Module P3.3 Electric charge, field and potential

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1 Opening items

1.1 Module introduction

It is difficult for us in the technologically developed world to imagine life without electricity. It provides light, heat, the power to run railways and underpins the whole electronics industry. All of these applications depend on electric *charges* moving in response to an *electric field*.

Electricity is the source of spectacular natural phenomena such as the aurora borealis and lightning. Lightning is of particular concern to people caught in thunderstorms in exposed places. The *Mountain Leadership Guide* includes the following advice to avoid being struck by lightning:

Move away from peaks. Do not shelter under trees or boulders. Find a place in the open about the same distance from a peak or projection as its height. Sit with your knees drawn up and your hands in your lap. Do not lean back or support your weight on your hands.

Is this good advice? If so why? Could it be improved on? What causes lightning in the first place? These questions will be discussed in Section 5 of this module but first we will establish the fundamentals of field, charge and potential.

We begin in Section 2 with electric *charge* and *Coulomb's law*, then in Section 3 we apply ideas about *electric fields* to an *electric dipole*, and see how *van der Waals forces* ensure that water is a liquid. In Section 4 we review and summarize some key ideas about *electrostatic potential energy*, *electric potential* and *equipotentials* before moving on to a discussion of *point discharge* and thunderstorms in Section 5.

Study comment Having read the introduction you may feel that you are already familiar with the material covered by this module and that you do not need to study it. If so, try the *Fast track questions* given in Subsection 1.2. If not, proceed directly to *Ready to study?* in Subsection 1.3.

1.2 Fast track questions

Study comment Can you answer the following *Fast track questions*?. If you answer the questions successfully you need only glance through the module before looking at the *Module summary* (Subsection 6.1) and the *Achievements* listed in Subsection 6.2. If you are sure that you can meet each of these achievements, try the *Exit test* in Subsection 6.3. If you have difficulty with only one or two of the questions you should follow the guidance given in the answers and read the relevant parts of the module. However, *if you have difficulty with more than two of the Exit questions you are strongly advised to study the whole module*.



Figure 1 See Question F1.

Question F1

Three charges of $+2.00 \,\mu\text{C}$, $-1.00 \,\mu\text{C}$ and $+1.00 \,\mu\text{C}$ are positioned at the corners A, B and C of a square as shown in Figure 1. Calculate (a) the *x*- and *y*-components of the electric field at point D, (b) the electric potential at D, and (c) the *x*- and *y*-components of the force experienced by a charge of $-2.00 \,\mu\text{C}$ at D.

Question F2

(a) Calculate the electric dipole moment of an electron and proton separated by 1.2×10^{-10} m. (b) At a certain point 1.0×10^{-8} m from the dipole, the electric field has magnitude 5.0×10^{5} NC⁻¹. What would be the magnitude of the field at a distance of 1.0×10^{-7} m from the dipole in the same direction?

Question F3

(a) What is the distinction between the electrostatic potential energy of a charge q at some point r in an electric field E(r) and the electric potential at the same point? (b) The potential of a certain distribution of charges depends only on the distance r from a fixed reference point. If the potential is given by V(r) = -ar, where a is a positive constant, what will be the force on a point charge of 2.00×10^{-6} C at a distance of 4.00 m from the reference point?



Study comment Having seen the *Fast track questions* you may feel that it would be wiser to follow the normal route through the module and to proceed directly to <u>*Ready to study?*</u> in Subsection 1.3.

Alternatively, you may still be sufficiently comfortable with the material covered by the module to proceed directly to the *Closing items*.

1.3 Ready to study?

Study comment In order to study this module you will need to be familiar with the following terms: <u>atom, Cartesian</u> <u>coordinates, displacement, force, kinetic energy, nucleus, potential energy, work</u> and be able to apply the <u>principle of</u> <u>conservation of energy. Vectors</u> are used extensively in this module: you must be familiar with the use of <u>components</u> and with the procedure for adding vectors (<u>vector addition</u>) and multiplying them by scalars (<u>scaling</u>). It would also be useful (though not essential) to have met the notions of <u>scalar product</u> before. You should also be familiar with the basic ideas of <u>calculus</u>, including the use of <u>derivatives</u> to represent <u>gradients</u> of graphs, and (<u>definite</u>) <u>integrals</u> to represent certain <u>sums</u>. However, you will not be required to evaluate integrals for yourself, so you do not need to be familiar with the techniques for doing so. Several of these terms are reviewed briefly in the course of this module but unless you are already fairly familiar with them you should consult the *FLAP* modules where they are discussed more fully. You can review them now by referring to the *Glossary* which will indicate where in *FLAP* they are developed. The following *Ready to study questions* will allow you to establish whether or not you need to review some of these topics before embarking on this module.

Question R1

Two vectors A and B lie in the (x, y) plane. The magnitude of A is A = |A| = 2, and A makes an angle of 30° with the positive *x*-axis. B = |B| = 4 and B makes an angle of 30° with the positive *y*-axis (both angles are measured clockwise from the axis mentioned). Calculate the *x*- and *y*-components of C and D where C = A + B and D = A - B.



Question R2

An archer (unwisely) fires an arrow vertically upwards. If the mass of the arrow is 25 g and the bow gives it an initial <u>kinetic energy</u> of 60 J, what is the <u>potential energy</u> of the arrow when it reaches its maximum height? What will be its <u>speed</u> on returning to the point from which it was fired?

(You can assume <u>air resistance</u> is negligible.)



2 Electric charge

2.1 A brief history of electric charge

At some time in your life you must have rubbed a balloon on a woollen pullover and then stuck it to the wall or ceiling (if you haven't, we suggest you try it at the first available opportunity). When you do this you are observing directly the force between electric charges. Similar observations were made by the ancient Greeks, who noticed that rubbing a piece of amber endowed it with strange properties including the ability to attract small particles. Indeed the word 'electricity' derives from the Greek *elektron* meaning amber.

Later experiments, especially in the 18th century, established that many materials exhibit such properties and that there are 'two kinds of electricity'. Objects with the same kind repel each other; objects with different kinds attract each other. The 'two kinds of electricity' can also neutralize each other if brought together and for this reason the pioneering scientist Benjamin Franklin (1706–1790) labelled them positive and negative. The quantity of either kind of electricity in a body is now known as the <u>electric charge</u> of the body and may be expressed as a multiple of the SI unit of charge, the <u>coulomb</u> (C).

Electrically charged particles play a fundamental part in nature; all the familiar forms of matter are ultimately composed of them. The *atoms* that make up gases, liquids and solids consist of negatively-charged particles called <u>electrons</u> swarming around a central *nucleus* that contains positively-charged particles called <u>protons</u> and uncharged <u>neutrons</u>. The *atoms* around a central *nucleus* that contains positively-charged particles called <u>protons</u> and uncharged <u>neutrons</u>. Although the charges of protons and electrons differ in sign they both have the same magnitude, usually denoted by the (positive) quantity *e*. In 1909, the American physicist Robert Millikan (1868–1953) began a series of experiments with electrons which indicated that the magnitude of the charge on the electron is $e \approx 1.6 \times 10^{-19}$ C. Thus Numerous experiments since then have confirmed this result and the most accurate value to date is

 $e = (1.602\,177\,38 \pm 0.000\,000\,48) \times 10^{-19}\,\mathrm{C}$

At the present time, every charged particle that has been experimentally isolated has been found to have a charge that is equal to *e* multiplied by some positive or negative whole number. This phenomenon is often described as the **quantization of charge** since it indicates that *e* is a fundamental 'unit' or 'quantum' of charge. The origin of charge quantization is still not fully understood, and there is strong evidence that various particles, including protons and neutrons, contain constituents called *quarks*, with charges 2e/3 and -e/3. However, it does not appear to be possible to isolate individual quarks, so the *directly observable* fundamental charge remains *e* rather than e/3.

Under normal circumstances, a sample of ordinary matter contains the same number of protons and electrons so that the effects of their charges cancel exactly and most objects are electrically neutral. Yet it is worth reflecting that the amount of charge contained in macroscopic objects is enormous. Your body contains something like 10^{28} protons and electrons, equivalent to 10^9 C of each kind of charge.

Another observed property of electric charge is that the net quantity of electric charge in the universe never changes. Charged particles can be created or destroyed in nuclear and subnuclear processes, but whenever this happens it always involves groups of two or more particles and occurs in such a way that the net charge at the start of the process is equal to the net charge at the end of the process. This observation is embodied in the **principle of conservation of charge**:

The net amount of charge in the universe is constant. \square

Defining the coulomb

This module is concerned primarily with <u>electrostatics</u> — the study of situations in which electric charges are stationary, or as near stationary as to make no difference. However, the formal definition of the coulomb, the SI unit of charge, is based on the concept of an *electric current* which involves the *movement* of charge. The SI unit of electric current is the *ampere* which may defined as follows:

The **ampere** (A) is that constant current which, if maintained in each of two infinitely long, straight, parallel wires of negligible cross section, placed 1 metre apart, in a vacuum, will cause each wire to experience a force of magnitude 2×10^{-7} N per metre of its length.

Now, if a constant current flowing in a wire causes a quantity of charge $\Delta q \leq t$ to flow past a fixed point in time Δt , we say that the current in the wire (expressed in amperes, or amps for short) is

$$I = \frac{\Delta q}{\Delta t}$$

It follows from this that we may define the coulomb as follows:

1 coulomb is the amount of charge transferred when an electric current of 1 ampere flows for 1 second, so 1 C = 1 A s.

• It was mentioned earlier that your body contains about 10^9 C of each kind of charge. How long would it take for a current of 1.00 A flowing past a fixed point on your body to transfer 1.00×10^9 C?



2.2 Charging by contact and by induction

When two suitable materials (for example glass and silk, or polythene and wool) are rubbed together, one acquires a net positive charge and the other a net negative charge. The exact mechanism is not completely understood but it is clear that electrons or ions must have been transferred between the two materials. If the materials were originally electrically neutral, they will acquire equal and opposite net charges as a result of the charge transfer. These net charges will each be an extremely tiny fraction (perhaps $1/10^{15}$) of the total positive and negative charges available within the two objects.

Materials can be broadly classified according to the freedom with which charge can move within them. An <u>electrical conductor</u> is a material in which charge moves freely; examples include metals, bodily fluids and tap water. Metals are good conductors because they contain many electrons which are only weakly held to individual atoms. In other materials such as salt solution or bodily fluids, it is ions rather than electrons which are free to move. A material in which charge does not move freely is called an <u>electrical insulator</u>. Dry wood, rubber and most plastics are examples of insulators. $\underline{\textcircled{P}}$ Excess charge deposited on an insulator tends to remain localized near the point of deposition.

If a conductor carries a net charge, positive or negative, the mutual repulsion between like charges will tend to force the excess charges to the surface of the conductor. Consequently, given the highly mobile nature of charges in a conductor, we never expect to find any net static charge inside an isolated conductor. This also ensures that there is no net electrical force within the conductor. If a conductor is brought near to a charged object, the charges within the conductor will redistribute themselves in such a way that the net force on each charge is zero. This redistribution of charge again ensures that there are no net electrical forces anywhere inside a conductor — and this applies to the space inside a *hollow* conductor as well as to a solid conducting body.

This useful result is exploited in <u>electrostatic screening</u>: if you want to protect something (such as a delicate piece of apparatus) from any external electrical influences, you simply enclose it in a metal container.

If a charged conductor is brought into contact with an uncharged conductor, the excess charge will be distributed between the two; this <u>charge sharing</u> means that the second conductor becomes charged. Once again the redistribution will eliminate net electrical forces within the conductor.

Charge can also be accumulated on conductors by a process known as <u>electrostatic induction</u>. If, say, a negatively charged body is brought close to a conductor, negative charges on the conductor will be repelled and positive charges will be attracted. The net effect will be a separation of charge on the conductor. If the side opposite the charged body is now connected to the Earth (which can be regarded as a huge conductor), negative charge will flow to the Earth leaving the conductor with an overall positive charge. This process is illustrated in Figure 2.

Note The term *induction* is also used to describe the production of electric current from a changing magnetic field. This is a completely unrelated use of the word. In this module *induction* will always mean *electrostatic induction*.





✤ Figure 2 illustrates the fact that when a charged body is connected to the Earth, free excess charge will flow from the body to the Earth. Why does this happen?

When a body is connected to the Earth by a conducting pathway we say it is <u>earthed</u>, or *connected to earth*. The process of connecting a body to earth is called <u>earthing</u>.

Figure 2 Charging by induction. (a) A negatively charged body (A) and a neutral conductor (B). (b) The conductor is brought close to the charged body giving rise to some separation of positive and negative charge. (c) The side furthest from the charged body is connected to the Earth (as indicated by the special earth symbol) and the excess negative charge migrates along this connection. (d) The conductor is now positively charged. The charged body retains its original negative charge.



2.3 Coulomb's law and electrostatic forces

The nature of the force between charges was first investigated by Charles Augustin de Coulomb (1736–1806) in 1785. In a series of painstaking experiments he looked at the way in which the force between charged bodies varies with their separation and with the size and sign of the charge. His work resulted in <u>Coulomb's law</u> which can be stated as follows: $\underline{\textcircled{M}}$

The magnitude of the electric force between two charged particles is proportional to the product of their charges and inversely proportional to the square of the distance between them. The force is directed along a line joining the particles and is repulsive for charges of the same sign and attractive for charges of opposite sign.

The force between charged particles is referred to as the **<u>Coulomb force</u>** or the <u>electrostatic force</u>.

Coulomb's law is expressed in terms of charged particles (that is charges whose dimensions are vanishingly small). However, a *spherically symmetric* charge distribution of total charge q has exactly the same effect at any point beyond its own surface as a point charge q located at the geometric centre of the distribution. Thus, any results that are derived for point charges will also apply to spherically symmetric charge distributions, such as charged spheres, *provided we confine our attention to points that are outside the distribution concerned*.

In mathematical terms, Coulomb's law implies that the *magnitude* of the electrostatic force on a point charge q_2 due to another point charge q_1 a distance r away is

$$F_{\rm el} = \left[\text{positive constant}\right] \times \frac{|q_1 q_2|}{r^2} \tag{1a}$$

Because of the $1/r^2$ dependence, we say that the Coulomb force satisfies an <u>inverse square law</u>. Note that F_{el} is a magnitude, and therefore a positive quantity, that is why Equation 1a involves the *modulus* $|q_1q_2|$ which is positive, rather than simply q_1q_2 which might be positive or negative. Of course, to describe the Coulomb force fully, we need to specify its direction as well as its magnitude. We can do this by introducing a vector \mathbf{r} that points from q_1 to q_2 and using it to define a <u>unit vector</u> $\hat{\mathbf{r}} = \mathbf{r}/|\mathbf{r}|$. Defined in this way, by dividing the vector \mathbf{r} by its own magnitude r, the unit vector points in the same direction as \mathbf{r} (i.e. from q_1 to q_2) but it has magnitude 1 and is therefore dimensionless and unitless. Using the unit vector we can write the electrostatic force on q_2 due to q_1 as

$$\boldsymbol{F}_{21} = [\text{positive constant}] \times \frac{q_1 q_2}{r^2} \hat{\boldsymbol{r}}$$
 (1b)

If q_1 and q_2 have the same sign q_1q_2 will be positive and the force on q_2 due to q_1 will point in the same direction as $\hat{\boldsymbol{r}}$, i.e. away from q_1 . However, if q_1 and q_2 have opposite signs q_1q_2 will be negative and \boldsymbol{F}_{21} will point towards q_1 . In the SI system, the positive constant is written as $1/(4\pi\epsilon)$, where ϵ is known as the <u>permittivity</u> of the medium between the charges, so

$$\boldsymbol{F} = \frac{q_1 q_2}{4\pi \varepsilon r^2} \,\hat{\boldsymbol{r}} \tag{1c}$$

The seemingly unnecessarily complicated form of the constant actually makes calculations easier (honestly!). The factor of 4π often cancels with other factors of 4π which appear because of the spherical symmetry of Coulomb's law. For charges in a vacuum, ε has the (approximate) value $8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$. This is known as the **permittivity of free space** and is written ε_0 . The presence of a medium reduces the force by a factor $1/\varepsilon_r$, where ε_r (a dimensionless number, greater than 1) is the called **relative permittivity** of the medium and is defined by

 $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_0 \boldsymbol{\varepsilon}_r$

Air has a relative permittivity of $\varepsilon_r = 1.005$, so $\varepsilon = 8.898 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$ for air.

Coulomb forces, like all other forces, are vectors and therefore they can be added according to the normal rules of <u>vector addition</u>. To find the force on a particular charged body due to a number of other charged bodies, we simply add the individual Coulomb forces vectorially.

Question T1

A hydrogen atom consists of 1 electron and 1 proton arranged such that the proton can be considered to be at the centre of a sphere with the electron at a distance of 5.0×10^{-11} m away. What is the magnitude of the Coulomb force between them? You may assume that the relevant value of the permittivity is ε_0 in this case. \square

Question T2

What is the magnitude of the force of attraction between the protons in your body and the electrons in the body of a friend standing 1 km away from you? Why don't you hurtle towards each other?



Question T3

For the arrangement of charges shown in Figure 3, find the force on the charge at C due to the charges at A and B. Recalculate the force if the $+1.0 \,\mu\text{C}$ charge at A is replaced by a charge of $-1.0 \,\mu\text{C}$.



Figure 3 See Questions T3 and T5.

3 Electric fields

3.1 Electric field: definition and representation

Study comment Fields (electric and gravitational) are introduced and discussed more fully elsewhere in *FLAP*. Consult the *Glossary* for references to those fuller treatments if you need them.

If you place an electric charge at some distance from another charge or from a group of charges, it experiences an electrostatic force, which acts 'at a distance' since there is no material connection between the charges. The question arises: how is the force transmitted? One plausible answer is that the region around a charge has a special property which causes a force to be exerted on other charges within it. This is the attitude conventionally adopted by physicists, and the property concerned is called the <u>electric field</u>.

Thus, rather than thinking of a system of charges genuinely exerting a force 'at a distance' on another charge, we prefer to think of a system of charges generating an electric field throughout space and the field acting on any charge placed within it. This may appear to be a rather subtle distinction but this way of thinking has many advantages. In particular, if you can specify the electric field throughout a region in space, you can work out the force on any charge at any point within the region without having to refer back to the system of charges which actually caused the force.

Having established the general idea that an electric field exists at all points throughout a region of space, we need to know how to quantify it, that is to assign a value to it at every point. Now, the role of the electric field is to tell us the force that would act on a charged particle placed at any point within it, so it makes sense to define the field in terms of that force. The usual procedure is to imagine that we are in possession of a <u>test charge</u> q that is sufficiently small that it will not significantly disturb the field it is being used to measure, and which we can place at any point. If we locate the test charge q at a point with position coordinates (x, y, z) then we can measure the electrostatic force \mathbf{F}_{el} (on q at (x, y, z)) that acts on it at that point. Now you should be able to see from Coulomb's law that this force will always be proportional to the q itself, so it is *not* purely characteristic of the charges that produce the electric field. However, if we divide \mathbf{F}_{el} (on q at (x, y, z)) by q then the resulting vector quantity will be independent of q and will be determined by the original charge distribution alone. It is this quantity, \mathbf{F}_{el} (on q at (x, y, z)) divided by q, that defines the electric field at (x, y, z).

Since the electric field is a vector quantity, and since its value varies from point to point, we may symbolize it E(x, y, z), thus we arrive at the basic definition of the electric field:

$$\boldsymbol{E}(x, y, z) = \frac{\boldsymbol{F}_{el}(\text{on } q \text{ at } (x, y, z))}{q} \quad \text{or} \quad \boldsymbol{F}_{el} = q\boldsymbol{E}$$
(1d)

This is such a fundamental definition that it deserves to be spelt out in words, so that its meaning is absolutely clear.

The electric field at any point is the electrostatic force per unit (positive) charge that would be experienced by a test charge placed at that point.

Note that the electric field is defined in terms of the force that *would* be experienced by an appropriately located test charge, so the field exists whether or not a test charge is actually present to experience its effect.

• What would be suitable SI units for the measurement of an electric field?

It can become rather tedious to repeatedly write E(x, y, z) each time we want to refer to the electric field at a point, so it is common to use the position vector \mathbf{r} as a shorthand for the coordinates (x, y, z) and write the field as $E(\mathbf{r})$. We will usually adopt this convention, but it is important to realize that $E(\mathbf{r})$ does not necessarily point in the direction of the position vector \mathbf{r} . The role of the parenthetical \mathbf{r} is simply to remind you that the magnitude and direction of the electric field will generally vary from place to place. Thus, if we were to write it out in full, in terms of its Cartesian components, we could equally well write the electric field in either of the following forms:

 $\boldsymbol{E}(\boldsymbol{r}) = (E_x(\boldsymbol{r}), E_y(\boldsymbol{r}), E_z(\boldsymbol{r}))$

or $\boldsymbol{E}(x, y, z) = (E_x(x, y, z), E_y(x, y, z), E_z(x, y, z))$

Many authors avoid such complications entirely by writing the electric field simply as E or (E_x, E_y, E_z) as appropriate, but in doing so they are leaving it to you to remember that the field and its components may vary with position.

Having discussed the general definition of the electric field we can now consider the electric field of a specific charge distribution. The simplest such distribution consists of a single point charge Q at the origin of a Cartesian coordinate system. From Coulomb's law, the electrostatic force on a test charge q at the point with position vector $\mathbf{r} = (x, y, z)$ is then:

$$\boldsymbol{F} = \frac{Qq}{4\pi\varepsilon r^2}\,\hat{\boldsymbol{r}}$$

where $r = |\mathbf{r}| = \sqrt{x^2 + y^2 + z^2}$, and $\hat{\mathbf{r}}$ is the *unit vector* defined by $\hat{\mathbf{r}} = \mathbf{r}/|\mathbf{r}|$. From this expression for the electrostatic force, and by comparing Equations 1a and 1d

$$F_{el} = [\text{positive constant}] \times \frac{|q_1 q_2|}{r^2}$$
(Eqn 1a)
$$\boldsymbol{E}(x, y, z) = \frac{\boldsymbol{F}_{el} (\text{on } q \text{ at } (x, y, z))}{q} \text{ or } \boldsymbol{F}_{el} = q\boldsymbol{E}$$
(Eqn 1d)

you should be able to see that at any point r the electric field of the point charge Q is:

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{Q}{4\pi\varepsilon r^2}\,\hat{\boldsymbol{r}} \tag{2}$$

Since the electric field is a vector quantity, the total field at a given position due to a system of point charges can be found using vector addition to add together the electric fields due to each of the charges individually.

Question T4

What is the electric field at a distance of 1.00 m from a point charge of +1.00 C?

Question T5

Calculate the electric field at point C in Figure 3 due to the charges at A and B. (You can ignore the charge at C.) \Box



Figure 3 See Questions T3 and T5.

We can represent an electric field diagramatically using <u>electric field lines</u>. These are <u>directed lines</u> (i.e. lines with arrows on them) that have the following properties:

- \circ The lines may begin and end on charges, but are otherwise continuous.
- \circ The lines are drawn so that at any point the field is tangential to the lines.
- At any point the direction of the lines shows the direction of the field. (Since the field is the force per unit positive test charge, the lines are directed away from positive charges and towards negative charges.)
- The density of the lines is proportional to the field strength. ('Density' is taken to mean the number of lines per unit area perpendicular to the field direction.)

Figure 4a shows field lines associated with a (positive) point charge, and Figure 4b shows the field line representation of the electric field surrounding a pair of charges of equal magnitude but opposite sign. Field directions throughout the region of interest, and areas of high and low field strength, can be seen at a glance. Field line diagrams also indicate the relative sizes of charges. If we double the size of the point charge in Figure 4a, we double the field strength everywhere and therefore we need to double the line density everywhere. This means we double the number of lines emanating from the charge.



Figure 4 Field lines of (a) an isolated positive charge, (b) two charges of equal magnitude and opposite sign.

Question T6

If the positive charge in Figure 5 is +1.0 C, what are the sizes of the other charges in the figure? \Box

It is worth remarking that the quantitative interpretation of field lines only works because the electric field due to a point charge obeys an *inverse square law*. It can easily be shown that the density of lines drawn radially from a point also obeys an inverse square law and so is a true representation of the field. As any field can be built up by summing the fields due to a distribution of point charges, so any field can be represented by field lines. If the electric field of a point charge were proportional to $1/r^3$, say, rather than $1/r^2$, the field line representation would not work.

• The electric field at a certain point is given by

 $\boldsymbol{E} = (E_x, E_y, E_z) = (10 \text{ N C}^{-1}, 15 \text{ N C}^{-1}, -20 \text{ N C}^{-1})$

What is the force \boldsymbol{F} on a particle of charge -4 C located at the point?



Figure 5 See Question T6.



3.2 The field of an electric dipole

As a further example of the electric field of a distribution of point charges, we will now examine another simple case — the field due to an electric dipole in a vacuum.

An <u>electric dipole</u> consists of two charges of equal magnitude but opposite sign which are very close together.

Such a pair is shown in Figure 6. For convenience, we choose a coordinate system such that the charges lie on the *x*-axis, equidistant from the origin, with the charge +q at x = +a and the charge -q at x = -a. Because of the symmetry of this charge distribution we need only consider the field in the (x, y) plane; whatever pattern we find there will be repeated in every other plane that includes the *x*-axis. We can therefore find the full three-dimensional field pattern by rotating the field in the (x, y) plane about the *x*-axis. Having effectively reduced the problem to



Figure 6 An electric dipole and the position vector *r* of some arbitrary point A.

two dimensions we can start by deriving an expression for the field at some point along the *x*-axis, let us call it (x, 0). As both charges lie on the *x*-axis, the field vector at (x, 0) will always point in the *x*-direction, so $E_y(x, 0)$ will be zero and we only need to work out the *x*-component, $E_x(x, 0)$.

Using subscripts 1 and 2 for the charges at -a and +a, respectively, application of Equation 2,

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{Q}{4\pi\varepsilon r^2}\,\hat{\boldsymbol{r}} \tag{Eqn 2}$$

in a vacuum where $\varepsilon = \varepsilon_0$, gives:

$$E_{x1}(x,0) = \frac{-q}{4\pi\varepsilon_0(x+a)^2}$$

and
$$E_{x2}(x,0) = \frac{+q}{4\pi\varepsilon_0(x-a)^2}$$

so
$$E_x(x,0) = E_{x1} + E_{x2} = \frac{q}{4\pi\varepsilon_0} \left[\frac{1}{(x-a)^2} - \frac{1}{(x+a)^2} \right]$$

i.e.
$$E_x(x,0) = \frac{q}{4\pi\varepsilon_0} \left[\frac{(x+a)^2 - (x-a)^2}{(x-a)^2(x+a)^2} \right]$$

so
$$E_x(x,0) = \frac{q}{4\pi\varepsilon_0} \left[\frac{4ax}{(x-a)^2(x+a)^2} \right]$$

If the charges are very close together, and if we take |x| >> a then, to a good approximation

$$E_x(x,0) = \frac{4aqx}{4\pi\varepsilon_0 x^4} = \frac{aq}{\pi\varepsilon_0 |x|^3}$$
(3)

When dealing with the field of a dipole the product of the charge magnitude and separation is called the electric **dipole moment** p. So, for two charges of magnitude q separated by a distance d:

$$p = qd \tag{4}$$

In this case the separation is 2a, so p = 2aq and Equation 3 may be written

$$E_x(x,0) = \frac{p}{2\pi\varepsilon_0 |x|^3}$$
(5a)

Recalling that $E_{y}(x, 0) = 0$ (5b)

we now know the electric field at any point on the *x*-axis where |x| >> a.

Question T7

 $E_{\rm v}(0, y) = 0$

and

For the dipole in Figure 6, show that at any point on the y-axis with coordinates (0, y), where |y| >> a,

$$E_x(0,y) = \frac{-p}{4\pi\varepsilon_0 |y|^3} \tag{6a}$$

The derivation of the dipole field on the *x*- and *y*-axes (Equations 5 and 6)

$$E_x(x,0) = \frac{p}{2\pi\varepsilon_0 |x|^3}$$
 (Eqn 5a)
$$E_y(x,0) = 0$$
 (Eqn 5b)

is relatively straightforward because of the symmetry of the charges.

However, it is not so much more difficult (just a bit more long-winded) to derive a general expression for $E(x, y) = (E_x(x, y), E_y(x, y))$ at any point in the (x, y) plane such as A in Figure 6, provided $r = \sqrt{x^2 + y^2} >> a$.

(6b)



Figure 6 An electric dipole and the position vector *r* of some arbitrary

point A.

The final result is:

$$E_x(x,y) = \frac{p}{4\pi\varepsilon_0 r^3} (2\cos^2\theta - \sin^2\theta)$$
(7a)

$$E_y(x,y) = \frac{p}{4\pi\varepsilon_0 r^3} (3\cos\theta\sin\theta)$$
(7b)

where $\mathbf{r} = (x, y)$ and θ are as shown in Figure 6.

Equations 7a and 7b show that:

At distances r which are large compared with the charge separation, the electric field of a dipole decreases in proportion to $1/r^3$ and also depends on θ .

Although we now have a complete description of the field in the (x, y) plane, it is still quite difficult to visualize the field. This is where field lines are useful. In fact you have already seen the dipole field in Figure 4b. Figure 4b gives us a 'feel' for the way the field behaves. Equations 7a and 7b allow us to do quantitative calculations for the field at any point in space.
Question T8

Show that Equations 7a and 7b

$$E_x(x, y) = \frac{p}{4\pi\varepsilon_0 r^3} (2\cos^2\theta - \sin^2\theta)$$
(Eqn 7a)
$$E_y(x, y) = \frac{p}{4\pi\varepsilon_0 r^3} (3\cos\theta\sin\theta)$$
(Eqn 7b)

are consistent with Equations 5 and 6.

$$E_x(x,0) = \frac{p}{2\pi\varepsilon_0 |x|^3}$$
(Eqn 5a)

$$E_{y}(x,0) = 0 \tag{Eqn 5b}$$

$$E_x(0, y) = \frac{-p}{4\pi\varepsilon_0 |y|^3}$$
(Eqn 6a)
$$E_y(0, y) = 0$$
(Eqn 6b)



3.3 The van der Waals force

There are many situations in the real world which can be approximated by electric dipoles. You have come across one already in Subsection 2.2 in the description of charging by induction. The displaced positive and negative charges on the conductor in Figure 2 behave to a good approximation like two point charges separated by a small distance — an electric dipole.



Figure 2 Charging by induction. (a) A negatively charged body (A) and a neutral conductor (B). (b) The conductor is brought close to the charged body giving rise to some separation of positive and negative charge. (c) The side furthest from the charged body is connected to the Earth (as indicated by the special earth symbol) and the excess negative charge migrates along this connection. (d) The conductor is now positively charged. The charged body retains its original negative charge.

Many *molecules* exhibit dipolar behaviour due to asymmetries in the distribution of their electrons. Water is a good example of this. The arrangement of atoms in a water molecule is shown in Figure 7. Electrons spend more time on average near the oxygen atom (O) than near the hydrogen atoms (H) that are bound to it, so the O side of the molecule has a net negative charge while the H side has a net positive charge. In effect it is a dipole.

The mutual attraction between charges of opposite sign on different molecules causes them to clump together (provided their kinetic energy is not too high) eventually forming crystals of solid H_2O or ice. It is this electrostatic attraction between molecular dipoles that allows water to be a liquid in a temperature range where similar sized molecules with little or no dipole moment form gases. This is very fortunate for us. If it were not for the fact that water is a liquid in just the right temperature range, life as we know it would never have developed on Earth!



Figure 7 The water molecule (H_2O) . (a) The positions of the atoms. (The small atoms are the hydrogen atoms, as indicated by the orientation of the water molecule shown.) (b) The equivalent dipole. The electrostatic forces between molecular dipoles are known as **van der Waals forces** after Johannes van der Waals (1837–1923) . Van der Waals forces can be important even for atoms and molecules which do not have a permanent dipole moment. For example, these forces are responsible for the formation of solids from the neutral atoms of the *noble gases.* This occurs because the motion of electrons within the atoms can give rise to instantaneous asymmetries in the charge distribution. The resulting transient dipoles can then induce dipoles in nearby atoms (see Figure 8). These act to reinforce the original transient dipole as well as inducing dipoles in other nearby atoms in their turn. Thus a network of mutually attracting dipolar atoms is built up, eventually binding together to form a solid if their kinetic energies are sufficiently low.

Figure 8 (a) An instantaneous asymmetry in the charge distribution of an atom gives rise to (b) an induced dipole in a nearby atom which reinforces the original dipole and also (c) induces a dipolar distribution in a third atom.



4 Electric potential and electric field

Calculations with vector quantities such as forces and fields can become very complicated, especially if vector addition has to be used repeatedly. This difficulty can be avoided sometimes by thinking in terms of *energy* (which is a scalar quantity) rather than force (which is a vector). This section introduces the related concepts of *electrostatic potential energy* and *electric potential* that are used in such situations.

4.1 Electrostatic potential energy

Figure 9 shows a particle of positive charge +q in a <u>uniform</u> electric field, i.e. in a field that has the same magnitude and direction at every point. Coordinates have been chosen in such a way that the field points along the *x*-axis, so its only non-zero component is E_x , and even that is the same everywhere due to the uniformity of the field. The charged particle is initially stationary, but it will be subject to an electrostatic force $F_x = qE_x$ in the *x*-direction, so if free to move it will accelerate in the direction of the force in accordance with Newton's second law of motion P. As a result of such motion the particle will gain <u>kinetic energy</u>. The kinetic energy E_{kin} gained in any movement, is equal to the *work* W done by the electrostatic force on the charge P.



Figure 9 A positive charge +q in a uniform electric field E.

In moving through a displacement Δx , the work done will be $qE_x\Delta x$, so this will change the kinetic energy by an amount ΔE_{kin} given by:

$$\Delta E_{\rm kin} = q E_x \Delta x \tag{8}$$

An alternative way of describing the same situation is to say that the charge possesses a certain amount of <u>electrostatic potential energy</u> (or <u>electrical energy</u> for short) which depends on its position in the field. The increase in kinetic energy is exactly balanced by a decrease in electrostatic potential energy so that the total energy change is zero and we may write.

$$\Delta E_{\rm el} + \Delta E_{\rm kin} = 0 \tag{9}$$

So, $\Delta E_{\rm el} = -\Delta E_{\rm kin}$, but we already know that $\Delta E_{\rm kin} = qE_x\Delta x$

Hence, in a uniform field the change in electrostatic potential energy as a result of a displacement parallel to the field is:

$$\Delta E_{\rm el} = -qE_x \Delta x \tag{10}$$

Question T9

In an electron gun in a television set, the electrons which strike the screen to create the picture are accelerated by travelling a distance of 1.00 cm in a uniform electric field of $3.00 \times 10^4 \,\mathrm{N\,C^{-1}}$. Assuming the electrons are initially at rest, calculate (a) the potential energy lost by the electrons when they are accelerating and (b) their final speed.

For a charge moving in a non-uniform field $E(\mathbf{r})$, the calculation of electrostatic potential energy is more complicated. As a first step towards finding the general expression for the potential energy of such a charge suppose that the motion is restricted to the *x*-direction, so we only have to worry about the field component in the *x*-direction, $E_x(\mathbf{r})$. \leq We can imagine that the charge *q* moves in short steps of length Δx that are small enough that $E_x(\mathbf{r})$ can be regarded as constant for each step. The work done in a single small step starting at \mathbf{r} will be $\Delta W = qE_x(\mathbf{r}) \Delta x$, and the total work done in moving from x_1 to x_2 will be approximately equal to the sum of the work in each small step, which we can write

 $W\approx \sum q E_x(\boldsymbol{r})\Delta x$

This approximation will become increasingly accurate as the steps become smaller and smaller, with the consequence that in the limit as the steps become vanishingly small we can write

$$W = \lim_{\Delta x \to 0} \left[\sum q E_x(\boldsymbol{r}) \Delta x \right] = q \lim_{\Delta x \to 0} \left[\sum E_x(\boldsymbol{r}) \Delta x \right]$$

Now, limits of sums of this kind are known as definite integrals, and are conventionally written as follows

$$W = q \int_{x_1}^{x_2} E_x(\boldsymbol{r}) dx \tag{11}$$

hence
$$\Delta E_{\rm el} = -W = -q \int_{x_1}^{x_2} E_x(\mathbf{r}) dx$$
 (12a)

In this case, we chose to consider motion in the x-direction so that we only had to consider a single component (E_x) of the non-uniform field. More generally, in two or three dimensions, the motion may be from any point \mathbf{r}_1 to any other point \mathbf{r}_2 and the movement may be along any pathway between those two points. For convenience we may consider this motion to be along x then along y and finally along z.

A simple extension of Equation 12a

$$\Delta E_{\rm el} = -W = -q \int_{x_1}^{x_2} E_x(\boldsymbol{r}) \, dx \tag{Eqn 12a}$$

then leads to

$$\Delta E_{\rm el} = -W = -q \left[\int_{x_1}^{x_2} E_x(\mathbf{r}) \, dx + \int_{y_1}^{y_2} E_y(\mathbf{r}) \, dy + \int_{z_1}^{z_2} E_z(\mathbf{r}) \, dz \right]$$

The quantity in square brackets can be expressed as the scalar product

$$\int_{r_1}^{r_2} \boldsymbol{E}(\boldsymbol{r}) \cdot d\boldsymbol{r}$$

which is called a *line integral*.

Under these circumstances the change in the electrostatic potential energy is given by:

$$\Delta E_{\rm el} = -q \int_{\boldsymbol{r}_1}^{\boldsymbol{r}_2} \boldsymbol{E}(\boldsymbol{r}) \cdot d\boldsymbol{r}$$
(12b)

Like other integrals this is just a shorthand for a limit of a sum, though in this case the quantity being summed is the <u>scalar product</u> $E(\mathbf{r}) \cdot \Delta \mathbf{r}$ where $\Delta \mathbf{r}$ is a small displacement from one point to another on the relevant path from \mathbf{r}_1 to \mathbf{r}_2 , and $E(\mathbf{r}) \cdot \Delta \mathbf{r} = |E(\mathbf{r})| |\Delta \mathbf{r}| \cos \theta$, where θ is the angle between $E(\mathbf{r})$ and $\Delta \mathbf{r}$ at any point \mathbf{r} .

To show the importance of Equation 12b we will now use it to develop an expression for the electrostatic potential energy of a point test charge q in the field of another point charge Q. The non-uniform field of a fixed point charge, Q, at the origin of coordinates was introduced earlier, in a vacuum it takes the form

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{Q}{4\pi\varepsilon_0 r^2} \,\hat{\boldsymbol{r}} \tag{Eqn 2}$$

This field is directed radially (towards or away from the origin) at all points. It follows that the work done by the electrostatic force when a particle moves from a point A to some other point B (Figure 10) is determined by the radial component of the field, $E_r(\mathbf{r}) = Q/(4\pi\varepsilon_0 r^2)$, the charge q, and the difference in the radial coordinates r_A and r_B of A and B. (Not, e it does *not* depend on the particular path that the charge follows from A to B.) It follows from Equation 12b

$$\Delta E_{\rm el} = -q \int_{\boldsymbol{r}_{\rm l}}^{\boldsymbol{r}_{\rm 2}} \boldsymbol{E}(\boldsymbol{r}) \cdot d\boldsymbol{r} \qquad ({\rm Eqn} \ 12{\rm b})$$

that in this case

thus

i.e.

$$\Delta E_{\rm el} = -q \int_{r_{\rm A}}^{r_{\rm B}} E_r(\boldsymbol{r}) dr \qquad (12c)$$

Figure 10 A charge q in a radial field due to a point charge Q.

$$\Delta E_{\rm el} = -q \int_{r_{\rm A}}^{r_{\rm B}} \frac{Q}{4\pi\varepsilon_0 r^2} dr = \frac{-qQ}{4\pi\varepsilon_0} \int_{r_{\rm A}}^{r_{\rm B}} \frac{1}{r^2} dr = \frac{-qQ}{4\pi\varepsilon_0} \left[\frac{-1}{r}\right]_{r_{\rm A}}^{r_{\rm B}}$$
$$\Delta E_{\rm el} = \frac{qQ}{4\pi\varepsilon_0} \left(\frac{1}{r_{\rm B}} - \frac{1}{r_{\rm A}}\right)$$



(12d)

$$\Delta E_{\rm el} = \frac{qQ}{4\pi\varepsilon_0} \left(\frac{1}{r_{\rm B}} - \frac{1}{r_{\rm A}} \right) \tag{Eqn 12d}$$

This expression for the *change* in the potential energy of the charge q between two positions is useful since it is such *changes* in potential energy that are physically meaningful. Nonetheless, it is sometimes convenient to be able to assign a definite value to the electrostatic potential energy of q when it is at some particular distance r from the origin. In order to do this we need to arbitrarily choose a reference point at which $E_{el} = 0$. The potential energy at any point is then the value of ΔE_{el} corresponding to the displacement from the reference point to the point of interest. When dealing with the field from a point charge, it is convenient to choose $E_{el} = 0$ where $E(\mathbf{r}) = 0$, i.e. when $r \to \infty$. We can then find the electrostatic potential energy of q at a distance r from Q by letting $r_A \to \infty$ and $r_B = r$ in Equation 12d:

$$E_{\rm el}({\rm of } q {\rm at } r) = \frac{qQ}{4\pi\varepsilon_0} \frac{1}{r}$$
(13)

Using Equation 12b

$$\Delta E_{\rm el} = -q \int_{\boldsymbol{r}_1}^{\boldsymbol{r}_2} \boldsymbol{E}(\boldsymbol{r}) \cdot d\boldsymbol{r}$$
 (Eqn 12b)

similar calculations may be carried out for any other electrostatic field, though the mathematics will usually be more complicated and the result will generally depend on the position vector \mathbf{r} of q rather than just its magnitude r. For such general cases we can represent the electrostatic potential energy of the test charge by $E_{\rm el}$ (of q at \mathbf{r}), but its value will still depend on some arbitrary choice of reference position

Question T10

A sphere carries a charge of $+40 \,\mu\text{C}$. A small particle with a charge of $+2.0 \,\mu\text{C}$ is moved from a position outside the sphere and 1.0 m from its centre to a position 1.5 m from its centre.

- (a) Without doing a calculation, decide whether the change in potential energy will be positive or negative.
- (b) Calculate the change in potential energy. Will it make any difference if the particle moves in a straight line between the two positions or follows some complicated path? □

Question T11

What is the electrical potential energy of the electron in a hydrogen atom as described in Question T1?



4.2 Electric potential: definition and representation

In much the same way that we defined the electric field at any point in terms of the force per unit positive charge, it is useful to define a quantity, called the <u>electric potential</u>, which represents the potential energy per unit charge that a test charge would have at any point:

$$V(\boldsymbol{r}) = \frac{E_{\rm el}(\text{of } q \text{ at } \boldsymbol{r})}{q} \tag{14}$$

Note that like the electric field, the electric potential can be simultaneously assigned a value at every point, but $V(\mathbf{r})$, unlike the electric field, is a scalar quantity.

In one dimension, the difference in potential between two positions x_1 and x_2 is (from Equation 12a):

$$\Delta E_{el} = -W = -q \int_{x_1}^{x_2} E_x(\mathbf{r}) dx \qquad (Eqn 12a)$$
$$\Delta V = -\int_{x_1}^{x_2} E_x dx \qquad (15a)$$

and in three dimensions (from Equation 12b):

$$\Delta E_{\rm el} = -q \int_{r_1}^{r_2} \boldsymbol{E}(\boldsymbol{r}) \cdot d\boldsymbol{r}$$
 (Eqn 12b)
$$W = -\int_{r_1}^{r_2} \boldsymbol{E}(\boldsymbol{r}) \cdot d\boldsymbol{r}$$
 (15b)

As with potential energy, we can arbitrarily define some point to have zero potential and then use Equation 15b to work out the potential at any other point. In the particular case of a point charge Q fixed at the origin, it is usual to choose V = 0 when $r \to \infty$, then for any point at a distance r from Q

$$V(r) = \frac{Q}{4\pi\varepsilon_0 r} \tag{16}$$

Since a spherically symmetric charge distribution produces the same external electrical effects as a point charge at its centre, Equation 16 also describes the potential at or beyond the surface of a conducting sphere that carries a net charge Q.

Electric potential is such a frequently used concept that its SI unit has been given a special name — the <u>volt</u> (V). Thus $1 \text{ V} = 1 \text{ J} \text{ C}^{-1}$. We are all familiar with this unit from its association with mains electricity, batteries, etc. Indeed differences in electric potential are often called *voltages*.

♦ Use Equation 14

$$V(\boldsymbol{r}) = \frac{E_{\rm el}(\text{of } q \text{ at } \boldsymbol{r})}{q}$$
(Eqn 14)

to write down an expression for the change in electric potential energy ΔE_{el} when a particle with charge q moves through a potential difference ΔV .

We are sometimes interested in the behaviour of individual electrons or ions in electric fields (as in the electron gun of Question T9). A single electron moving through a potential difference of, say, 2000 V has $\Delta E_{\rm el} = 2000 \,\mathrm{J} \,\mathrm{C}^{-1} \times 1.602 \times 10^{-19} \,\mathrm{C} = 3.204 \times 10^{-16} \,\mathrm{J}$ —a very small energy change. Rather than always working in joules and having to deal with such very small numbers, it is convenient to introduce a different energy unit.

One <u>electronvolt</u> (eV) is defined as the change in the electrostatic potential energy of a single electron when it moves through a potential difference of 1 V. 1 eV = 1.602×10^{-19} C × 1 V = 1.602×10^{-19} J.

The electronvolt is not an SI unit, but it is very widely used to measure small energies. ≤ 2 To find ΔE_{el} in eV for a single charged particle moving through a potential difference, you simply multiply its charge (measured in units of *e not* in coulomb) by the potential difference (measured in volts). For example, an electron moving through a potential difference of 2000 V will change its electrical potential energy by 2000 eV (often written 2 keV). Similarly, if an oxygen ion of charge 2*e*, formed from an oxygen atom that has lost two of its electrons, moves through 2000 V then ΔE_{el} is 4 keV.

Question T12

For a battery, the 'negative' terminal is usually taken to be at zero potential. If a 12 V car battery is connected to two plates within a vacuum, and an electron released from the negative plate travels to the positive plate, what will be the change in the electron's electrostatic potential energy (give your answer in eV and in J)? \Box



The importance of the potential

One of the great advantages of dealing with potentials is the ease with which they may be added together. The electric potential at a point in space due to a number of charges is simply the sum of the potentials due to the individual charges. Thus for a distribution of point charges, Equation 16

$$V(r) = \frac{Q}{4\pi\varepsilon_0 r}$$
(Eqn 16)

can be used repeatedly to calculate the total potential at any point.

The scalar nature of the potential means that it is often relatively straightforward to determine the potential due to a given charge distribution.

Question T13

Calculate the electric potential at point B in Figure 11 due to the charges at A and C.

The other crucial feature of the potential $V(\mathbf{r})$ is that it can be used to derive the associated electrostatic field. We will introduce the relationship between the field and the potential by considering a deliberately simple case in which the electric potential $V(\mathbf{r})$ depends on x but is actually independent of y and z. Under these circumstances we may represent the potential by V(x). If a charge q undergoes a small displacement from x to x + Δx , the electrostatic force that acts throughout the displacement will be approximately given by $qE_x(x)$, and the corresponding change in electrostatic potential energy of the charge will be $\Delta E_{el} \approx -qE_x(x)\Delta x$. It follows that the potential difference between x and $x + \Delta x$ is $\Delta V = \Delta E_{el}/q \approx$ $-E_x(x)\Delta x$. Rearranging this we see that

$$E_x(x) \approx -\frac{\Delta V}{\Delta x}$$



$+2.0\,\mu\text{C} \bullet \text{C}$ $50\,\text{cm}$ $50\,\text{cm}$



This approximation will become increasingly accurate as Δx is reduced, with the result that in the limit as Δx becomes vanishingly small

$$E_x(x) = -\lim_{\Delta x \to 0} \left(\frac{\Delta V}{\Delta x}\right)$$

Now the limit on the right-hand side defines the <u>gradient</u> of the graph of V against x, that is the <u>derivative</u> of V(x) with respect to x:

$E_x(x) = -\frac{dV}{dx}$	(17)	<u> </u>
---------------------------	------	----------

So, the *x*-component of the electric field is given by *minus the gradient of the potential*.

Equation 17 emerged from the study of a deliberately simplified case; more generally the potential will depend on x, y and z, and we will have to work out all three components of the electric field at each point \mathbf{r} . However, similar principles apply even though the mathematics is more complicated. We can summarize this:

- 1 Once the electric potential is known in some region it is always possible to determine the corresponding electrostatic field by taking appropriate derivatives of the potential. The component of the electric field in any direction is minus the potential gradient in that direction.
- 2 The *direction* of the electric field at a point is given by the direction in which the potential decreases most rapidly.
- 3 The *magnitude* of the electric field at a point is the magnitude of the maximum potential gradient, measured along the direction of the electric field.

Equation 17

$$E_x(x) = -\frac{dV}{dx}$$
(Eqn 17)

also shows that the SI unit of electric field can be written in the form $V m^{-1}$ which emphasizes the fact that the field is related to the potential gradient.

• Show that $1 \text{ V m}^{-1} = 1 \text{ N C}^{-1}$.



Equipotentials and the representation of potentials

In <u>Subsection 3.1</u>, you saw how electric field lines provide a useful means of visualizing electric fields. A different kind of visualization can be achieved using the concept of electric potential. In two dimensions a line passing only through points of equal electric potential is known as an <u>equipotential</u> <u>line</u>, and in three dimensions we can similarly define <u>equipotential surfaces</u>. From <u>point 2</u> above we can see that the *electric field at any point must always be perpendicular to such lines or surfaces*.

Figure 12 shows equipotential surfaces surrounding a point charge. The surfaces are separated by equal intervals of potential. The surfaces form concentric spheres showing that the field must be radial everywhere.

Figure 12 Equipotential surfaces surrounding a point charge.



The fact that the surfaces crowd together near the charge shows that the field is stronger in this region. The fact that the field is related to *minus* the gradient of the potential shows that the field points away from regions of relatively high potential, towards regions of lower potential.

Question T14

Figure 13 shows an equipotential plot for a particular charge distribution.

In Figure 13:

- (a) indicate positions of positive and negative charge;
- (b) say whether charges are approximately the same size or, if not, which is the bigger;
- (c) draw lines of electric field on the diagram. \Box



Figure 13 See Question T14.

4.3 Millikan's experiments to measure *e*

In Subsection 2.1 we referred to Millikan's experiments which supplied evidence for the quantization of charge and which provided a value for e, the electronic charge. To arrive at his results, Millikan studied the behaviour of small charged oil droplets in an electric field. Figure 14a shows a schematic diagram of his apparatus.

Figure 14a Schematic diagram of Millikan's apparatus.



A fine spray of oil droplets is produced at the top of the apparatus and, in the process, the droplets become charged by friction. Falling under gravity, 🔄 some of the charged droplets enter the illuminated space between two metal plates that have an adjustable potential difference between them.

The experimenter, observing through the microscope, selects one of the droplets and adjusts the potential difference so that the droplet remains stationary in mid air; the downward force due to gravity on the droplet then being exactly balanced by an upward electrostatic force (Figure 14b). The electric field between the plates is uniform, so (from Equation 17)

$$E_x(x) = -\frac{dV}{dx}$$
(Eqn 17)

it has magnitude

$$E = V/d \tag{18}$$

where V is the balancing potential difference and d the distance between the plates.



Figure 14b The behaviour of a negatively charged oil droplet between the charged plates.

The magnitude of the electrostatic force on a droplet of charge magnitude |q| is $F_{el} = |q| E$ (from Equation 2),

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{Q}{4\pi\varepsilon r^2}\,\hat{\boldsymbol{r}} \tag{Eqn 2}$$

so for a stationary droplet

$$|q|V/d = mg \tag{19}$$

The mass *m* of the droplet can be found either by measuring its diameter (by observing it against a graduated scale) and calculating *m* using the density of the oil, or by measuring the droplet's *terminal velocity* $\leq m$ as it falls through the air in the absence of any electric field. Knowing *V*, *d*, *g* and *m* for any particular droplet, Equation 19 can be used to determine the magnitude and sign of *q*.

Millikan carried out his experiments over several years, and introduced various refinements to improve accuracy. He used a non-volatile oil, so that the mass of a droplet would not change due to evaporation, and convection currents were reduced by enclosing the apparatus in a constant temperature oil bath. In a further improvement, he used X-rays to alter the charge so that he could make several different charge measurements on any one droplet.

By repeating his experiments with a very large number of droplets, Millikan found that the charge magnitude on a droplet was always a whole number multiple of a basic charge e, where $e = 1.6 \times 10^{-19}$ C (to two significant figures).

Question T15

(a) What could you deduce about the basic unit of charge if you measured the magnitude of the charges on some oil drops to be 3.2×10^{-19} C, 6.4×10^{-19} C and 1.28×10^{-18} C? (b) How might your conclusion be affected if you measured a charge magnitude of 4.8×10^{-19} C?

Question T16

An oil drop of mass 9.8×10^{-15} kg is suspended between two plates 1.0 cm apart with a potential difference of 2000 V across the gap. If the drop is negatively charged due to an excess of electrons, how many such electrons must there be on the drop? \square





5 Point discharge and thunderstorms

5.1 Point discharge

Lightning tends to strike tall pointed objects (such as trees and church spires). We can understand why by thinking about the electric field around an irregularly-shaped charged conductor.

If a conductor is charged, the excess charge will distribute itself on the surface so as to minimize the repulsive forces between like charges — it moves until there is no electric field within the conductor. We can put this another way: charge will move so as to minimize its electric potential energy — it will move until there is no potential difference between the various parts of the conductor. 💇 It follows that:

The surface of a conductor will be an *equipotential* surface.

This has some important consequences for the associated field and charge distribution.

Although most of the atmosphere consists of neutral atoms, there are always a few stray electrons and ions, and these charged particles are available to be accelerated by an electric field. Around a charged object such as that in Figure 15, the movement of charge will be greatest in the air near the pointed end where the field is strongest. Positive ions are repelled from the point, and electrons are attracted to it, so the conductor becomes neutralized by charge of the opposite type — the process is known as **point discharge**.



Figure 15 Field lines around an irregularly-shaped conductor that carries a net positive charge.

A conductor with any sharp corners will lose its charge more rapidly than one whose surface is a smooth curve. This is one of the reasons why scientific equipment used in the study of electrostatics often includes spherical conductors (Figure 16).

5.2 Lightning and thunder

We now have the tools with which to tackle the questions about lightning that were posed in the introduction to this module. First of all, what is lightning?

As mentioned in <u>Subsection 5.1</u>, there are always a few charged particles in the air that are available to be accelerated by an electric field. If the field is strong enough they can be accelerated to such high energies that when they collide with atoms they liberate more electrons. The ions and freed electrons are then accelerated in their turn, colliding with more atoms and creating a cascade of charged particles. This converts very large amounts of electrical potential energy into kinetic energy in a very short time giving rise to intense local heating of the air.



Figure 16 A schematic diagram of a Leyden jar (an antique electrical storage device) with a spherical conductor to minimize point discharge.

Temperatures can reach 3×10^4 K which is about five times hotter than the surface of the Sun! We see this as a visible flash. The sudden heating has another effect: it causes the air to expand explosively generating a shock wave which creates the sound of thunder.

The electric field required to cause <u>electrical breakdown</u> of the air in this manner is of the order of 3×10^6 V m⁻¹.



Figure 17 Charge separation in a thunder cloud. (a) The distribution of charges. Field lines between the cloud and Earth are also shown. (b) The charge distribution replaced by point charges.

We know that large fields must be associated with large concentrations of charge. It is apparent therefore that thunderclouds must be highly charged. How does this happen?

The precise mechanism of the charging process is not fully understood but it has to do with the strong rising currents of moist air associated with thunderstorm activity. Ice crystals formed within the cloud fall through a spray of warm, rising water droplets and become negatively charged leaving the droplets with a net positive charge.

This results in a charge separation in the cloud as shown in Figure 17a.

Although the distribution of charges is complex, we can get a rough idea of what is going on by assuming the positive and negatively charged regions are spherical and replacing them with point charges. A typical thunderstorm modelled in this way is shown in Figure 17b. It has negative and positive charges of roughly 50 C at heights of 5 km and 10 km, respectively.



Figure 17 Charge separation in a thunder cloud. (a) The distribution of charges. Field lines between the cloud and Earth are also shown. (b) The charge distribution replaced by point charges.

Question T17

Using the information above

(It has negative and positive charges of roughly 50 C at heights of 5 km and 10 km, respectively.}

calculate the field strength (a) halfway between the dipole charges in the cloud shown in Figure 17b and, (b) at the position of the Earth's surface due to the dipole, assuming the presence of the Earth does not affect the calculation. \Box



Figure 17 Charge separation in a thunder cloud. (a) The distribution of charges. Field lines between the cloud and Earth are also shown. (b) The charge distribution replaced by point charges.

The assumption in the last part of Question T17 is not in fact correct. The Earth behaves as a good conductor and the negative charges at the base of the cloud induce a net positive charge in the Earth's surface. The effect of this is to double the calculated value of E at the Earth's surface. There will also, of course, be local variations. As a conductor, the Earth's surface is an equipotential. For high ground, the potential difference between the Earth and the cloud base will be the same as for low ground, but the distance between them will be shorter giving a larger potential gradient (see Figure 17a).



Figure 17 Charge separation in a thunder cloud. (a) The distribution of charges. Field lines between the cloud and Earth are also shown. (b) The charge distribution replaced by point charges.

Added to this, as mentioned in <u>Subsection 5.1</u>, the ground charge will tend to concentrate at regions of high curvature. Thus fields become intensified near pointed objects. Even so, fields close to the Earth are usually less than the required figure of $\sim 3 \times 10^6$ V m⁻¹ for the initiation of lightning. Most lightning flashes begin with localized high fields inside the cloud. Once the initial ionization starts, the 'flash' moves in short bursts, at each stage following the local potential gradient. This occurs both for flashes between the charges in the cloud and between the base of the cloud and Earth. Typically 30C of charge is transferred in less than 1 second.

The other question posed in the *Introduction* is whether the advice given to mountaineers in the *Mountain Leadership Guide* makes sense. The electric field close to the Earth is stronger near high ground and pointed or sharply curving objects. Consequently, the advice to avoid trees, boulders and peaks and to crouch down or squat is good. Figure 17a shows that the concentration of field lines on peaks and projections means the field some distance away will be reduced — as a rule of thumb, find a position about the same distance away as the projection is high.

When lightning strikes the ground there is an enormous transfer of charge in a very short time. This means there are very large fields through the ground in the vicinity of the strike. If you have two points of contact with the ground separated by a distance Δx , the potential difference between those points is given by $\Delta V = E\Delta x$. So the greater the separation between the points of contact, the greater potential difference. As your body is a good conductor, charge will flow through you between the points of contact with the ground. You minimize the danger by keeping your points of contact as close together as possible and making sure that the conducting path does not include your heart or brain. You can achieve this by squatting and keeping your hands off the ground.
Figure 18a shows a sensible position. You could improve on the *Mountain Leadership Guide* advice by maintaining only one point of contact with the ground, while still remembering to keep as low as possible

This is illustrated in Figure 18b but maybe being struck by lightning would be preferable to keeping this up for any length of time!



Figure 18 How to minimize the danger from a lightning strike. (a) The recommended position and (b) the optimum position.

6 Closing items

6.1 Module summary

- 1 There are two kinds of <u>electric charge</u> labelled positive and negative. Charge is <u>quantized</u> in units of e where $e = 1.603 \times 10^{-19}$ C. The total charge in the universe is a <u>conserved</u> quantity.
- 2 Materials can be broadly classified as electrical *conductors* or *insulators*, according to the ease with which charge moves within them. (There is also a class of materials known as semiconductors which behave as conductors in some circumstances and insulators in others.)
- 3 Transfer of charge between two initially neutral objects made of appropriate materials may be effected by rubbing the two objects together. Conductors may be charged by *electrostatic induction*.
- 4 The <u>electrostatic force</u> between charged particles (or spherically symmetric bodies) is governed by <u>Coulomb's law</u>:

$$\boldsymbol{F} = \frac{q_1 q_2}{4\pi \varepsilon r^2} \, \hat{\boldsymbol{r}} \tag{Eqn 1c}$$

where $\hat{\boldsymbol{r}} = \boldsymbol{r}/|\boldsymbol{r}|$ and $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_0 \boldsymbol{\varepsilon}_{\mathrm{r}}$.

The electrostatic force due to a distribution of charges is the vector sum of the electrostatic forces of the individual charges.

5 The <u>electric field</u> E at any point is the electrostatic force per unit (positive) charge that would be experienced by a test charge placed at that point:

$$\boldsymbol{E}(x, y, z) = \frac{\boldsymbol{F}_{el}(\text{on } q \text{ at } (x, y, z))}{q}$$

The electric field at any point specified by $\mathbf{r} = (x, y, z)$ due to a point charge Q at the origin is

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{Q}{4\pi\varepsilon r^2}\,\hat{\boldsymbol{r}} \tag{Eqn 2}$$

- 6 *Field lines* are a useful, semiquantitative way of representing the electric field in a region of space. They emerge from positive charges, point in the direction of the electric field at any point, and are drawn in such a way that their density in any region is proportional to the strength of the electric field in that region.
- 7 An <u>electric dipole</u> is a pair of charges with equal magnitude and opposite sign separated by a small displacement. The <u>dipole moment</u> is the product of the charge magnitude and separation. Coulomb forces between molecular or atomic dipoles (called <u>van der Waals forces</u>) are responsible for intermolecular bonding in many solids.

8 The change in <u>electrostatic potential energy</u> of a charge q in an electric field E(r) when it is moved from r_1 to r_2 is the negative of the work done by the electrostatic force during the displacement. It is independent of the path followed from r_1 to r_2 and is given by

$$\Delta E_{\rm el} = -q \int_{\boldsymbol{r}_1}^{\boldsymbol{r}_2} \boldsymbol{E}(\boldsymbol{r}) \cdot d\boldsymbol{r}$$
 (Eqn 12b)

For a point charge q at a distance r from a charge Q we can say that

$$E_{\rm el}({\rm of } q {\rm at } r) = \frac{qQ}{4\pi\varepsilon_0} \frac{1}{r}$$
(Eqn 13)

provided we adopt the arbitrary convention that $E_{\rm el} = 0$ where $r \rightarrow \infty$.

9 For any charge distribution, the *electric potential* at any point in space is the electrostatic potential energy per unit charge that a test charge would have at that point

$$V(\boldsymbol{r}) = \frac{E_{\rm el}(\text{of } q \text{ at } \boldsymbol{r})}{q}$$
(Eqn 14)

For a point charge q at a distance r from a charge Q, $V(\mathbf{r})$ depends only on $r = |\mathbf{r}|$ and may be written

$$V(r) = \frac{Q}{4\pi\varepsilon_0 r}$$
(Eqn 16)

provided $E_{\rm el} = 0$ where $r \rightarrow \infty$.

10 The unit of electric potential is the <u>volt</u> (V), where $1 \text{ V} = 1 \text{ J C}^{-1}$. The <u>electronvolt</u> (eV) is the change in the electrostatic potential energy of a single electron when it moves through a potential difference of 1V. Since $\Delta E_{el} = q\Delta V$, $1 \text{ eV} = 1.602 \times 10^{-19} \text{ C} \times 1 \text{ V} = 1.602 \times 10^{-19} \text{ J}$.

11 The (scalar) electric potential due to a distribution of charge is often relatively easy to determine. At any point, the component in any direction of the corresponding electric field is minus the gradient of the potential, in that direction. In the case of an electric field that varies only in the *x*-direction, V depends only on *x* and

$$E_x(x) = -\frac{dV}{dx}$$
(Eqn 17)

- 12 *Equipotential lines* are lines passing only through points of equal potential; *equipotential surfaces* are surfaces passing only through points of equal potential. The electric field is everywhere perpendicular to equipotential lines and surfaces and points in the direction in which the potential decreases most rapidly.
- 13 The surface of a conductor will be an equipotential surface.

6.2 Achievements

Having completed this module, you should be able to:

- A1 Define the terms that are emboldened and flagged in the margins of the module.
- A2 Explain how objects may be charged by contact and by induction.
- A3 State and use Coulomb's law.
- A4 Explain the origin of van der Waals forces in terms of forces between induced dipoles.
- A5 Explain the concept of electric field, write down an expression for the field due to a point charge and describe the electric fields that arise in a range of simple situations (e.g. the uniform electric field).
- A6 Describe the field due to an electric dipole and say how it differs from that due to a point charge.
- A7 Perform simple calculations involving electric fields.
- A8 Draw and interpret electric field line patterns for simple charge distributions.
- A9 Distinguish between electrostatic potential energy and electric potential and perform simple calculations based on these concepts.
- A10 Describe the relationship between electric field and electric potential, and use it in cases where the potential depends on only one variable.

- A11 Explain the significance of equipotentials and their relationship to field lines.
- A12 Explain the phenomenon of point discharge in terms of charge, field and potential.
- A13 Explain in terms of charge, field and potential the cause of lightning and simple precautions which can be taken to minimize risk of injury due to lightning strikes.

Study comment You may now wish to take the *Exit_test* for this module which tests these Achievements. If you prefer to study the module further before taking this test then return to the *Module contents* to review some of the topics.

6.3 Exit test

Study comment Having completed this module, you should be able to answer the following questions each of which tests one or more of the Achievements.

Question E1

(A2 and A3) A spherical conductor carrying a charge of 4.00×10^{-5} C is momentarily connected to a second, identical, initially neutral, spherical conductor. After this, the spheres are positioned a certain distance apart where the force between them is measured as 0.200 N. Calculate their separation. Is the force between the spheres repulsive or attractive?



Question E2

(A5, A7, A9 and A10) Two point charges A and B of +2Q and +Q, respectively, are separated by a distance D (Figure 19).

(a) Show that there is a point on the line joining the two charges, at a distance of $\left(\frac{\sqrt{2}}{1+\sqrt{2}}\right)D$ from A, where the field is exactly zero.



Figure 19 See Question E2.

(b) Show that the electric potential at this point is $\frac{Q(3+2\sqrt{2})}{4\pi\epsilon_0 D}$ assuming zero potential is an infinite distance

away.

(c) If $Q = 0.100 \,\mu\text{C}$ and $D = 2.00 \,\text{m}$, calculate the work done by the field in bringing an electron from very far away to the zero field point.

(d) Calculate the speed of the electron when it reaches this point assuming the only force it experiences is that due to the two charges. (Mass of an electon is 9.11×10^{-31} kg.)



Question E3

(A8, A10 and A11) Each of the field line diagrams in Figure 20 is supposed to represent a region of space containing a single, stationary point charge. Only one of the configurations is physically possible. Say which it is and explain why the others are not possible. On the possible configuration, draw in the associated pattern of equipotentials.



Figure 20 See Question E3.

Question E4

(A13) In mediaeval times in Belgium, bell ringers were often sent to ring bells in church towers to frighten off approaching thunderstorms. Explain the low life expectancy among mediaeval, Belgian bell ringers. Why are conditions safer today?



Study comment This is the final *Exit test* question. When you have completed the *Exit test* go back to Subsection 1.2 and try the *Fast track questions* if you have not already done so.

If you have completed **both** the *Fast track questions* and the *Exit test*, then you have finished the module and may leave it here.

