre, and take its radius to be $R_E = 6.40 \times 10^6$ m, its mass $m_E = 6.00 \times 10^{24}$ kg and $G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$.

***Study comment*** Having seen the *Fast track questions* you may feel that it would be wiser to follow the normal route through the module and to proceed directly to *Ready to study?* in Subsection 1.3.

Alternatively, you may still be sufficiently comfortable with the material covered by the module to proceed directly to the *Closing items*.

## 1.3 Ready to study?

***Study comment***

In order to study this module you will need to be familiar with the following terms: *acceleration*, *Cartesian coordinate system*, *charge*, *displacement*, *energy*, *force*, *function*, *gradient* (of a graph), *mass*, *magnitude* (of a vector), *Newton's laws of motion*, *scalar*, *tangent* (to a curve), *vector*, *velocity* and *work*. You should be familiar with vector notation, and be able to express a vector in terms of its (scalar) *components*, you should also be able to carry out *vector addition*. This module makes some use of calculus notation, but you are *not* required to have a detailed knowledge of the techniques of *differentiation* and *integration.* In particular it is assumed you know that a *derivative* such as $dy/dx$ represents the rate of change of $y$ with respect to $x$, and that it may be interpreted in terms of the gradient of a graph of $y$ against $x$. It would also be an advantage if you were familiar with the idea that a *definite integral* such as $y = \int_a^b \frac{c}{x^2}\,dx$ represents the limit of a sum and may be interpreted as the area under a graph of $c/x^2$ between $x = a$ and $x = b$. However, this is not an essential prerequisite since it is explained at the appropriate point in the text. *SI units* are used throughout the module. If you need help with any of these concepts or procedures, refer to the *Glossary* which will indicate where in *FLAP* they are developed. The following *Ready to study questions* will help you establish whether you need to review some of the above topics before embarking on this module.

## Question R1

Write down in terms of kg, m, and s the SI units of (a) acceleration, (b) force, (c) energy. Where appropriate, give the name of the unit.

## Question R2

Draw a sketch showing the addition of two vectors of magnitude 1 unit and 2 units, inclined at an angle of 60° to each other.

## Question R3

Plot a graph of $y = 1/x$ over the range $x = 0$ to $x = 3$.

## Question R4

When a particle initially located at a point with position vector $\boldsymbol{r}_1$ is moved to a different point with position vector $\boldsymbol{r}_2$, the particle is said to have undergone a displacement $\boldsymbol{s}_{12}$. Express $\boldsymbol{s}_{12}$ in terms of $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$. What is the physical significance of the magnitude of $\boldsymbol{s}_{12}$?

# 2 Forces and fields

## 2.1 Fields

What is a *field*? The term is used in many different ways in everyday English as shown by the following extract from *Chambers Twentieth Century Dictionary*:

> … country or open country in general: a piece of ground enclosed for … pasture or sport … speciality: area of knowledge, interest, …: the locality of a battle …
>
> Chambers 1981

The list continues with ‘a wide expanse’ (geologists refer to large areas covered with ice as ice *fields*), and ‘area visible to an observer at one time’ (we may speak of the *field* of view of an optical instrument such as a microscope or camera and also of its depth of *field*). This is followed by the dictionary’s closest approach to the meaning that will concern us in this module: ‘region of space in which forces are at work’.

In physics we are accustomed to dealing with systems that exert forces at all points throughout a region of space. The Earth is a good example of such a system. No humans, not even the astronauts who have walked on the Moon, have ever been entirely free of the gravitational force that the Earth exerts on objects in its vicinity. We describe this situation by saying that the Earth is the source of a *gravitational field*, and that any body located within this field will be acted upon by a gravitational force due to the Earth.

In a similar way, a positive or negative electric charge will exert an electrical force on any other charge in its vicinity, so it may be said to produce an *electric field*, and a magnet may be said to produce a *magnetic field*. Gravitational and electrical forces only act on bodies with mass or charge, but the presence of a gravitational or an electric field in some region of space tells us that a force will act if an appropriate body is placed in that region. Later in this section we will replace these rather vague notions of gravitational fields and electric fields by precise definitions based on quantitative relationships between the fields and the forces they describe. However, before linking fields and forces in that way it is worth examining the general concept of a field in order to emphasize that fields are not only of use when describing force distributions.

If you watch television weather forecasts you will see several examples of fields that have nothing to do with forces. Figure 2 shows the distribution of ground-level temperatures across the British Isles at a certain time, this constitutes a *temperature field*. The word *field* is used in this context to emphasize that the temperature has some particular value at every point throughout the region of interest at the time in question.

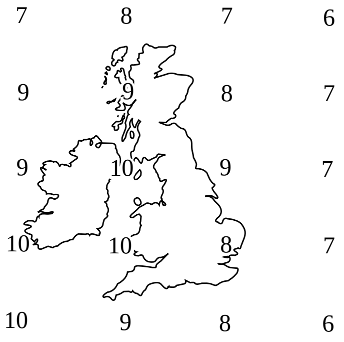



**Figure 2** The temperature field observed across the British Isles one day in January 1981.

In a similar way, a weather forecast might provide information about the distribution of air pressure, hours of sunshine in the past week or even inches of rainfall in the past month. In each of these cases it is possible to associate a value of the named physical quantity with each point in a given region at a particular time. It is this feature that is common to all fields and which we will adopt as the defining characteristic of a field.

> A **field** is a physical quantity to which a definite value can be ascribed at each point in some region of space at a particular time.

The temperatures in Figure 2 are only shown at regularly-spaced intervals on a two-dimensional grid, but the implication that there is a unique temperature at every point is clear. We can express this mathematically by saying that the temperature is a *function* of position, and we can indicate that by writing the temperature $T$ at a point with coordinates $(x, y)$ as $T(x, y)$. If we wanted to discuss temperatures at different heights $z$ above the surface of the British Isles, we would write $T(x, y, z)$, and if we wanted to discuss the way this temperature changed with time we might write $T(x, y, z, t)$. From this point of view, a field is simply a function of position and (possibly) time.

## Scalar and vector fields

Temperature is a *scalar* quantity — its value at any point in space and time is fully specified by a single number together with an appropriate unit of measurement, e.g. the kelvin (K), or the degree Celsius (°C). For this reason a temperature field is said to be a **scalar field**. Similarly, any other field whose value at any given point in space and time is specified by a single scalar quantity is said to be a scalar field.

Although a two-dimensional scalar field such as the temperature field $T(x, y)$ can be represented by numbers as in Figure 2, it is generally much easier to interpret the alternative representation shown in Figure 3. In this case lines called *isotherms* ☞ passing only through points of equal temperature have been used to represent the field. Other scalar fields may be depicted in similar ways (e.g. all points on an *isobar* have the same pressure, and all points on a *contour line* have the same altitude above sea-level). The ability to form a mental image of a field on the basis of such a pictorial representation is a useful skill to develop.

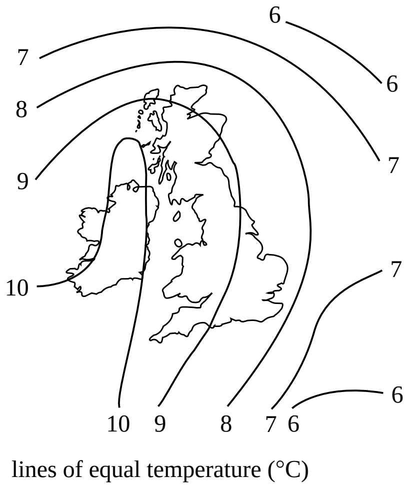



**Figure 3** Isotherms plotted using the same data as Figure 2.

## Question T1

A flat sheet of metal is heated near its edge, and its temperature is measured at points in a grid-like array, as shown on Figure 4. Sketch on the figure the isotherms for 100 °C, 200 °C and 300 °C. (Assume that the temperature varies smoothly across the sheet so that you can interpolate values, e.g. if two neighbouring grid points are at 170 °C and 230 °C then the temperature halfway between them is 200 °C.) ❑

| | | | | |
|---|---|---|---|---|
| 65 | 95 | 115 | 150 | 185 |
| 100 | 150 | 200 | 250 | 300 |
| 130 | 170 | 235 | 275 | 350 |
| 125 | 170 | 230 | 250 | 295 |
| 90 | 120 | 160 | 170 | 175 |

**Figure 4** See Question T1.

Fields that specify *forces*, such as gravitational and electric fields (defined more fully in Subsections 2.3 and 2.4, respectively) cannot be scalar fields since they must define a force which has direction as well as magnitude at every point. Since force is a vector quantity, any such field is said to be a **vector field**. Velocity is also a vector quantity, so the weather maps that use arrows to show wind velocities across the British Isles are pictorial representations of the wind velocity field (see Figure 5). Since vectors are conventionally denoted in print by a bold typeface, it is usual to also use bold type for vector fields. Thus, the ground-level wind velocity field of Figure 5 may be denoted $\boldsymbol{v}(x, y)$, the gravitational field a height $z$ above the surface of the Earth $\boldsymbol{g}(x, y, z)$ and the rapidly changing electric field inside a device by $\boldsymbol{E}(x, y, z, t)$.

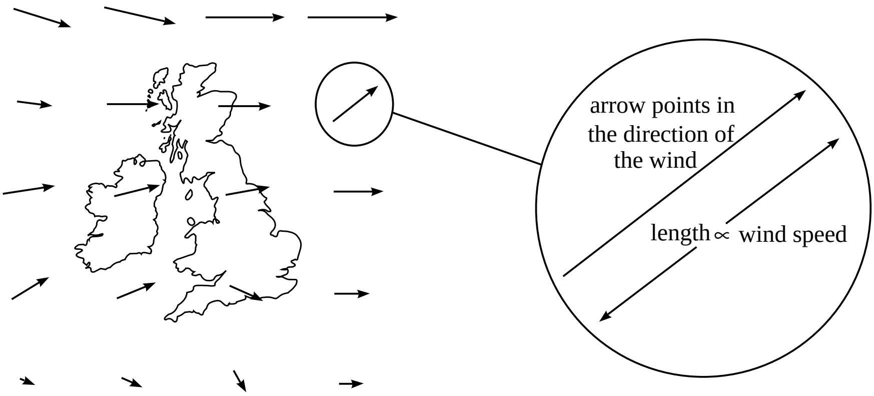



**Figure 5** The wind velocity field for the same day as illustrated in Figure 2. The symbol ∝ is read as ‘is proportional to’.

✥ Classify the following as scalar fields or vector fields: the temperature throughout a room, the weight of a cubic centimetre of air at any point in the Earth's atmosphere, wind speeds throughout central Scotland, the depth below sea-level of each point on the floor of the Atlantic Ocean.

## 2.2 Gravitational forces

***Study comment*** One of our immediate aims is to write down the general definition of the gravitational field, however before doing so we need to introduce the vector description of the gravitational force. If you are already familiar with this way of describing gravitational forces you need only glance through the content of this section, pausing only to answer the text questions that it contains.

We are all familiar with the gravitational force of the Earth pulling downwards on objects situated on or above its surface. It is what keeps our feet on the ground. In a similar way objects on the Moon feel the gravitational force of the Moon.

Isaac Newton (1642–1727) ☞ suggested that the force which one 'heavenly' body exerts on another is of the same nature as the gravitational force that the Earth exerts on a terrestrial object such as a person or an apple. In fact, according to Newton, gravitational forces act between *all* masses. This led him to formulate the following law:

**law of universal gravitation**

Every particle of matter in the Universe attracts every other particle with a force whose magnitude is directly proportional to the product of the masses of the particles and inversely proportional to the square of the distance between them.

Thus the magnitude of the **gravitational force** that any particle of mass $m_1$ exerts on a particle of mass $m_2$ a distance $r$ away is given by

$$F_{\text{grav}} = \frac{Gm_1m_2}{r^2}$$

where $G = 6.673 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2}$ is the **universal gravitational constant**.

There are several points to note about the gravitational force between two particles:

1 It is a **distant-action force**, i.e. it always exists between two particles without their being in contact and regardless of any intervening matter.

2 The force magnitude is proportional to the mass of each particle.

3 The force magnitude obeys an **inverse square law**, i.e. it is proportional to $1/r^2$ and thus decreases rapidly as the separation of the particles increases.

4 The force is always *attractive*, i.e. when two particles exert a gravitational force on each other the direction of the force on each particle is towards the other. (Since the magnitudes of these two oppositely directed forces are equal it is clear that they constitute a third law pair of the kind described by *Newton's third law of motion*.) ☞

**Question T2**

Confirm that the units of $G$ are $\mathrm{N\,m^2\,kg^{-2}}$. ❏ ☞

### Question T3

Sketch a graph of $F_{grav}$ against $r$ over the range $r = 0$ to $r = 3\,$m. ❏

$$F_{grav} = \frac{Gm_1m_2}{r^2}$$

By making use of vector notation we can express all the information about the magnitude and direction of the gravitational force in a single equation. This equation takes its simplest form if we imagine that the particle of mass $m_1$ is located at the origin O of a *Cartesian coordinate system,* while the particle of mass $m_2$ is at a point P with position coordinates $(x, y, z)$. The point P can be specified by a *position vector* $\boldsymbol{r}$ (see Figure 6b) which may be written in terms of its components as $(x, y, z)$, and which has magnitude $r = |\boldsymbol{r}| = \sqrt{x^2 + y^2 + z^2}$. ☞

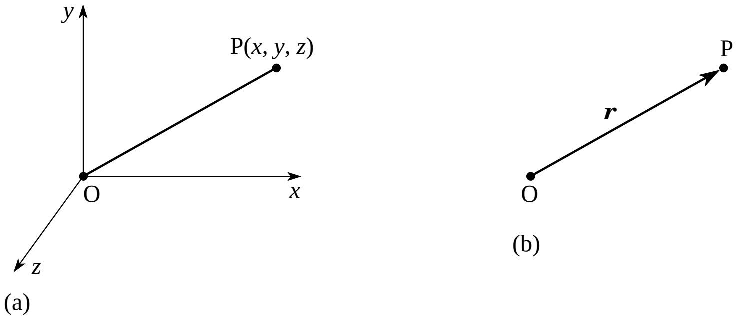



**Figure 6** Alternative, but equivalent, ways of defining the position of point P, (a) by its coordinates $(x, y, z)$, (b) by the position vector $\boldsymbol{r}$.

We can then introduce a **unit vector** $\hat{\boldsymbol{r}}$, defined by the relation

$$\hat{\boldsymbol{r}} = \frac{\boldsymbol{r}}{|\boldsymbol{r}|} \quad \text{i.e.} \quad \hat{\boldsymbol{r}} = \frac{\boldsymbol{r}}{\sqrt{x^2 + y^2 + z^2}}$$

The unit vector $\hat{\boldsymbol{r}}$ points in the same direction as $\boldsymbol{r}$ but its magnitude is just 1; it is therefore a dimensionless quantity whose only purpose is to indicate the direction from the mass $m_1$ at the origin to the mass $m_2$ at the point P.

We can now write the gravitational force acting on $m_2$ due to $m_1$ as follows:

$$\boldsymbol{F}_{21} = -\frac{Gm_1m_2}{r^2}\hat{\boldsymbol{r}} \tag{1}$$

The negative sign in Equation 1 indicates that $\boldsymbol{F}_{21}$ points in the opposite direction to the unit vector $\hat{\boldsymbol{r}}$. It is worth noting that the force $\boldsymbol{F}_{12}$ on $m_1$ due to $m_2$ points in the same direction as $\hat{\boldsymbol{r}}$, and is given by $\boldsymbol{F}_{12} = -\boldsymbol{F}_{21}$.

## Question T4

Suppose that the masses $m_1$ and $m_2$ are located at points specified by the position vectors $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$, respectively. Write down a formula similar to Equation 1

$$\boldsymbol{F}_{21} = -\frac{Gm_1m_2}{r^2}\,\boldsymbol{\hat{r}} \qquad \text{(Eqn 1)}$$

for the gravitational force on $m_2$ due to $m_1$, taking care to define any new terms that you introduce. (*Hint*: You may find it useful to recall that the *displacement* from $m_1$ to $m_2$ is $\boldsymbol{s}_{12} = \boldsymbol{r}_2 - \boldsymbol{r}_1$. See Question R4.) ❑

## Question T5

Consider two small lead spheres of masses $m_1$ and $m_2$, where $m_1$ is greater than $m_2$, isolated in space from all other bodies. Since the gravitational force between the spheres is attractive they will accelerate towards each other.

(a) Will their accelerations be equal? If not, which is larger?

(b) As the spheres move towards each other, will their accelerations remain constant? If not, will they increase or decrease? ❑

**Question T6**

Two small objects of masses 1 g and 1 kg are placed 1 m apart. Given that $G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$, calculate the magnitude of the gravitational force that each of the masses exerts on the other. ❏

## 2.3 Gravitational fields

Suppose that the 1 g mass in Question T6 is replaced by a 2 g mass. How will this affect the magnitude of the force acting on the mass? We could calculate the new magnitude using Equation 1,

$$\boldsymbol{F}_{21} = -\frac{Gm_1m_2}{r^2}\,\hat{\boldsymbol{r}} \qquad \text{(Eqn 1)}$$

but there is really no need to do so. We know that the gravitational force on a particle is proportional to its mass, so doubling the mass will double the magnitude of the gravitational force that acts upon it. In fact, once we have calculated the force on *one* known mass at a particular point P due to any fixed arrangement of masses, we can immediately calculate the gravitational force on any other mass placed at P. The calculation is easy because once we know the gravitational force on one mass at P we can work out *the gravitational force per unit mass* placed at that point. If the coordinates of P are $(x, y, z)$, we can denote the gravitational force per unit mass at P by $\boldsymbol{g}(x, y, z)$. The gravitational force on any mass $m$ placed at a point P with coordinates $(x, y, z)$ will then be

$$\boldsymbol{F}_{\text{grav}}\,(\text{on } m \text{ at P}) = m\boldsymbol{g}(x, y, z)$$

Of course, P can be any point within a region, so we can regard this vector quantity $\boldsymbol{g}(x, y, z)$ that varies from place to place as a vector field, and it is then called the **gravitational field**. ☞

> At any point, the value of the gravitational field $\boldsymbol{g}(x, y, z)$ is the gravitational force per unit mass that would act on a particle placed at that point.

The gravitational field at any point P can be determined experimentally by placing a **test mass** $m$ at P, measuring the gravitational force on it, and then dividing that force by $m$. (A *test mass* $m$ should be thought of as a particle of sufficiently small mass $m$ that it will not significantly influence the bodies that give rise to the gravitational field and therefore will not disturb the field, though its mass should be sufficiently great that the force exerted on it by the field can be measured.) It should be noted that the above definition refers to the force that *would* act on a particle, so it makes sense to speak of the field even when there is no particle present to experience its effect.

When discussing the gravitational field, or any other field come to that, it is rather tedious to repeatedly write $\boldsymbol{g}(x, y, z)$, so we will follow the usual practice of representing the position coordinates $x$, $y$ and $z$ by the single vector quantity $\boldsymbol{r}$, we may then indicate the gravitational field by the symbol $\boldsymbol{g}(\boldsymbol{r})$.

We then have the following general definition of the gravitational field:

$$\boldsymbol{g}(\boldsymbol{r}) = \frac{\boldsymbol{F}_{\text{grav}}\ (\text{on } m \text{ at } \boldsymbol{r})}{m} \tag{2}$$

where $\boldsymbol{F}_{\text{grav}}$ is the force that *would* act on a test mass $m$ if it were located at the point with position vector $\boldsymbol{r}$. Note that in writing the field as $\boldsymbol{g}(\boldsymbol{r})$ we are *not* implying that it points in the direction of $\boldsymbol{r}$ at every point, only that the magnitude and direction of $\boldsymbol{g}$ depend on position.

In the particular case that the source of the gravitational field is an isolated point mass $m_0$ at the origin then, from

Equations 1 $\quad \boldsymbol{F}_{21} = -\dfrac{Gm_1m_2}{r^2}\,\hat{\boldsymbol{r}}$ (Eqn 1)

and 2, we obtain: $\quad \boldsymbol{g}(\boldsymbol{r}) = -\dfrac{Gm_0}{r^2}\,\hat{\boldsymbol{r}}$ (3)

Note the negative sign indicating that in this particular case $\boldsymbol{g}(\boldsymbol{r})$ is in the opposite direction to $\hat{\boldsymbol{r}}$. Also note that this is only an *example* of a gravitational field, albeit an important one. The general definition is still that of Equation 2.

✦ Write down an expression for the magnitude $g(\boldsymbol{r})$ of the vector field $\boldsymbol{g}(\boldsymbol{r})$ given in Equation 3, at the point $\boldsymbol{r}$.

$$\boldsymbol{g}(\boldsymbol{r}) = -\frac{Gm_0}{r^2}\,\hat{\boldsymbol{r}} \qquad \text{(Eqn 3)}$$

**Question T7**

Calculate the magnitude of the gravitational field $g(r)$ at a distance of $2\,\text{m}$ from an isolated $5\,\text{kg}$ mass. ☞ Make sure you include appropriate SI units in your answer. ❏

The gravitational field in a region, like the wind velocity field of Figure 5, can be represented pictorially by a set of arrows whose direction is that of the field and whose length is proportional to the field strength (i.e. magnitude) at the points where they are drawn. An alternative pictorial representation of a vector field is provided by **field lines**. These are directed lines, possibly curved, that have arrowheads on them drawn so that at any point the tangent to the field line represents the direction of the field at that point, and the spacing of the lines is related to the field strength (where the lines are close together the field is strong and where they are further apart the field is weaker).

Gravitational field lines surrounding an isolated point mass $m$ are shown in Figure 7a. These extend radially in all directions. The direction of the field is shown by the arrowheads. A test mass placed at point A for example will experience a force towards the central mass. If the central mass had been $2m$ we would have drawn twice as many field lines since the field would have been twice as strong at every point.

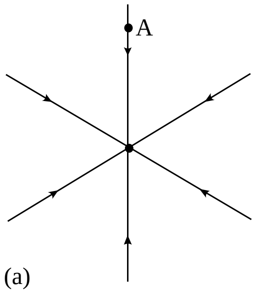
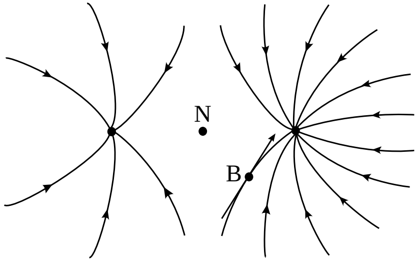



**Figure 7a** Gravitational field lines for mass $m$.

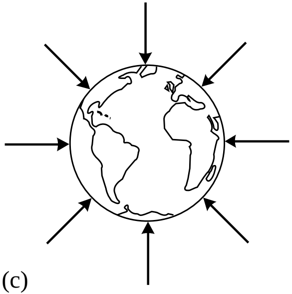
What happens to the field lines if we bring two masses $m$ and $2m$ closer together? As they approach, the field lines of each isolated mass will be modified by the presence of the other mass. This causes the field lines to curve (see Figure 7b). Where the lines are curved, as at point B, the direction of the field is along the tangent to the curve at that point. Between the masses it is as if the field lines 'repel one another', and there is a region where the field lines are far apart, indicating that the field is weak. There is even a point N, called a **neutral point**, at which the forces on a test mass due to $m$ and $2m$ are equal and opposite so there is no resultant force at all and the gravitational field is zero at that point.




(b)

**Figure 7b** Gravitational field lines for masses $m$ and $2m$ close together, the neutral point will be closer to the mass $m$.

Figure 7c shows the gravitational field lines associated with the Earth. (You must of course imagine the field lines spreading out in three dimensions.) The magnitude of the field will be proportional to the density of field lines in any region. Notice that the lines are closer together near the surface of the Earth where the field is stronger. Further away from the Earth, the radial field lines will be modified by the field lines associated with the Moon.

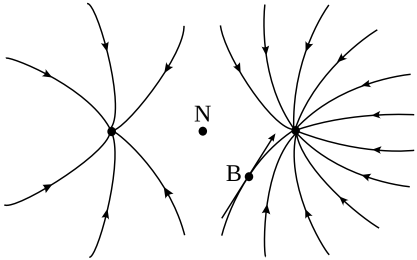


(c)

**Figure 7c** Gravitational field lines for the Earth's gravitational field.

Field lines are a useful aid to visualizing the distribution of a field in space. Be careful though not to fall into the trap of thinking that the field lines actually exist. They don’t!

At the neutral point N of Figure 7b we can determine the gravitational field due to two masses by working out the forces that each mass would separately exert on a test mass and adding those forces together vectorially.




(b)

**Figure 7b** Gravitational field lines for masses $m$ and $2m$ close together, the neutral point will be closer to the mass $m$.

However, there is really no need to worry about forces and test masses at all. We can find the combined effect of two gravitational fields by adding the fields themselves, provided we remember that the fields add vectorially at every point. The statement that gravitational fields may be added in this straightforward way is called the **principle of superposition**.

❖ Find the magnitude and direction of the gravitational field at point O due to two 4 kg masses placed at distances of 2 m from O as shown in Figure 8. ($G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$.)

From the resultant field at O we can calculate the magnitude of the gravitational force acting on, say, a 1 g test mass placed at O:

$$|\boldsymbol{F}\text{ (on 1 g at O)}| = \text{test mass} \times \text{field strength at O} = 10^{-3}\,\mathrm{kg} \times g(0, 0)$$

$$= 10^{-3}\,\mathrm{kg} \times 9.43 \times 10^{-11}\,\mathrm{N\,kg^{-1}}$$

$$= 9.43 \times 10^{-14}\,\mathrm{N}$$

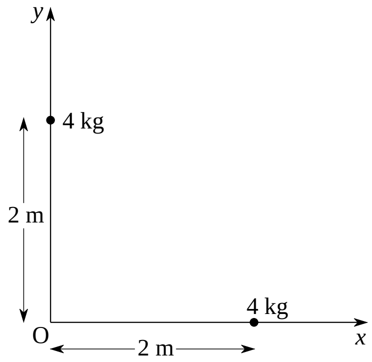



**Figure 8** Arrangement of two 4 kg masses about point O.

The **weight** of a body near the surface of the Earth is usually taken to mean the gravitational force that the Earth exerts on the body. ☞ You will almost certainly have seen calculations in which a body of mass $m$ has been associated with a weight of magnitude $W = mg$, where $g$ is called the *magnitude of the acceleration due to gravity*. The value of $g$ varies a little from place to place, but is usually taken to be $9.81\,\mathrm{m\,s^{-2}}$ across the UK. In view of what has been said about gravitational fields in this subsection you should now be able to give a somewhat different interpretation to the equation $W = mg$. By rearranging the equation to obtain $g = W/m$, and then identifying $W$ with the magnitude of the gravitational force on the body we find

$$g = \frac{F_{\rm grav}}{m}$$

However $F_{\rm grav}/m$ represents the magnitude of the Earth's gravitational field; so, taking into account the downward direction of both the field and the acceleration due to gravity, we can say:

> The gravitational field of the Earth is equal to the acceleration due to the Earth's gravity at any point.

**Question T8**

Show that the SI units of gravitational field are *exactly* equivalent to those of acceleration. ❏

## 2.4 Electrostatic forces and fields

The **electrostatic force** that exists between charged particles is similar in many ways to the gravitational force between masses, e.g. they are both action-at-a-distance forces. However, electric charge, which is measured in units of coulomb (C), may be positive or negative. This allows electrostatic forces to differ from gravitational forces in one major respect. Whereas the electrostatic force between oppositely charged particles is attractive, like the gravitational force, the force between similarly charged particles (both positive or both negative) is repulsive.

**Coulomb's law** ☞ is the electrostatic equivalent of Newton's law of gravitation; it provides a quantitative description of the force between electrically charged particles. In its simplest vector form it says that the electrostatic force on a point charge $q_2$ at a position $\boldsymbol{r}$ due to a point charge $q_1$ at the origin is

$$\boxed{\boldsymbol{F}_{21} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}\,\hat{\boldsymbol{r}} \qquad (4)}$$ ☞

where $\hat{\boldsymbol{r}}$ is a unit vector pointing from $q_1$ to $q_2$, and $\varepsilon_0 = 8.854 \times 10^{-12}\,\mathrm{C^2\,N^{-1}\,m^{-2}}$ is a constant called the **permittivity of free space**.

If the charges $q_1$ and $q_2$ have the same sign $q_1q_2/(4\pi\varepsilon_0 r^2)$ will be positive and Equation 4 will imply that $\boldsymbol{F}_{21}$ points in the same direction as $\hat{\boldsymbol{r}}$, i.e. it is a repulsive force pushing $q_2$ away from the origin. If $q_1$ and $q_2$ have opposite signs $q_1q_2/(4\pi\varepsilon_0 r^2)$ will be negative and the force on $q_2$ will point in the opposite direction to $\hat{\boldsymbol{r}}$, towards the origin. Thus, substituting values of $q_1$ and $q_2$ *with their correct signs* into Equation 4 gives the direction of $\boldsymbol{F}_{21}$ as well as its magnitude.

$$\boldsymbol{F}_{21} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}\hat{\boldsymbol{r}} \qquad \text{(Eqn 4)}$$

By analogy with the gravitational constant $G$ in Newton's law of gravitation, the constant $1/(4\pi\varepsilon_0)$ in Coulomb's law is sometimes called the **electrostatic constant**.

❖ The force $\boldsymbol{F}_{12}$ that acts on $q_1$ due to $q_2$ is equal in magnitude but opposite in direction to $\boldsymbol{F}_{21}$. Write down an expression for the common magnitude $F_{el}$ of these two forces and use it to determine appropriate SI units for $1/(4\pi\varepsilon_0)$.

This answer shows that the force magnitude $F_{el}$ varies as $1/r^2$, i.e. like the magnitude of the gravitational force, it obeys an *inverse square law*.

A distribution of electric charges that might exert electrostatic forces on **test charges** will create an **electric field** in the same way that a distribution of masses creates a gravitational field.

At any point, the value of the electric field $\boldsymbol{E}(\boldsymbol{r})$ is the electrostatic force per unit positive charge that would act on a test charge placed at that point.

Note that the electric field may vary from point to point throughout a region, so we write it as $\boldsymbol{E}(\boldsymbol{r})$ or sometimes $\boldsymbol{E}(x, y, z)$ to emphasize its dependence on position. (Many authors will simply denote it by $\boldsymbol{E}$ and leave it to you to remember that it is a function of position.)

The general definition of an electric field given above may be also be expressed as an equation:

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{\boldsymbol{F}_{\rm el}\ (\text{on } q \text{ at } \boldsymbol{r})}{q} \tag{5}$$

☞

where $\boldsymbol{F}_{\rm el}$ (on $q$ at $\boldsymbol{r}$) represents the electrostatic force that would act on a test charge $q$ if it were located at the point with position vector $\boldsymbol{r}$. The similarity between electric field and gravitational field, as expressed by Equations 5 and 2, respectively, should be noted. However, the units of the two are quite different. The electric field is the force per unit positive charge whilst the gravitational field is the force per unit mass; it is *only* the gravitational field which has the same dimensions and units as acceleration.

It follows from Equations 4 and 5

$$\boldsymbol{F}_{21} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}\,\hat{\boldsymbol{r}} \qquad \text{(Eqn 4)}$$

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{\boldsymbol{F}_{\text{el}}\ (\text{on } q \text{ at } \boldsymbol{r})}{q} \qquad \text{(Eqn 5)}$$

that in the particular case that the sole source of the electric field in some region is an isolated point charge $q_0$ at the origin:

$$\boxed{\boldsymbol{E}(\boldsymbol{r}) = \frac{q_0}{4\pi\varepsilon_0 r^2}\,\hat{\boldsymbol{r}} \qquad (6)}$$

It follows from Equation 5 that the SI units of electric field are $\mathrm{N\,C^{-1}}$. However it is more conventional to express electric field strengths in terms of an equivalent unit, the volt per metre ($\mathrm{V\,m^{-1}}$). The use of volts as a unit of measurement will be discussed more fully in Subsection 3.3.

## Question T9

Consider two charges, $-q$ situated at O and $+q$ situated at P, where $q$ is a positive quantity.

(a) In which direction does the electrostatic force on the charge at P act?

(b) Is it in the same direction as the electric field at P?

Suppose we change the charge at O to $+q$ and the charge at P to $-q$

(c) In which direction does the electrostatic force now act at point P?

(d) Is it in the same direction as the electric field at P? ❑

## Question T10

Two charges $+q$ and $-q$ are situated as shown in Figure 10, where OA = OB. Find the direction of the electric field at point P. ❑

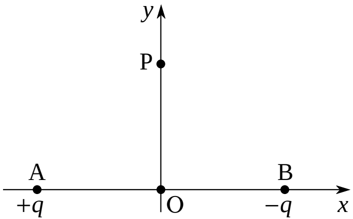



**Figure 10** Arrangement of two charges $+q$ and $-q$ about O.

Just as with gravitational fields, we can use field lines to represent an electric field. Figure 11a shows the electric field lines surrounding a negative point charge, and Figure 11b those surrounding a positive point charge. Note that electric fields, like gravitational fields, obey the principle of superposition.

❖ What is the difference between Figures 11a and 11b?

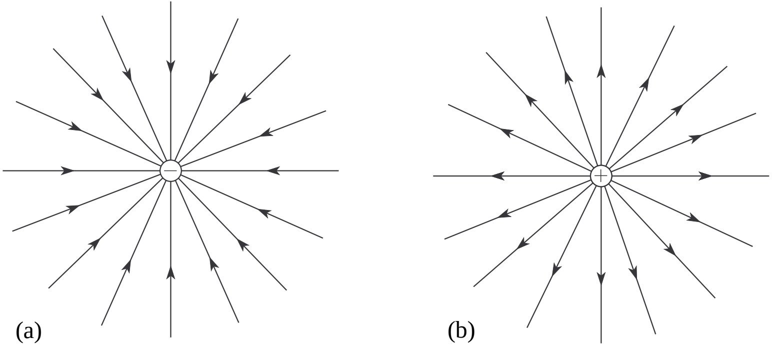



**Figure 11a/b** Field lines for (a) a negative point charge $-q$, (b) a positive point charge $+q$

If we bring two charges close together, the electric field lines of each isolated charge appear to be modified by the presence of the other, just as we found in the case of gravitational field lines. For example, in Figure 11c we illustrate a positive charge and a negative charge of equal magnitude. We see that some of the field lines emerge from $+q$ and end on $-q$.

In Figure 11d two positive charges of equal magnitude are illustrated, and we see that the field lines appear to repel one another. It is also clear that there is a neutral point between the two charges where the resultant force is zero.

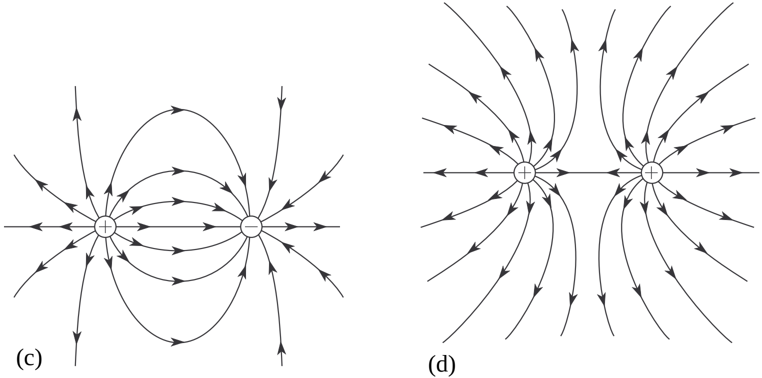



**Figure 11c/d** Field lines for (c) point charges $+q$ and $-q$ close together, (d) point charges $+q$ and $+q$ close together. These are of course two-dimensional representations of the three-dimensional fields surrounding the charges.

Note that electric fields, like gravitational fields, obey the principle of superposition. That is to say that if we know that a certain distribution of charge produces an electric field $\boldsymbol{E}_1(\boldsymbol{r})$, and that some other distribution of charge produces a field $\boldsymbol{E}_2(\boldsymbol{r})$, then when both charge distributions are present simultaneously the resultant electric field at any point $\boldsymbol{r}$ will be the vector sum $\boldsymbol{E}_1(\boldsymbol{r}) + \boldsymbol{E}_2(\boldsymbol{r})$.

**Question T11**

Consider three charges, $-q$, $+2q$, $-q$ all in line, with equal spacing between them. Sketch the electric field lines surrounding the charges. (Remember that if the charge is doubled, then we must draw twice as many lines.) ❏

**Question T12**

Two positive and two negative charges of equal magnitude are arranged at the corners of a square with the two positive charges one above the other and the two negative charges one above the other.

(a) Draw arrows at several points in and around the square to represent the direction and relative strength of the electric field at those points due to the charges. Identify any positions where the field is zero. (Let the length of the arrow you draw at any point indicate the relative strength of the electric field at that point.)

(b) On a separate sketch draw the electric field lines associated with the arrangement of charges. Make sure your answers to (a) and (b) are consistent. ❏

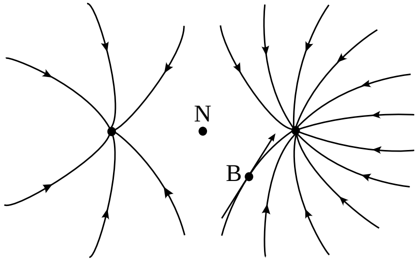
It is possible to indicate the presence of an electric field in some region by using small bits of thread suspended in oil or grass seeds scattered on a sheet of paper. Close to a charged body the threads (or seeds) will tend to form linear patterns similar to those of the field lines that might be used to represent the electric field in that region. ☞. However, these lines should not be interpreted as being *the* field lines — field lines may be drawn at any point where the field is not zero and are simply a convenient way of depicting the field pictorially; they are not physically 'real' and are certainly not limited to those locations where the threads or seeds happen to be.

## 2.5 Uniform fields and constant fields

If a scalar quantity such as temperature has just a single value over an entire region then it may be represented by a scalar field of the form $T(\boldsymbol{r})$ = constant. Such a field is said to be a **uniform field**. Similarly, a vector field is said to be uniform if it has the same magnitude and direction at all points. ☞

✧ What must the field lines look like for a uniform vector field?

We have seen (in Figure 7c) that the gravitational field varies as we move away from the Earth. However, if we consider a small enough region the field lines within that region will be approximately parallel. The gravitational field within a room or building or even a town is, to all intents and purposes, the same everywhere, both in magnitude and direction. The gravitational field in such a region is therefore *uniform*, and a given mass will experience the same gravitational force at any point within the region. (See Figure 12.) If $\boldsymbol{\hat{z}}$ is a unit vector in the vertically upwards direction, we may represent the uniform downward gravitational field in a room by $\boldsymbol{g}(\boldsymbol{r}) = -g\boldsymbol{\hat{z}}$, where the constant $g$ is the local magnitude of the acceleration due to gravity.




(b)

**Figure 7b** Gravitational field lines for (b) masses $m$ and $2m$ close together, the neutral point will be closer to the mass $m$.

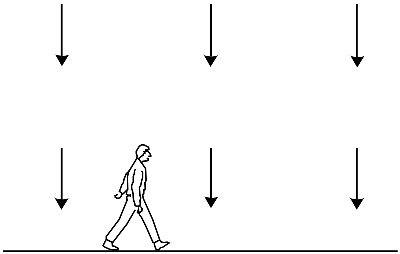


**Figure 12** A uniform gravitational field — one that doesn't change with position.

A uniform electric field would exist at all points above an infinite plane sheet of uniformly distributed charge. At each point the field would be perpendicular to the sheet and would have magnitude $\sigma/(2\varepsilon_0)$, where $\sigma$ ☞ is the charge per unit area of the sheet (measured in $\mathrm{C\,m^{-2}}$). Another uniform electric field would exist at all points below the infinite sheet; it would have the same magnitude as the field above the sheet, but it would point in the opposite direction. Figure 13 shows the field lines around such a sheet of positive charge; if the charge had been negative all of the field lines would have to be reversed.

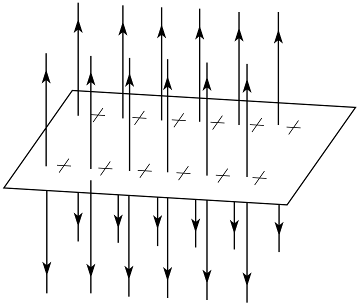



**Figure 13** The electric field due to an infinite sheet of uniform positive charge.

## Question T13

Suppose we have an infinite sheet of positive charge of surface (charge) density $\sigma$ and an infinite sheet of negative charge of surface (charge) density $-\sigma$, placed one above the other so that they are parallel. Add the fields of these two charge distributions together and hence determine the magnitude of the resultant field (a) between the parallel sheets, and (b) outside the sheets. (*Hint*: Note that the units of surface charge density will be $\mathrm{C\,m^{-2}}$, and remember that the fields add vectorially.) ❏

*Note* You may be puzzled why the electric field above a charged plate does *not* depend on a distance from the plate. The explanation lies in the restriction to an 'infinite plate' — for which any actual distance above the plate is negligible in comparison to the size of the plate. In practice, these simple expressions are also approximately valid *close* to a real finite charged plate, providing the distance is small compared to the size of the plate and we are well away from the ends of the plates, where the field becomes non-uniform. A similar restriction of course applies also to the gravitational field of the Earth. It is only close to the Earth's surface that the field is uniform.

Whether uniform or not, any field that does not vary with *time* is said to be a **constant field**. The electrostatic fields created by fixed distributions of charge are examples of constant fields.

# 3 Potential energy and potential

## 3.1 Work and energy in a uniform field

If an appropriate test particle (a test mass or a test charge) is located at any point in a gravitational or an electric field, a force will act upon it due to the field. If the test particle is displaced from one position to another within the field then the force that the field exerts will do work on the test particle. In the particular case that the field is uniform (and constant) the force that it exerts on a given test particle will be the same everywhere (and at all times), and may be represented by a single constant vector $\boldsymbol{F}$. Under these circumstances, if the test particle undergoes a displacement $\boldsymbol{s}$ (from position $\boldsymbol{r}_1$ say to position $\boldsymbol{r}_2$, so that $\boldsymbol{s} = \boldsymbol{r}_2 - \boldsymbol{r}_1$) then the *work* $W$ done by the force may be expressed in any of the following equivalent ways:

1 $W$ = magnitude of the force × component of the displacement in the direction of the force

2 $W$ = magnitude of the displacement × component of the force in the direction of the displacement

3 $W = \boldsymbol{F} \cdot \boldsymbol{s} = Fs\cos\theta = F_x s_x + F_y s_y + F_z s_z$

In the last of these expressions $\boldsymbol{F} \cdot \boldsymbol{s}$ denotes the so-called *scalar product* of $\boldsymbol{F}$ and $\boldsymbol{s}$, the angle $\theta$ is that between $\boldsymbol{F}$ and $\boldsymbol{s}$, and the various subscripted quantities are the components of $\boldsymbol{F}$ and $\boldsymbol{s}$ in an arbitrarily chosen Cartesian coordinates system.

For the situations we consider in this module, force and displacement are always along the same line, so $\theta = 0°$ or 180°. For convenience we may choose this common line to be the $x$-axis of a Cartesian coordinate system, in which case the work done by the force is simply $W = F_x s_x$. In such (effectively) one-dimensional cases we can say that the work done is simply the product of the force and the displacement. If the displacement is small we may represent it by $\Delta x$ ☞ and the correspondingly small amount of work done may then be represented by $\Delta W = F_x \Delta x$.

| |
|---|
| The work done when a constant (one-dimensional) force $F_x$ acts over a displacement $\Delta x$ is $\Delta W = F_x \Delta x$. ☞ |

When describing the (nearly) uniform gravitational field that acts over any small region near the surface of the Earth it is conventional to use a coordinate system in which the $z$-axis points vertically upwards. The force that the field exerts on a particle of mass $m$ may then be represented by $F_z = -mg$, and the work done by that force when the mass undergoes a displacement $\Delta z$ is $\Delta W = -mg\Delta z$. If the mass is raised, the displacement $\Delta z$ is positive and the work done by the gravitational force is negative; if the mass is lowered $\Delta z$ is negative, the gravitational force and the displacement are in the same direction, and the work done by the gravitational force is positive. Note that throughout this discussion the work we have been considering is that done *by the gravitational force*, not that done by any external agency working to overcome the gravitational force. If *you* had to raise an object *you* would have to do some positive work, but the work done by the gravitational force while you were raising the object would be negative.

The **potential energy** ($E_{pot}$) of a body describes its capacity to do work by virtue of its position in a field. By lifting a mass, we increase its potential energy since the gravitational force acting on the mass does positive work when the mass falls. If the mass had been lowered, i.e. displaced in the direction of the gravitational force, it would have lost part of its capacity to do work by falling and its potential energy would have been correspondingly reduced. It follows from these considerations that the change in potential energy when a test mass is displaced is generally equal to *minus* the work done by the gravitational force during the displacement, ☞ so we may write

$$\Delta E_{pot} = mg\Delta z \tag{7a}$$

Since in this case $E_{pot}$ arises by virtue of position within a gravitational field, it is called **gravitational potential energy**, a name that is often abbreviated to **gravitational energy**, and may be denoted $E_{grav}$.

Note that Equation 7a relates *changes* in potential energy to changes in position; it does not completely determine the value of the potential energy at any point. This means we are free to *choose* a point at which $E_{grav} = 0$ for an object, and Equation 7a will then determine the potential energy of the object at all other points.

For example, you may choose to say that a body at a certain point on a horizontal table top has $E_{grav} = 0$, but it is then inevitable that the body will also have $E_{grav} = 0$ at every other point on the table top, that its gravitational potential energy will be increasingly positive as it is raised above the table top, and that its $E_{grav}$ will become negative if the body falls from the table. You could decide, instead, to let $E_{grav} = 0$ at some point on the floor. $E_{grav}$ would then be positive at all places above floor level, and it would increase with height.

The similarities between gravitational and electrostatic forces mean that all the preceding discussion may also be applied to uniform electric fields, provided we take due care over signs. Figure 14 shows a test charge $q$ (which may be positive or negative) in a uniform constant electric field $E_x$. If we take the direction from left to right as the direction of increasing $x$, then the electrostatic force on the charge is specified by the single component $F_x = qE_x$ and the work done by that force when the charge is displaced by an amount $\Delta x$ is $\Delta W = F_x\,\Delta x = qE_x\,\Delta x$. It follows that the change in potential energy when the charge is displaced by $\Delta x$ is given by

$$\Delta E_{pot} = -qE_x\,\Delta x \qquad (7b)$$

☞

In this case the potential energy is called the **electrostatic potential energy**, which is often abbreviated to **electrical energy,** and may be denoted $E_{el}$. As in the gravitational case, we can arbitrarily choose a point within the field at which we let $E_{el} = 0$, but having made that choice Equation 7b will automatically determine the value of $E_{el}$ at every other point within the field.

## Question T14

(a) Suppose we choose point C in Figure 14 as the place where $E_{\rm el} = 0$. If $q$ is positive, what is its electrical energy $E_{\rm el}$ at point A, and what can be said about $E_{\rm el}$ at point B?

(b) If we now define point A to be where $E_{\rm el} = 0$, what can be said about $E_{\rm el}$ at points B and C? ❏

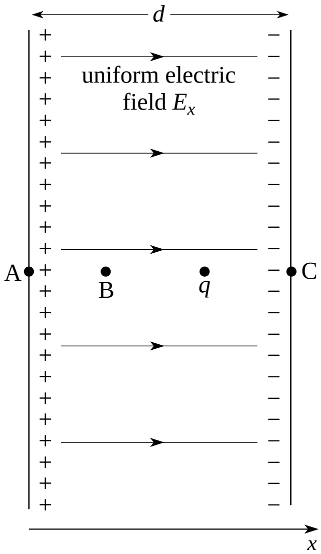



**Figure 14** A test charge $q$ charge in a uniform electric field $E_x$. The points A, B and C represent three of the charge's possible positions. A and C are separated by a distance $d$.

The general conclusion that may be drawn from this discussion of potential energy and uniform fields is as follows:

> If a uniform field describes a constant (one-dimensional) force $F_x$ acting on a test particle, then the work done by that force when the test particle is displaced by an amount $\Delta x$ will be $\Delta W = F_x \Delta x$, and the corresponding change in the potential energy of the test particle will be
>
> $$\Delta E_{\rm pot} = -\Delta W = -F_x \Delta x \qquad (7c)$$

Equations 7a and 7b

$$\Delta E_{\rm pot} = mg\Delta z \qquad \text{(Eqn 7a)}$$

$$\Delta E_{\rm pot} = -qE_x \Delta x \qquad \text{(Eqn 7b)}$$

are particular examples of this general relation.

## 3.2 Work and energy in a non-uniform field

Among the most common non-uniform fields are those due to an isolated mass or charge located at the origin (see Equations 3 and 6, respectively).

$$\boldsymbol{g}(\boldsymbol{r}) = -\frac{Gm_0}{r^2}\,\hat{\boldsymbol{r}} \qquad \text{(Eqn 3)}$$

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{q_0}{4\pi\varepsilon_0 r^2}\,\hat{\boldsymbol{r}} \qquad \text{(Eqn 6)}$$

In such cases the force on a test particle may be described by a single non-zero component, $F_r$, that acts along a radial direction measured outward from the origin. ($F_r$ will be positive if the force is directed away from the origin, and negative if it is directed towards the origin.) Because the field is non-uniform, the value of $F_r$ may be expected to depend on the radial position coordinate of the test particle, $r$. In other words, the force on the test particle will depend on its distance from the origin, and may be represented by $F_r(r)$.

✥ Write down an expression for the radial component of the gravitational force on a test particle of mass $m$ at a radial distance $r$ from an isolated point mass $m_0$ located at the origin.

Although gravitational and electrostatic forces generally vary with position, if the (radial) displacement ($\Delta r$) of the test particle is small enough that $F_r(r)$ does not change significantly, then we can still write the work done as $\Delta W = F_r\Delta r$.

Suppose now the particle undergoes a large radial displacement from a point $P_1$, at a distance $r_1$ from the origin, to a point $P_2$, at a distance $r_2$ from the origin. We can estimate the work done by $F_r(r)$ over such a displacement by dividing the radial path from $P_1$ to $P_2$ into a large number of very small steps and then adding together the small amount of work done in each of these steps. This estimate can be made into an accurate evaluation of the work done if we make even the largest of the steps 'vanishingly' small. This process of determining a quantity by considering a limit of a sum is known as *definite integration* and is fully explained elsewhere in *FLAP*. For our present purposes it is sufficient to note the following points:

1 The total work done by the force $F_r(r)$ when the test particle is displaced from $r_1$ to $r_2$ is given by the **definite integral**:

$$W = \int_{r_1}^{r_2} F_r(r)\,dr \qquad (8)$$

2 It is possible to interpret such an integral in terms of the 'area under a graph', measured in the scale units appropriate to the graph. In this case, given a graph of $F_r(r)$ against $r$, the value of $W$ is equal to the area between the graph and the $r$-axis between $r_1$ and $r_2$, as indicated in Figure 15. In this case the curve representing $F_r(r)$ happens to lie below the $r$-axis, so the area $W$ is a negative quantity. (If the curve is plotted on graph paper this area can be found by counting squares, though doing so is tedious.)

3 In many cases the value of $W$ may be found more simply by applying standard mathematical techniques that allow definite integrals to be evaluated once the function $F_r(r)$ is specified. These are fully developed elsewhere in *FLAP*.

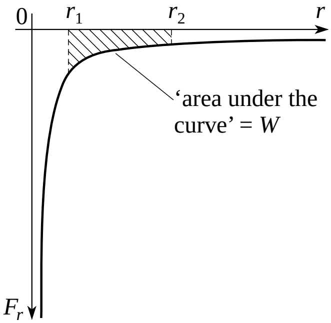



**Figure 15** The graph of the gravitational force $F_r(r)$ on a test particle at a radial distance $r$ from a point mass $m_0$ at the origin. The integral in Equation 8 can be interpreted as the area under this graph between $r_1$ and $r_2$, which is negative for areas below the $r$-axis.

Let us apply these ideas to the case of a test particle of mass $m$ in the gravitational field due to a mass $m_0$ at the origin. (This situation is represented in Figure 15.) In this case we know that

$$F_r(r) = -\frac{Gmm_0}{r^2}$$

so, $$W = \int_{r_1}^{r_2} -\frac{Gmm_0}{r^2}\,dr$$

However $G$, $m$ and $m_0$ are all constants, so according to the rules of integration this may be written as

$$W = -Gmm_0 \int_{r_1}^{r_2} \frac{1}{r^2}\,dr$$

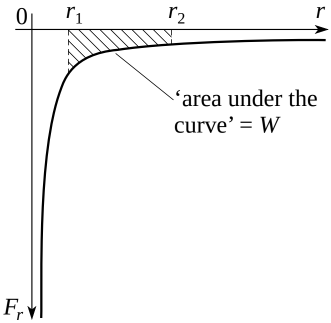



**Figure 15** The graph of the gravitational force $F_r(r)$ on a test particle at a radial distance $r$ from a point mass $m_0$ at the origin. The integral in Equation 8 can be interpreted as the area under this graph between $r_1$ and $r_2$, which is negative for areas below the $r$-axis.

Now the result of evaluating this integral (as you can confirm using standard techniques if you know them, or by drawing the graph of $1/r^2$ and measuring the area between the curve and the $r$-axis from some value $r_1$ to some other value $r_2$ as just described) is

$$W = Gmm_0\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = Gmm_0\,\frac{r_1 - r_2}{r_1 r_2}$$

In this case, $F_r(r)$ was negative because the gravitational force on the test mass pointed towards the origin, so over a positive displacement, in which $r_2$ is greater than $r_1$, we would expect the work done by the gravitational force to be negative. This is exactly what the above expression indicates. (Remember, $W$ is the work done *by* the gravitational force when the particle is displaced, *not* the work you would do if you had to cause the displacement.)

Using Equation 8

$$W = \int_{r_1}^{r_2} F_r(r)\,dr \qquad \text{(Eqn 8)}$$

we can now find a general expression for the change in the *potential energy* of a test particle when it is displaced from $r_1$ to $r_2$. We know from the definition of potential energy, that the change in the potential energy will be equal to *minus* the work done by the field force over the displacement.

Thus in general terms:

$$\Delta E_{\rm pot} = -W = -\int_{r_1}^{r_2} F_r(r)\,dr \tag{9}$$

Note that this is an expression for the *change* in the potential energy of the test particle. As usual, we are free to choose an arbitrary point in the field at which $E_{\rm pot} = 0$, but having made that choice the expression for $\Delta E_{\rm pot}$ will determine the value of $E_{\rm pot}$ at every other point.

Applying Equation 9 to the particular case of the gravitational field due to a point mass $m_0$ at the origin we see that

$$\Delta E_{\rm grav} = -Gmm_0\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

In this particular case it is conventional to choose the point at which the mass $m$ has $E_{\rm grav} = 0$ to be a point at infinity ($\infty$), i.e. one for which $r \rightarrow \infty$. Taking this as the starting point for a displacement we can determine the full value of the gravitational potential energy of a mass $m$ at any distance $r$ from the mass of $m_0$ at the origin by letting $r_1 \rightarrow \infty$ and $r_2 = r$. It follows that:

$$E_{\rm grav}\ (\text{of } m \text{ at } r) = -\frac{Gmm_0}{r} \qquad (10)$$

Note that in this situation the gravitational potential energy of the mass $m$ is *negative*. $E_{\rm grav}$ is a small negative quantity for large $r$ and becomes more negative as $r$ decreases (see Figure 16).

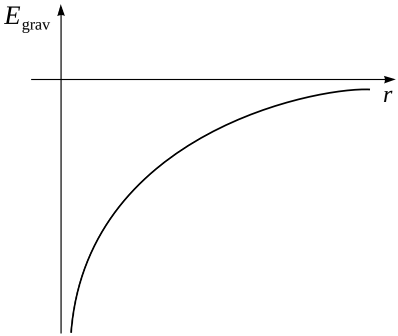



**Figure 16** Gravitational potential energy, $E_{\rm grav}$, of a mass $m$ at a distance $r$ from a mass $m_0$.

We have dealt here with gravitational forces, but the above ideas may also be applied to electrostatic forces.

**Question T15**

Apply Equation 9

$$\Delta E_{\text{pot}} = -W = -\int_{r_1}^{r_2} F_r(r)\,dr \qquad \text{(Eqn 9)}$$

to two charges $q$ and $q_0$ separated by a distance $r$ to find the equivalent electrical expression to Equation 10.

$$E_{\text{grav}}\ (\text{of } m \text{ at } r) = -\frac{Gmm_0}{r} \qquad \text{(Eqn 10)}$$

Sketch $E_{\text{pot}}$ (of $q$ at $r$) as a function of $r$ if $q$ and $q_0$ are both positive. ❏

We have seen that a definite integral of the force $F_r(r)$ described by a field allows us to deduce the associated potential energy $E_{\text{pot}}(r)$, but we can reverse that process to reveal another important link between (conservative) forces and their associated potential energy functions. We saw earlier in this subsection that even in a non-uniform field the work done over a small displacement is given by an expression of the form $\Delta W = F_r\Delta r$. It follows that if $F_r(r)$ is the force at the starting point of the displacement then, at least approximately

$$\Delta E_{\text{pot}} \approx -F_r(r)\Delta r$$

Rearranging this and considering the limit in which $\Delta r$ becomes vanishingly small, so that the approximation becomes increasingly accurate, we have

$$F_r(r) = \lim_{\Delta r \to 0}\left(\frac{-\Delta E_{\rm pot}}{\Delta r}\right)$$

$$\text{i.e.}\qquad F_r(r) = -\frac{dE_{\rm pot}}{dr} \qquad (11)$$

In words, the radial component of the force on a test particle at a distance $r$ from the mass of $m_0$ (or charge $q_0$) at the origin is equal to *minus* the derivative of the particle's potential energy at that point. Since $dE_{\rm pot}/dr$ may be interpreted as the gradient of the graph of $E_{\rm pot}$ against $r$, we may describe Equation 11 as:

The component of the force in any direction is equal to minus the gradient of the potential energy in that direction.

The results obtained in this subsection have been derived in the context of the purely radial field of a point mass or charge at the origin, but this last result is actually much more general as we have stated. In any (conservative) field the component of the force that the field exerts in any direction on a test particle at any point is given by *minus* the gradient of the corresponding potential energy in that direction. ☞ In these general cases the potential energy of the test particle is not restricted to depending on $r$ alone (that was a particular property of the field of a point source) so we should represent it as $E_{\rm pot}(\boldsymbol{r})$ rather than $E_{\rm pot}(r)$. Nonetheless at any point we can still identify a direction in which the potential energy is *declining most rapidly*, and that is the direction in which the force on the test particle will act. The existence of such forces explains the tendency of a system to minimize its potential energy if it is free to do so.

✥ By considering the gradient of Figure 16, describe the way that $F_r(r)$ depends on $r$, paying due attention to its direction.

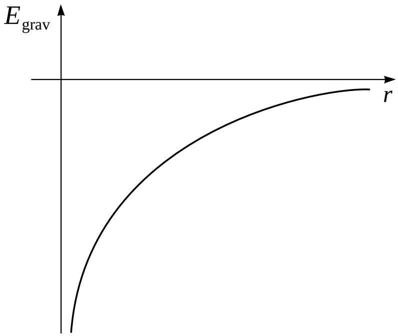



**Figure 16** Gravitational potential energy, $E_{\rm grav}$, of a mass $m$ at a distance $r$ from a mass $m_0$.

**Question T16**

Sketch graphs to show how force and potential energy vary with position in a *uniform* electrostatic field (e.g. show how $F_x$ and $E_{\rm el}$ for a positive charge $q$ depend on $x$ in Figure 14). ❑

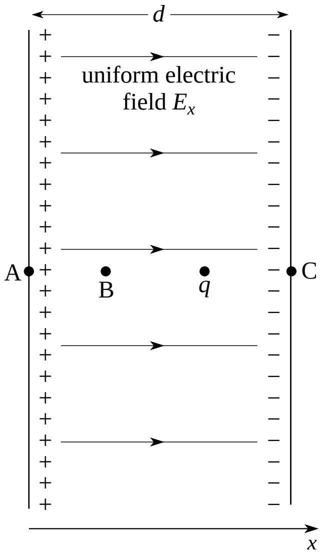



**Figure 14** A test charge $q$ charge in a uniform electric field $E_x$. The points A, B and C represent three of the charge's possible positions. A and C are separated by a distance $d$.

## 3.3 Potential

One of the main reasons for introducing the gravitational and electrostatic fields in the way that we did, as the force per unit mass and the force per unit positive charge on a test particle, was that the resulting quantities $\boldsymbol{g}(\boldsymbol{r})$ and $\boldsymbol{E}(\boldsymbol{r})$ are *independent of the test charge*. The force on the test particle is proportional to the mass or charge of the test particle, but the field that it helps to define does not depend on those properties. This subsection introduces a widely used quantity called the **potential**, that is related to the potential energy in the same way that the field is related to the force.

If the gravitational potential energy of a test mass $m$ at some point $\boldsymbol{r}$ in a gravitational field is $E_{\rm grav}$ (of $m$ at $\boldsymbol{r}$), then the **gravitational potential** $V_{\rm grav}(\boldsymbol{r})$ at that point is the gravitational potential energy *per unit mass* at that point:

i.e. $$V_{\rm grav}(\boldsymbol{r}) = \frac{E_{\rm grav}\ (\text{on } m \text{ at } \boldsymbol{r})}{m} \tag{12a}$$

☞

Similarly, we can define the **electric potential** $V_{\rm el}(\boldsymbol{r})$ at any point in an electrostatic field as the electrostatic potential energy per unit charge at that point

$$\text{i.e.}\qquad V_{\rm el}(\boldsymbol{r}) = \frac{E_{\rm el}(\text{on } q \text{ at } \boldsymbol{r})}{q} \tag{12b}$$

✧ What are appropriate SI units of (a) gravitational potential and (b) electrical potential?

Electric potential is such a widely used quantity that its SI unit has a special name, the **volt** (V). Thus, $1\,\text{V} = 1\,\text{J}\,\text{C}^{-1}$. It is a common practice to refer to the difference in electric potential between two points as the 'voltage' or the 'voltage difference' between those points.

✧ A particle of charge $e = 1.60 \times 10^{-19}\,\text{C}$ is released from a point in an electric field at which the electric potential is $V_1 = 100\,\text{V}$. If the only forces on the particle are those due to the electric field, how much work will have been done on the particle when it arrives at a point where the potential is $V_2 = -100\,\text{V}$?

There are two main reasons why the concept of potential is useful and important. First, the potential energies $E_{\rm grav}$ and $E_{\rm el}$ in Equations 12a and 12b

$$V_{\rm grav}(\boldsymbol{r}) = \frac{E_{\rm grav}\ (\text{on } m \text{ at } \boldsymbol{r})}{m} \tag{12a}$$

$$V_{\rm el}(\boldsymbol{r}) = \frac{E_{\rm el}\,(\text{on } q \text{ at } \boldsymbol{r})}{q} \tag{12b}$$

are proportional to $m$ and $q$, respectively, so the potentials $V_{\rm grav}$ and $V_{\rm el}$ defined by those equations do *not* depend on $m$ and $q$. Hence the potentials (unlike the potential energies) are *independent of the test charges* used to determine them and *characterize the fields* themselves. Secondly, the potential is a *scalar* field, so it is easier to deal with mathematically than a *vector* field. (We will exploit this simplicity later.)

**Question T17**

Write down expressions for the gravitational potential due to a point mass at the origin, and for a point charge at the origin. Use the first expression to calculate the gravitational potential at a distance of 10 m from a 5 kg mass. ❏

In the context of gravitation, the description of a force as *minus* the gradient of potential energy (Equation 11)

$$F_r(r) = -\frac{dE_{\rm pot}}{dr} \qquad \text{(Eqn 11)}$$

implies that the 'force per unit mass' (i.e. the gravitational field) must be minus the gradient of the 'potential energy per unit mass' (i.e. the gravitational potential). A similar relationship links the electrostatic field and its potential. Hence, for the simple cases we have been considering in this module, where the position may be specified by a single variable such as $r$ or $x$, we can write for a gravitational field:

$$\boxed{g_x(x) = -\frac{dV_{\rm grav}(x)}{dx} \quad \text{or} \quad g_r(r) = -\frac{dV_{\rm grav}(r)}{dr}} \qquad (13)$$

and for an electrostatic field:

$$\boxed{E_x(x) = -\frac{dV_{\rm el}(x)}{dx} \quad \text{or} \quad E_r(r) = -\frac{dV_{\rm el}(r)}{dr}} \qquad (14)$$

Like the relationship between force and potential energy, these relationships may be generalized to the case where the potential is a function of $\boldsymbol{r}$ (i.e. a function of $x$, $y$ and $z$) rather than a function of a single variable. We will not explore this case mathematically, but as before we will state that generally:

> The component of the field in any direction is minus the gradient of the potential in that direction.
>
> The direction of the field is the direction in which the potential decreases most rapidly. ☞

Table 1 summarizes the relationship between the main characteristics of the fields that we have explored in this module.

**Table 1** The relationship between forces, fields, potential energies and potentials in (effectively) one-dimensional cases.

| | | |
|---|---|---|
| −(potential energy) | differentiate $\longrightarrow$<br>integrate $\longleftarrow$ | force |
| divide by mass $m$ or charge $q$ $\downarrow$ $\uparrow$ multiply by mass $m$ or charge $q$ | | divide by mass $m$ or charge $q$ $\downarrow$ $\uparrow$ multiply by mass $m$ or charge $q$ |
| −(potential) | differentiate $\longrightarrow$<br>integrate $\longleftarrow$ | field |

## Question T18

Make a table listing expressions for the field and potential of a point mass $m_0$, and the associated force and potential energy for a test mass $m$ at a distance $r$ from $m_0$. Check the entries in your table by confirming that they are related in the fashion indicated by Table 1. ❏

**Table 1** The relationship between forces, fields, potential energies and potentials in (effectively) one-dimensional cases.

$$\begin{tikzcd}[column sep=huge, row sep=huge]
-(\text{potential energy}) \arrow[r, shift left, "\text{differentiate}"] \arrow[d, shift right, "\text{divide by mass } m \text{ or charge } q"'] & \text{force} \arrow[l, shift left, "\text{integrate}"] \arrow[d, shift right, "\text{divide by mass } m \text{ or charge } q"'] \\
-(\text{potential}) \arrow[r, shift left, "\text{differentiate}"] \arrow[u, shift right, "\text{multiply by mass } m \text{ or charge } q"'] & \text{field} \arrow[l, shift left, "\text{integrate}"] \arrow[u, shift right, "\text{multiply by mass } m \text{ or charge } q"']
\end{tikzcd}$$

## 3.4 Using potential

We can now describe the effect of an isolated mass $m_0$ (or charge $q_0$) in terms of either its field or its potential. In general, which should we prefer? Both have their uses. If we want to know the force on an object, then it is more direct to think about the field. If we want to discuss work or energy it is better to deal with the potential. However, if we want to find the combined effect of several masses or charges, it is often easier to work with the potential, even if we are ultimately interested in the forces that such a combination exerts. The reason for this is that the potentials may be combined using scalar addition, but combining the fields involves the more complicated process of vector addition. ☞ So, to find the combined field it is often quicker to combine the potentials and then take the negative gradient of the combined potential.

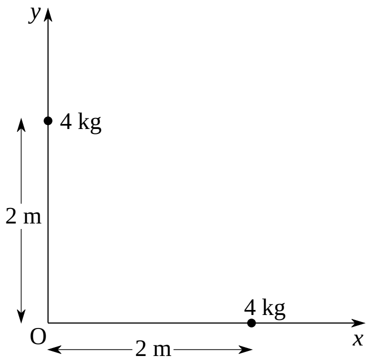



**Figure 8** Arrangement of two 4 kg masses about point O.

❖ Calculate the potential at point O due to the two 4 kg masses shown in Figure 8.

We could extend the above example by finding a general expression for $V(\boldsymbol{r})$ as a function of position, then differentiating to find the field. Alternatively, the potential can be calculated at positions on a grid surrounding the masses. We can then produce a ‘map’ of the potential by drawing **equipotential surfaces** (these ‘surfaces’ are just lines or **contours** in two dimensions) over which the potential has a fixed constant value. Equipotentials are usually drawn so that for consecutive surfaces (or contours) the potential changes by a fixed amount. ☞

Figure 17 shows some equipotential contours (solid curves) surrounding two masses.

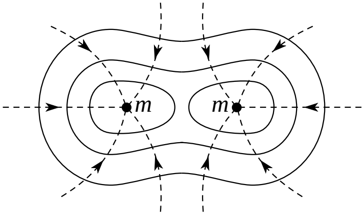



**Figure 17** Equipotential contours (full curves) surrounding two masses $m$ close together. Gravitational field lines (dashed lines).

Equipotentials have two important features, which we will illustrate with the example of an isolated point mass. Since the potential depends only on $r$, the equipotentials are a set of concentric spheres in this case (Figure 18). There are a couple of points worth noting about Figure 18:

- The equipotentials are close together near the mass, where the field is strong, and as the field becomes weaker the equipotential surfaces are further apart. In other words, the potential changes *more rapidly* with position where the field is *stronger*.
- The equipotentials are everywhere *perpendicular* to the field lines.

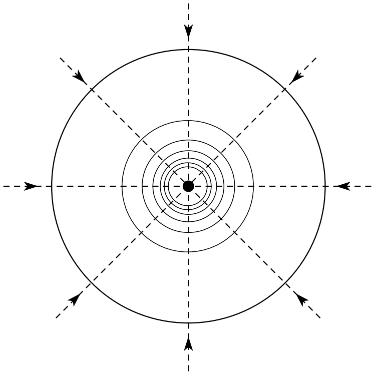


**Figure 18** Gravitational equipotentials (full curves) for a point mass.

Along any equipotential surface $V$ is, by definition, constant. Therefore, along an equipotential, $F_{\rm grav}$ must do no work and the field component must be zero. The direction of the field at any point will be that in which the potential is decreasing most rapidly, and that will always be perpendicular to the equipotential surfaces. Equipotentials can therefore be used to deduce the pattern of field lines, as shown in Figure 17, or vice versa.

❖ Describe the electrostatic equipotentials near a large flat sheet of uniformly distributed charge.

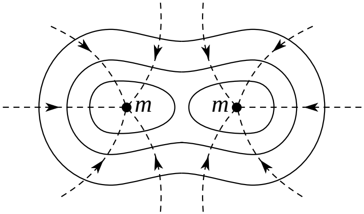



**Figure 17** Equipotential contours (full curves) surrounding two masses $m$ close together. Gravitational field lines (dashed lines).

## Question T19

Sketch the equipotentials for the charges and field lines shown in Figure 25 (see Answer T11). ❑

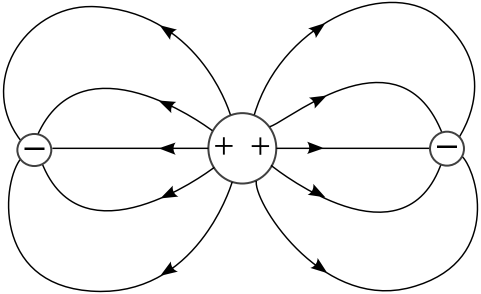



**Figure 25** See Answer T11.

# 4 Closing items

## 4.1 Module summary

1 A *field* is a physical quantity to which a definite value can be ascribed at each point in some region of space at a particular time. For a *scalar field* (e.g. $T(\boldsymbol{r})$) we need to specify only the numerical value (and the appropriate unit of measurement) of the quantity at all points in the region. For a *vector field* (e.g. $\boldsymbol{E}(\boldsymbol{r})$) we generally need to specify the three components of a vector at all points in the region.

2 According to *Newton's law of gravitation* the *gravitational force* on a point mass $m_2$ due to a point mass $m_1$ is given by

$$\boldsymbol{F}_{21} = -\frac{Gm_1m_2}{r^2}\boldsymbol{\hat{r}} \qquad \text{(Eqn 1)}$$

where $G$ is the *universal gravitational constant*, $r$ is the distance from $m_1$ to $m_2$ and $\boldsymbol{\hat{r}}$ is a *unit vector* that points from $m_1$ to $m_2$. (A unit vector parallel to a given vector $\boldsymbol{r}$ is defined by $\boldsymbol{\hat{r}} = \boldsymbol{r}/|\boldsymbol{r}|$, and has magnitude 1.)

3 According to *Coulomb's law* the *electrostatic force* on a point charge $q_2$ due to a point charge $q_1$ is given by

$$\boldsymbol{F}_{21} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}\,\hat{\boldsymbol{r}} \qquad \text{(Eqn 4)}$$

where $\varepsilon_0$ is the *permittivity of free space*, $r$ is the distance from $q_1$ to $q_2$, and $\hat{\boldsymbol{r}}$ is a *unit vector* that points from $q_1$ to $q_2$.

4 The *gravitational field* $\boldsymbol{g}(\boldsymbol{r})$ at a point is the gravitational force per unit mass on a *test mass* placed at that point:

$$\boldsymbol{g}(\boldsymbol{r}) = \frac{\boldsymbol{F}_{\rm grav}\ (\text{on } m \text{ at } \boldsymbol{r})}{m} \qquad \text{(Eqn 2)}$$

The *electric field* $\boldsymbol{E}(\boldsymbol{r})$ at a point is the electrostatic force per unit positive charge on a *test charge* placed at that point:

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{\boldsymbol{F}_{\rm el}\ (\text{on } q \text{ at } \boldsymbol{r})}{q} \qquad \text{(Eqn 5)}$$

5 The gravitational and electric fields due to a point particle of mass $m_0$ and/or charge $q_0$ at the origin are, respectively,

$$\boldsymbol{g}(\boldsymbol{r}) = -\frac{Gm_0}{r^2}\,\hat{\boldsymbol{r}} \qquad \text{(Eqn 3)}$$

and $$\boldsymbol{E}(\boldsymbol{r}) = \frac{q_0}{4\pi\varepsilon_0 r^2}\,\hat{\boldsymbol{r}} \qquad \text{(Eqn 6)}$$

6 When vector fields combine according to the *principle of superposition*, the resultant field is the vector sum of the individual fields at each point.

7 A vector field can be represented by directed *field lines*. The (directed) tangent to a field line represents the direction of the field and the density of the lines in any region represents the strength of the field in that region.

8 In a *uniform (gravitational* or *electric) field*, the field is everywhere the same in magnitude and direction. The uniform gravitational field in any small region close to the Earth's surface may be written $\boldsymbol{g}(\boldsymbol{r}) = -g\hat{\boldsymbol{z}}$, where $g$ is the magnitude of the acceleration due to gravity, and a uniform electric field in the $x$-direction may be written $\boldsymbol{E}(\boldsymbol{r}) = E_x\hat{\boldsymbol{x}}$.

9 If a force described by a (conservative) field does work $W$ on a test particle when that particle is displaced, then the *potential energy* that the particle possesses by virtue of its position in the field will change by an amount

$$\Delta E_{\rm pot} = -W$$

The work done by a uniform force $F_x$ over a displacement $\Delta x$ is $W = F_x \Delta x$, ☞ and that done by a varying force $F_r(r)$ (that depends on a single variable $r$) over a displacement from $r = r_1$ to $r = r_2$ is given by

$$W = \int_{r_1}^{r_2} F_r(r)\,dr \qquad \text{(Eqn 8)}$$

10 A point at which $E_{\rm pot} = 0$ may be chosen arbitrarily. Once that choice has been made, $E_{\rm pot}$ at all other places can be deduced from the appropriate expression for $\Delta E_{\rm pot}$.

11 In the field of a mass $m_0$ or charge $q_0$ at $r = 0$, with $E_{\rm pot} = 0$ as $r \to \infty$

$$E_{\rm grav}\ (\text{of } m \text{ at } r) = -\frac{Gmm_0}{r} \qquad \text{(Eqn 10)}$$

and $$E_{\rm el}\ (\text{of } q \text{ at } r) = \frac{qq_0}{4\pi\varepsilon_0 r}$$

In a uniform gravitational field $\boldsymbol{g}(\boldsymbol{r}) = -g\boldsymbol{\hat{z}}$, with $E_{\rm grav} = 0$ at $z = 0$

$$E_{\rm grav}\ (\text{of } m \text{ at } z) = mgz$$

In a uniform electrostatic field $\boldsymbol{E}(\boldsymbol{r}) = E_x\boldsymbol{\hat{x}}$, with $E_{\rm el} = 0$ at $x = 0$

$$E_{\rm el}\ (\text{of } q \text{ at } x) = -qE_xx$$

12 The gravitational or electrostatic force on a test particle is equal to minus the gradient of the particle's potential energy. So, in cases where the force may be specified by a single component that depends on a single variable such as $r$:

$$F_r(r) = -\frac{dE_{\rm pot}}{dr} \qquad \text{(Eqn 11)}$$

13 The *gravitational* (*or electric*) *potential* at any point is equal to the potential energy per unit mass (or charge) at that point.

$$V_{\rm grav}(\boldsymbol{r}) = \frac{E_{\rm grav}\ (\text{on } m \text{ at } \boldsymbol{r})}{m} \qquad \text{(Eqn 12a)}$$

and $$V_{\rm el}(\boldsymbol{r}) = \frac{E_{\rm el}(\text{on } q \text{ at } \boldsymbol{r})}{q} \qquad \text{(Eqn 12b)}$$

In the case of a point mass or charge at $r = 0$

$$V_{\rm grav}(r) = -\frac{Gm_0}{r} \quad \text{and} \quad V_{\rm el}(r) = \frac{q_0}{4\pi\varepsilon_0 r}$$

14 The gravitational or electrostatic field is equal to minus the gradient of its potential. So, in cases where the force may be specified by a single component that depends on a single variable such as $r$:

$$g_r(r) = -\frac{dV_{\rm grav}(r)}{dr} \qquad \text{(Eqn 13)}$$

and $$E_r(r) = -\frac{dV_{\rm el}(r)}{dr} \qquad \text{(Eqn 14)}$$

15 An *equipotential surface* (or *contour*) is one on which the potential has a constant value. Equipotential surfaces (or contours) are always at right angles to field lines, and are closer together where the field is stronger.

***Study comment*** You may now wish to take the *Exit test* for this module which tests these Achievements. If you prefer to study the module further before taking this test then return to the *Module contents* to review some of the topics.

## 4.2 Achievements

Having completed this module, you should be able to:

A1 Define the terms that are emboldened and flagged in the margins of the module.

A2 Distinguish between scalar and vector fields, and give examples of each.

A3 List the properties of gravitational forces.

A4 Determine the gravitational force between two point masses.

A5 Determine the gravitational field due to a simple arrangement of masses.

A6 Apply the principle of superposition to gravitational and electric fields.

A7 Determine the force on an appropriate test particle in a given gravitational or electric field.

A8 Determine the electrostatic force between two point charges.

A9 Determine the electrostatic field due to a simple arrangement of charges.

A10 Draw field lines for simple arrangements of point masses or charges.

A11 In simple cases, calculate the work done and the corresponding change in potential energy when a mass or charge is displaced in a gravitational or electrostatic field.

A12 Determine the potential due to a simple arrangement of masses or charges.

A13 Sketch equipotential surfaces (or contours) for a uniform gravitational or electrostatic field and for simple arrangements of point masses or charges.

A14 Relate the force that a field exerts on a particle to the gradient of the corresponding potential energy, and relate a field to the gradient of its potential.

## 4.3 Exit test

***Study comment*** Having completed this module, you should be able to answer the following questions, each of which tests one or more of the *Achievements*.

**Question E1**

(*A2*) (a) Is the electric field surrounding a point charge a scalar or a vector field?

(b) Do the contours on an Ordnance Survey map represent a field? If so, what kind of field?

**Question E2**

(*A4*) What is the gravitational force between two protons separated by a distance of $2.0 \times 10^{-15}$ m? Take the mass of a proton as $1.67 \times 10^{-27}$ kg.

## Question E3

(*A5*, *A6* and *A7*) Calculate the resultant gravitational field at point O due to two isolated masses one of mass 3 kg at a distance of 1 m and one of mass 4 kg at a distance of 2 m arranged as shown in Figure 19. Hence find the force on a 1 g mass placed at O.

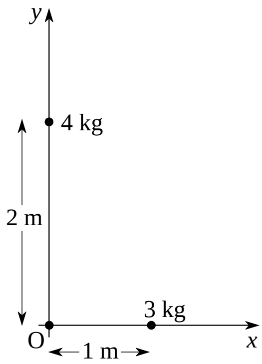



**Figure 19** See Question E3.

## Question E4

(*A6* and *A10*) Sketch in some field lines surrounding the Earth–Moon pair taking into account the fact that the Earth is approximately 80 times more massive than the Moon, and hence its gravitational field (at any given radius) correspondingly stronger. Mark on your sketch the point N where the resultant gravitational field is zero.

## Question E5

(*A4*, *A5* and *A6*) Use the data given in the margin ☞ to calculate the distance of point N (in Question E4) from the Earth. Treat the Earth and Moon as point masses. (*Hint*: If you call the distance of point N from the Earth $r_1$, and the distance of point N from the Moon $r_2$, you can calculate the ratio $r_1/r_2$ directly using $m_E$ and $m_M$. You can then calculate the actual value of $r_1$ using the Earth–Moon distance $R$.)

## Question E6

(*A6*, *A9* and *A10*) Two positive and two negative charges of equal magnitude are arranged at the corners of a square as shown in Figure 20. Draw the field lines associated with the arrangement. What is the magnitude of the electric field at the centre of the square? Indicate the direction of the resultant field at the point A halfway down one side of the square.

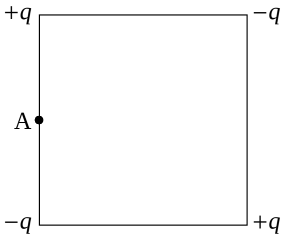



**Figure 20** See Question E6.

## Question E7

(*A12*) Calculate the gravitational potential at a distance of 10 m from a 2 kg mass.

## Question E8

(*A13*) Draw the equipotentials for the charges illustrated in Figure 20.

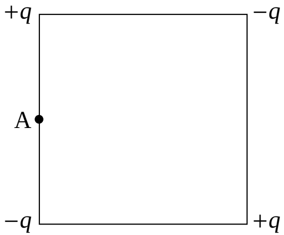



**Figure 20** See Question E8.

***Study comment*** This is the final *Exit test* question. When you have completed the *Exit test* go back to Subsection 1.2 and try the *Fast track questions* if you have not already done so.

If you have completed **both** the *Fast track questions* and the *Exit test*, then you have finished the module and may leave it here.