Module P9.1 Introducing atomic nuclei

Exit module

1 Opening items

1.1 Module introduction

The atom of any element is made up of a tiny, positively charged nucleus surrounded by negatively charged electrons. The nucleus, although small, contributes almost all of the mass of an atom. The chemical properties of materials are almost independent of the structure of the nucleus, since they arise from the interactions of the electrons in the atoms or molecules of the material. In contrast many other processes, such as the release of energy by the Sun and by terrestrial nuclear energy sources, depend crucially on the properties of the nucleus. Therefore a knowledge of the nucleus is essential for understanding these processes.

In this module we will discuss the basic properties of nuclei, starting with their charge and mass. In Section 2 we will discuss the basic nuclear constituents—*nucleons* (a collective name for *protons* and *neutrons*) and the meaning of terms such as *mass number* and *atomic number* used to describe nuclei. We will also explain the nature of *isotopes*, and introduce the *atomic mass unit*. Next (in Section 3), we will consider the forces between particles in the nucleus and introduce the *strong interaction between nucleons*. A discussion of *nuclear binding energy* and nuclear mass will enable us to relate the *mass defect* of a nucleus to the energy released in a nuclear reaction via *Einstein's energy–mass equation*. Finally, in Section 4, we will apply these ideas to unstable nuclei and radioactive decay, introduce the *Q-value* of a reaction and use it to predict which reactions may occur and which are forbidden.

Study comment Having read the introduction you may feel that you are already familiar with the material covered by this module and that you do not need to study it. If so, try the *Fast track questions* given in Subsection 1.2. If not, proceed directly to *Ready to study?* in Subsection 1.3.

1.2 Fast track questions

Study comment Can you answer the following *Fast track questions*?. If you answer the questions successfully you need only glance through the module before looking at the *Module summary* (Subsection 5.1) and the *Achievements* listed in Subsection 5.2. If you are sure that you can meet each of these achievements, try the *Exit test* in Subsection 5.3. If you have difficulty with only one or two of the questions you should follow the guidance given in the answers and read the relevant parts of the module. However, *if you have difficulty with more than two of the Exit questions you are strongly advised to study the whole module*.

Question F1

How many neutrons and protons are there in ${}^{22}_{10}\text{Ne}$ and ${}^{94}_{40}\text{Zr}$ nuclei?

Question F2

Calculate, using data from the table the binding energy of the nucleus of $^{208}_{82}\text{Pb}$ in units of MeV.
(1 u = 931.502 MeV/c^2)

Properties of some common nuclides

Z	Element	Symbol	A	Atomic mass/u	Relative abundance/%
1	hydrogen	H	1	1.007 825	99.985
			2	2.014 102	0.015
2	helium	He	4	4.002 603	99.999
79	gold	Au	197	196.966 560	100
82	lead	Pb	204	203.973 037	1.4
			206	205.974 455	24.1
			207	206.975 885	22.1
			208	207.976 641	52.4
83	bismuth	Bi	209	208.980 388	100

* The neutron, in the free state, is not stable; it has a mean lifetime of approximately 15 min. It has a mass of 1.008 665 u.
In contrast, a free proton is stable and has a mass of 1.007 276 u.

Study comment Having seen the *Fast track questions* you may feel that it would be wiser to follow the normal route through the module and to proceed directly to *Ready to study?* in Subsection 1.3.

Alternatively, you may still be sufficiently comfortable with the material covered by the module to proceed directly to the *Closing items*.

1.3 Ready to study?

Study comment In order to study this module you will need to understand the following terms: *atom*, *charge* (and its SI unit, the *coulomb*), *chemical symbol*, *electric field*, *electron*, *electrostatic force*, *electric potential difference*, *element*, *energy* (and its SI unit, the *joule*), *energy conservation*, *gravitational force*, *ion*, *kinetic energy*, *magnetic field* and *mass*. If you are uncertain about any of these terms then you can review them by reference to the *Glossary*, which will also indicate where in *FLAP* they are developed. The following *Ready to study questions* will allow you to establish whether you need to review some of the topics before embarking on this module.

Question R1

Do like charges attract or repel?

Question R2

An electron is accelerated from rest through a potential difference of 5 V. Determine the increase in kinetic energy of the electron in joules. ☞

Question R3

State Newton’s law of universal gravitation. Is the force involved attractive or repulsive?

2 The structure of the nucleus

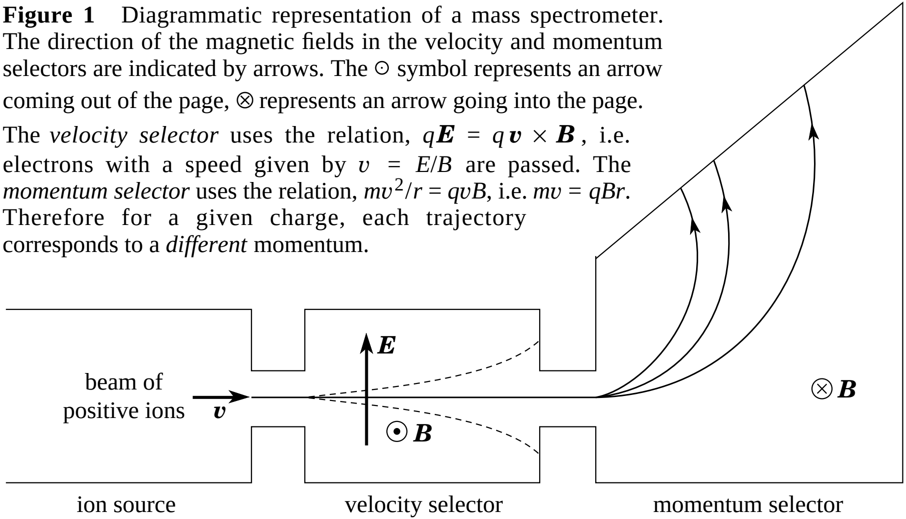
2.1 The contents of nuclei

The scattering of charged particles by a thin metal foil, a phenomenon investigated by Hans Geiger (1882–1945) and Ernest Marsden (1889–1970) in 1909 and interpreted by Ernest Rutherford (1871–1937) in 1911, revealed that the atomic nucleus is very small. The order of magnitude of a nuclear diameter, 10^{-14} m, is tiny compared with the typical atomic diameter of 10^{-10} m. ☞ Nonetheless, experiments have shown that the nucleus has its own internal structure. But what are the constituents of the nucleus? The nucleus of one particular kind of atom, the atom of hydrogen, is very simple. The single *electron* in a hydrogen atom may be detached to leave a nucleus that consists of a single, stable particle called the **proton**. ☞ A proton has a charge $+e$, ☞ and its mass, m_p, is 1.673×10^{-27} kg, i.e. about 1840 times that of the electron ☞. Protons are responsible for the net positive charge of each kind of nucleus, but is the proton the sole building block of all nuclei? We will consider this as a physicist should — by logical analysis, *tested by reference to experiment*. Consider, as an example, the nucleus of the element silicon. A silicon atom has 14 electrons, and since the atom as a whole is electrically *neutral*, the silicon nucleus must therefore have a charge of $+14e$. This means that the nucleus must contain 14 protons, and that its mass would be *about* $14m_p$ if it contained *only* protons. So we have a prediction; how may we test it?

The *principle* used in the measurement of the mass of a nucleus is that charged particles are deflected by both electric and magnetic fields. The instrument for measuring atomic and nuclear masses—a **mass spectrometer**— has three parts (see Figure 1).

Figure 1 Diagrammatic representation of a mass spectrometer. The direction of the magnetic fields in the velocity and momentum selectors are indicated by arrows. The ⊙ symbol represents an arrow coming out of the page, ⊗ represents an arrow going into the page.

The *velocity selector* uses the relation, $q\boldsymbol{E} = q\boldsymbol{v} \times \boldsymbol{B}$, i.e. electrons with a speed given by $v = E/B$ are passed. The *momentum selector* uses the relation, $mv^2/r = qvB$, i.e. $mv = qBr$. Therefore for a given charge, each trajectory corresponds to a *different* momentum.



The first part is the *ion source* in which the atoms of the element are stripped of one or more of their electrons. The positively charged ions produced are accelerated and emerge from the ion source into the *velocity selector*. The electric and magnetic fields in this region are uniform along the path of the beam and are mutually perpendicular to each other and to the direction of motion of the beam. The magnitudes of these fields are adjusted to select the speed of the ions that will pass into the final chamber (the *momentum selector*), where there is only a uniform magnetic field. As the ion beam passes through this magnetic field it is deflected in a direction at right angles to that of the field. (see Figure 1). The radius of curvature of the path along which the deflected ion travels in this region depends on its charge to mass ratio and hence (provided all the ions have the same charge) this radius enables its mass to be determined. Since the mass of an ion is due almost entirely to the nuclear mass, this may then be deduced from the ion mass.

Let us now return to our prediction. Is the mass of the nucleus of silicon equal to $14m_p$? You can imagine the disappointment of an early experimenter when it was discovered that this mass turned out to be about $28m_p$. *There must be some other constituent of the nucleus*.

The answer to the puzzle was not discovered until, in 1932, James Chadwick (1891–1974) detected another particle of about the same mass as the proton, *but with no charge*. This particle is the **neutron**. The mass of our nucleus of silicon is now explained if it contains 14 neutrons in addition to its 14 protons — then *both* charge *and* mass are accounted for.

It turns out that *all* elements (with the exception of hydrogen, where the nucleus is just a single proton ☞) have a nucleus consisting of protons and neutrons. Since they make up all nuclei, protons and neutrons are known collectively as **nucleons**. This is used in the following notation to characterize any nucleus.

> The **atomic number**, Z, specifies the number of protons in the nucleus. The total number of nucleons is called the **mass number**, A.

In our silicon example, $Z = 14$, $A = 28$. In a neutral atom, the charge on the nucleus is balanced by the charge of the surrounding electrons, and so there must be Z electrons. Since it is the electrons that determine the chemical properties of an atom, Z defines the *chemical element* and determines its place in the *Periodic Table of the elements*. ☞

Z specifies the charge on the nucleus in units of e, whereas A gives the total number of nucleons and so relates to its mass. This is illustrated in Figure 2.

❖ Do we need another *independent* number to specify the number of neutrons?

We denote the nucleus of any element by the chemical symbol of its atom with Z as a subscript and A as a superscript before the symbol, i.e. as ${}^{A}_{Z}\mathrm{X}$, where X is the chemical symbol of the element.

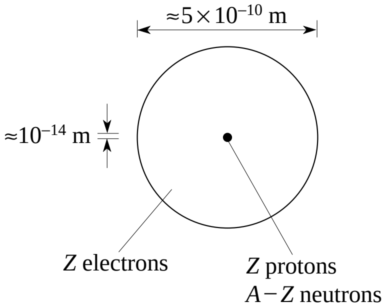


Figure 2 An atom contains Z electrons, Z protons and $(A - Z)$ neutrons, where Z is the atomic number of the nucleus and A its mass number.

Question T1

Use this notation to write the symbol for a silicon nucleus, given that $Z = 14$, $A = 28$ and its chemical symbol is Si. Is this notation as brief as possible? ❏

Following the development and refinement of the mass spectrometer in the 1930s, the masses of all available stable nuclei were measured. Some results were unexpected. It was found that the nuclei from atoms of a particular chemical element, characterized by a particular value of Z, did not all have the same mass number A. As they all have the same number of protons — namely Z — they must differ in their numbers of neutrons. Atoms of a particular element that differ only in the number of *neutrons* they contain are called **isotopes** of that element ☞. For example, silicon has three common isotopes, ${}^{28}_{14}\mathrm{Si}$, ${}^{29}_{14}\mathrm{Si}$ and ${}^{30}_{14}\mathrm{Si}$, which occur naturally with relative proportions of 92%, 5% and 3%, respectively. The term **nuclide** is often used to denote the atom of an element distinguished by the number of neutrons in its nucleus. ☞

Question T2

Consider the isotope ${}^{68}_{30}\mathrm{Zn}$ of the element zinc.

(a) How many protons does the nucleus contain?

(b) How many neutrons does the nucleus contain?

(c) Given that the charge on the electron is approximately -1.6×10^{-19} C, what is the charge on the nucleus?

(d) How many electrons are there in a neutral atom of zinc?

(e) A nucleus X has atomic number 31. Is X an isotope of zinc? ❑

Masses of nuclei are extremely small. The silicon isotope ${}^{28}_{14}Si$ has a mass of about 4.65×10^{-26} kg. Such small numbers would be very tiresome to work with so, just as charges are quoted in units of e, atomic masses are quoted not in kg but in their own more appropriately sized unit, the *atomic mass unit*, u.

The **atomic mass unit** (u) is defined so that the mass of the most common isotope of the carbon atom, with a ${}^{12}_{6}C$ nucleus, is *exactly* 12 u. So $1\,u = 1.6606 \times 10^{-27}$ kg.

Note that the unit is defined in terms of the mass of the *atom* and *not* the mass of the nucleus. The mass of an atom and its nucleus, for simple purposes, will be used in this module *as if* they were the same — they differ by approximately the mass of Z electrons.

Table 1 gives the atomic masses of a few elements, listed in order of increasing atomic number. ☞

Table 1 Properties of some common nuclides*.

Z	Element	Symbol	A	Atomic mass/u	Relative abundance/%
1	hydrogen	H	1	1.007 825	99.985
			2	2.014 102	0.015
2	helium	He	4	4.002 603	99.999
6	carbon	C	12	12.000 000	98.89
			13	13.003 355	1.11
8	oxygen	O	16	15.994 915	99.758
			17	16.999 131	0.038
			18	17.999 159	0.204
13	aluminium	Al	27	26.981 541	100

* The neutron, in the free state, is not stable; it has a mean lifetime of approximately 15 min. It has a mass of 1.008 665 u. In contrast, a free proton is stable and has a mass of 1.007 276 u.

Table 1 Properties of some common nuclides (continued)

Z	Element	Symbol	*A*	Atomic mass/u	Relative abundance/%
			37	36.965 903	24.23
30	zinc	Zn	64	63.929 145	48.6
			66	65.926 035	27.9
			67	66.927 129	4.1
			68	67.924 846	18.8
			70	69.925 325	0.6
79	gold	Au	197	196.966 560	100
82	lead	Pb	204	203.973 037	1.4
			206	205.974 455	24.1
			207	206.975 885	22.1
			208	207.976 641	52.4
83	bismuth	Bi	209	208.980 388	100

Question T3

If the mass of the electron is 9.110×10^{-31} kg, estimate the mass of the ${}^{12}_{6}\mathrm{C}$ *nucleus*.

By what percentage does this differ from 12 u? ❏

An important feature of nuclear structure can be visualized by plotting the number of neutrons ($N = A - Z$) in a **stable** ☞ nucleus against the number of protons (Z). This is shown in Figure 3, where each point represents a *completely* stable nucleus (i.e. one that *never* disintegrates). If there is no point shown at coordinate (Z, N) then no stable nucleus exists with that number of protons and neutrons. Figure 3 is therefore a useful chart which gives the number of stable isotopes for any given nucleus. For instance, we can see that chlorine with atomic number 17, has two entries on the vertical line through $Z = 17$, one at $A - Z = 18$ and the other at $A - Z = 20$. There are therefore two stable isotopes of chlorine, ${}^{35}_{17}\mathrm{Cl}$ and ${}^{37}_{17}\mathrm{Cl}$. But the question arises, why do neutrons and protons bind together to form nuclei? We will look at this in the next subsection.

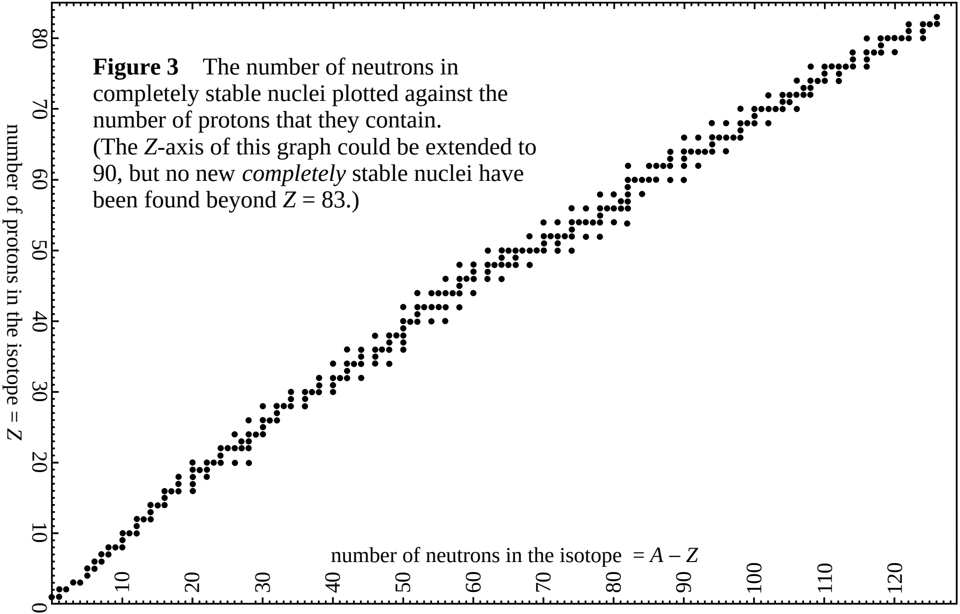


Figure 3 The number of neutrons in completely stable nuclei plotted against the number of protons that they contain.
(The Z-axis of this graph could be extended to 90, but no new *completely* stable nuclei have been found beyond Z = 83.)

2.2 The strong interaction between nucleons

The term **interaction** is used to describe a fundamental force underlying the behaviour of particles. ☞ When the existence of nuclei was discovered there were only two *known* ways in which neutrons and protons within a nucleus might interact. One interaction produced the gravitational force — all bodies with mass attract each other. The other produced the electrostatic force between charged particles —attractive or repulsive. Both protons and neutrons have mass, so the gravitational interaction tends to hold them together. Only protons have charge and in the nucleus they repel each other, opposing stability. If these were the only forces within the nucleus, which would 'win'? We can answer this by calculating the strength of the attractive force between two protons due to gravity, and comparing the result with their mutual repulsion due to their like charges.

Two protons, each of mass $m_{\rm p}$ and charge e, separated by a distance r, attract each other with a gravitational force of magnitude $F_{\rm grav}$ and repel each other with an electrostatic force of magnitude $F_{\rm el}$ where

$$F_{\rm grav} = \frac{Gm_{\rm p}{}^2}{r^2} \quad \text{and} \quad F_{\rm el} = \frac{e^2}{4\pi\varepsilon_0 r^2}$$

(G is the universal gravitational constant and ε_0 the permittivity of free space.) ☞

❖ Given that $\varepsilon_0 = 8.854 \times 10^{-12}\,{\rm C^2\,N^{-1}\,m^{-2}}$ and $G = 6.673 \times 10^{-11}\,{\rm N\,m^2\,kg^{-2}}$, work out the units of the ratio $F_{\rm grav}/F_{\rm el}$.

Calculation of the ratio of the gravitational force to the electrostatic force gives a value which is of the order of 10^{-36}. Notice that the separation r is immaterial since the factor $1/r^2$ cancels in the ratio. The gravitational force is *exceptionally* weak and quite unable to counteract the repulsion between the protons. Therefore there *must* be *another* attractive force, to account for stable nuclei.

The strong attractive force which is responsible for holding nuclei together is called, somewhat unimaginatively, the **strong nuclear force** or the **strong interaction between nucleons**. It arises from an interaction between the particles that is fundamentally different from the interactions that give rise to the electrostatic and gravitational forces — it depends neither on charge nor mass. ☞ Let us look at Figure 3 again to see what hints it can give us about the nature of this third interaction.

Place a ruler along the line $N = Z$ on Figure 3. This line joins points corresponding to *equal* numbers of protons and neutrons. What do you observe?

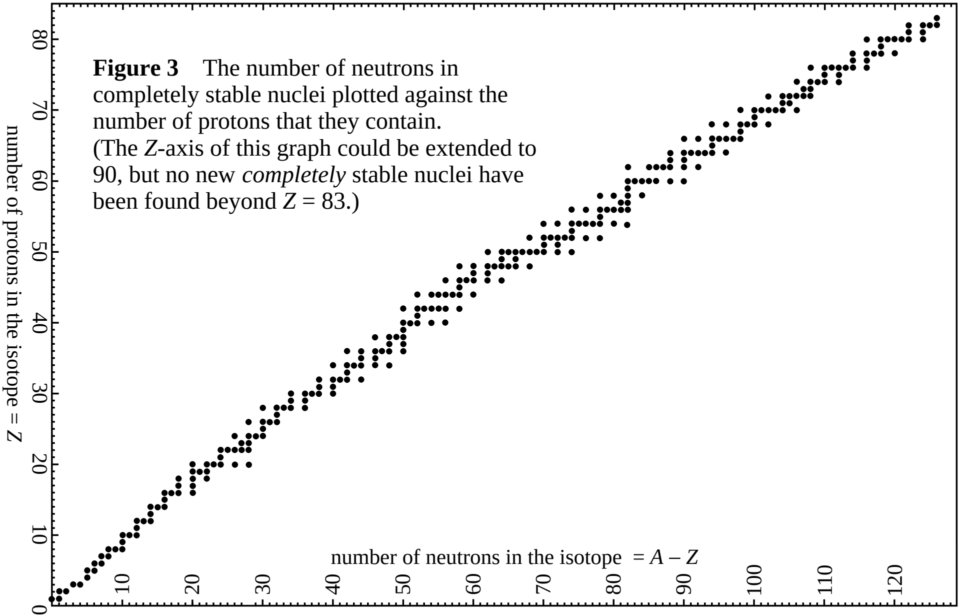


Figure 3 The number of neutrons in completely stable nuclei plotted against the number of protons that they contain. (The Z-axis of this graph could be extended to 90, but no new *completely* stable nuclei have been found beyond $Z = 83$.)

Question T4

From Figure 3, calculate the ratio of the number of neutrons to the number of protons for the nucleus with atomic number 69. What is the corresponding value for the nucleus with $Z = 79$? ❏

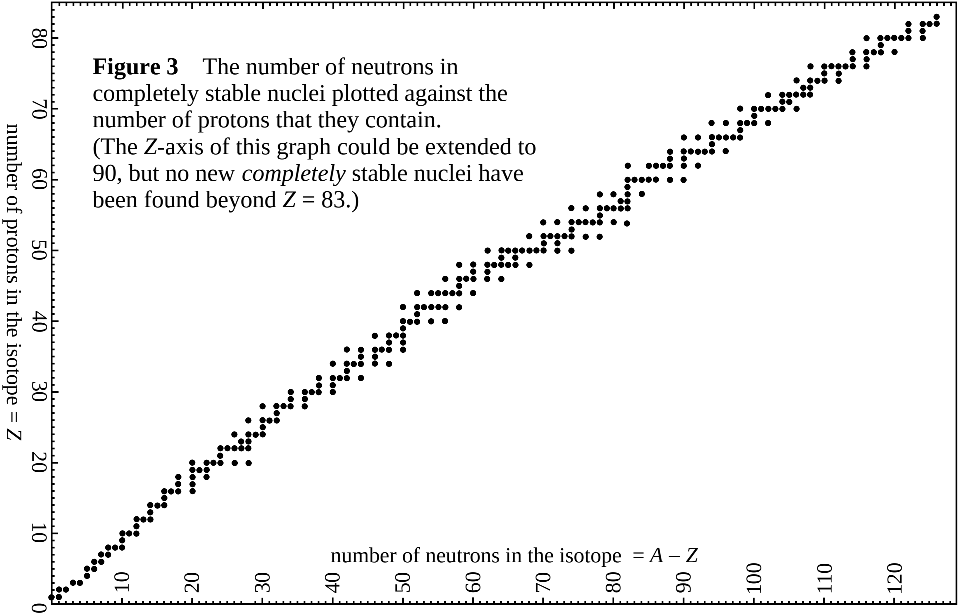


Figure 3 The number of neutrons in completely stable nuclei plotted against the number of protons that they contain. (The Z-axis of this graph could be extended to 90, but no new *completely* stable nuclei have been found beyond $Z = 83$.)

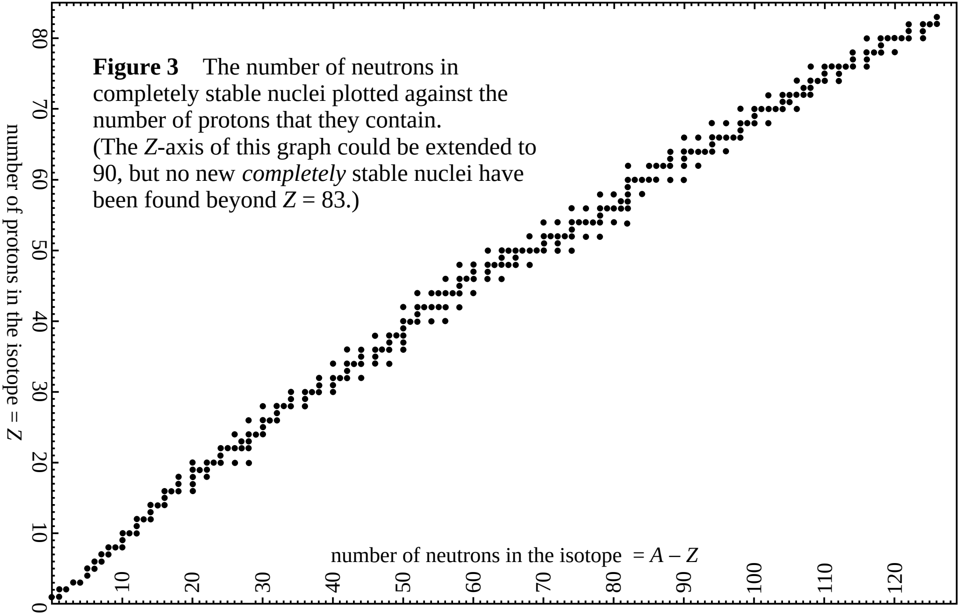


Figure 3 The number of neutrons in completely stable nuclei plotted against the number of protons that they contain.
(The Z-axis of this graph could be extended to 90, but no new *completely* stable nuclei have been found beyond $Z = 83$.)

Figure 3 shows that the larger the number of neutrons and protons in the nucleus, the greater must be the neutron to proton ratio. It also shows that no stable nuclei exist with $Z > 83$. ☞

What clues does Figure 3 give us about the nature of the strong interaction between nucleons?

- Since it is the number of neutrons that affects nuclear stability, we can deduce that neutrons also experience the strong nuclear interaction.
- For small nuclei there are about equal numbers of neutrons and protons (line $N = Z$). This suggests that the strong nuclear interaction does not discriminate between neutrons and protons — it is *charge-independent*.
- As the number of protons increases, their mutual electrostatic repulsion must increase so it requires proportionally more and more neutrons to stabilize the nucleus. The addition of more neutrons increases the nuclear attraction without adding electrostatic repulsion. Presumably also, this may increase the mean distance between the protons and so reduce the electrostatic repulsion. This explains why the stability line veers away from the line $N = Z$ as Z increases.
- As Z increases, eventually the nuclear force is unable to stabilize the nucleus against the electrostatic repulsion. No fully stable nucleus occurs with $Z > 83$. ☞

So, even the strong interaction between nucleons is insufficient to overcome the electrostatic repulsion in all circumstances. This hints at the limited range of effectiveness of the strong interaction.

The electrostatic force diminishes as the inverse square of the separation of the interacting charges (i.e. $F_{\rm el} \propto 1/r^2$). This is a relatively slow rate of reduction with distance; so the electrostatic force is effective over a relatively long range.

Suppose that the strong interaction decreases with distance with a higher power of r (so that it is effectively zero at about 10^{-14} m), what would be the consequence of this? In this case, unlike that of charge repulsion, nucleons would interact strongly only with their nearest neighbours in the nucleus. This means that each nucleon makes a more or less fixed contribution to the attractive force within a moderate to large nucleus, irrespective of the total number of nucleons present. Consequently, as the nucleus increases in size, the total attractive force due to the strong interaction increases approximately in proportion to the total number of nucleons (i.e. $\propto A$). However, the long-range electrostatic repulsion would allow each proton to repel *all* other protons in the nucleus, not just nearest neighbours. The total repulsive force therefore depends on the number of *pairs* of protons which experience the force. As the nuclear size increases this repulsion increases approximately in proportion to the *square* of the total number of protons (i.e. $\propto Z^2$). ☞ At some value of Z the strong attraction between nucleons will be unable to sustain the nucleus against the electrostatic repulsion. This seems to occur at $Z = 83$.

Let us review our *tentative* conclusions:

The strong interaction between nucleons has four characteristics:

- It is *attractive*.
- It is *independent* of the charge and mass of the interacting particles — the neutron–proton, proton–proton and the neutron–neutron *strong* interactions are the same.
- The interaction has a *very short range* (about 10^{-14} m).
- The interaction consequently must be *very strong*.

We will now look at some further evidence about the magnitude of the strong interaction. To do this we will need to introduce the concept of *binding energy*. This is the subject of the next section.

3 Nuclear energies and masses

3.1 Nuclear binding energy

In general, any system is stable when it has the lowest possible energy. A collection of nucleons bound together in a stable nucleus must therefore have a lower energy than these same nucleons as a collection of free particles. If this were not so, the nucleus would spontaneously disperse into its constituents; it would *decay* rather than be *stable*. The difference between the energy of the constituents if *just* free (that is with no kinetic energy) and the energy of the bound nucleus is called the nuclear **binding energy**. Expressed another way, the nuclear binding energy is the energy that must be given to the nucleus in order that *all* of its constituents may *just* break free. Binding energy is a similar concept to *ionization energy* — the energy needed just to separate an electron from an atom, leaving a positive ion ☞ — only with nuclear binding energy we are considering the complete dismantling of the nucleus into its constituent nucleons, rather than removing just one of them.

Let us look first at the magnitude of binding energies and, as with the nuclear masses earlier, define an appropriate unit for this measurement. To take an example, breaking up a helium nucleus, ${}^{4}_{2}\mathrm{He}$, into its two protons and two neutrons requires approximately 5×10^{-12} J. On an everyday-scale this is a very small amount of energy so it can be measured in terms of a very small energy unit.

The energy units customarily used in atomic and nuclear physics are based on the **electronvolt** (eV), which is the kinetic energy gained by an electron when it is accelerated through a potential difference of one volt. This is equal to 1.602×10^{-19} J. So 1 eV = 1.602×10^{-19} J.

The eV is the unit which is appropriate for atomic physics. For instance, it takes about 13.6 eV to ionize a hydrogen atom. But as you can see in the example of helium, the eV is much smaller than a typical nuclear binding energy, so in nuclear physics the megaelectronvolt, MeV, is used: 1 MeV = 10^6 eV = 1.602×10^{-13} J. The binding energy of ${}^{4}_{2}\mathrm{He}$ is 28.3 MeV. ☞

Question T5

A proton is accelerated through a potential difference of 1 kV. How much kinetic energy is transferred to the proton? Give the answer both in joules and MeV. ❏

Experimentally, binding energies of stable nuclei are found to vary from about 2 MeV to about 2000 MeV. As you will see later, data on binding energies can be used to consolidate our tentative inferences about the strong interaction between nucleons. First, however, we will see how binding energies may be *deduced* from measurements of nuclear masses with the help of Einstein's famous relationship between mass and energy.

3.2 Nuclear masses and Einstein's equation

If you were to approach a person in the street and ask them to tell you a scientific formula, they would probably mention the equation $E = mc^2$. This equation is a result from *Einstein's special theory of relativity*, the details of which do not concern us here; you will doubtless meet it elsewhere in your studies. ☞ The result may be stated in words as follows: 'An amount of energy (of any type) has an equivalent mass' or more specifically, the amount of energy E equivalent to a mass m is given by

Einstein's mass–energy equation:

$$E = mc^2 \tag{1}$$

where c is the speed of light in a vacuum, approximately $2.998 \times 10^8\,\mathrm{m\,s^{-1}}$. The energy that an object has purely by virtue of its mass is often called its **mass energy**. ☞ Equation 1 is so important in nuclear physics that it will be referred to by name. We will use the equation here to relate the *measured* masses of nuclei to their binding energies, and thus to deduce the binding energies from purely empirical data about masses.

But let us first gain some feeling for the size of the energies involved in relation to everyday energy units (joules) and to nuclear units (MeV). The mass which is equivalent to one joule is given by Equation 1 as:

$$\text{mass equivalent of 1 J of energy} = \frac{1\,\text{J}}{(2.998\times10^{8}\,\text{m}\,\text{s}^{-1})^{2}} \approx 10^{-17}\ \text{kg}$$

It is not surprising that Einstein's equation started life (in 1905) as a *theoretical* speculation. With the instrumentation available at the time, masses of this order could not have been detected experimentally.

But in the atomic and nuclear domain we are familiar with masses and energies which are many orders of magnitude less than the everyday scale. Let us look for instance at the proton mass, $m_{\rm p}$, which is 1.673×10^{-27} kg. The corresponding energy, $m_{\rm p}c^2$, is equal to 1.503×10^{-10} J or approximately 938 MeV in nuclear units.

Note that mass has the dimensions of energy divided by speed squared: this can be seen both from Einstein's equation and the familiar expression for kinetic energy, $E_{\rm kin} = \frac{1}{2}mv^2$. Einstein's equation can be rearranged to give $m = E/c^2$, and this leads to a convenient unit for mass.

When working in nuclear units, with energy in units of MeV, it is common to express masses of nuclear particles in units of MeV/c^2. The mass of a particle, in MeV/c^2, is numerically the same as its energy equivalent in MeV.

In our example the mass of the proton is 938 MeV/c^2. The energy equivalent of the mass is then obvious, it is 938 MeV.

Question T6

The mass of the electron, m_e, is 9.110×10^{-31} kg. What is the energy equivalent of this mass in J and in MeV? Express the mass of the electron in MeV/c^2. ❑

Question T7

We have seen in Subsection 2.1 that the atomic mass unit (1 u) is equal to 1.6606×10^{-27} kg. Using the same approach as in Question T6, express the magnitude of 1 u in units of MeV/c^2. ❑

In the context of the physics of the nucleus, Einstein’s equation provides an explanation for a phenomenon which would have sorely troubled a 19th-century physicist — the so-called **missing mass**. Let us examine this through an example. We will take the most common oxygen isotope $^{16}_{8}O$, which has eight protons and eight neutrons. From Table 1 we may calculate the mass of the constituents:

$$8m_p + 8m_n = (8 \times 1.007\,825\,\text{u}) + (8 \times 1.008\,665\,\text{u}) = 16.131\,920\,\text{u}$$

However, the *measured* mass of the $^{16}_{8}O$ nucleus is 15.994 915 u. This is *less* than the mass of the constituents. The difference, 0.137 005 u, is known as missing mass or **mass defect**. *Every* nucleus (with the exception of $^{1}_{1}H$) has a mass defect. This would have been incomprehensible to 19th-century physicists who regarded mass as the *quantity of matter* in a body. They would have said ‘how could matter have possibly disappeared from the constituents merely by their being part of a bound system?’ But if energy can be considered as having an equivalent mass then there is a simple explanation; the mass defect is simply the mass equivalent of the binding energy — the energy that has been ‘saved’ by binding the nuclear constituents together.

This explanation is based on a revised view of mass and energy conservation. In the 19th century we had two separate principles, the *conservation of energy* and the *conservation of mass*. Now, following Einstein’s introduction of the concept of mass energy, we have a single principle, the **conservation of relativistic energy** in which the total energy of a body of mass m includes its mass energy mc^2.

Applying this new principle to a nucleus we can write

$$\begin{pmatrix}\text{mass energy of}\\ \text{free particles}\end{pmatrix} = \begin{pmatrix}\text{mass energy of}\\ \text{nucleus}\end{pmatrix} + \begin{pmatrix}\text{binding energy of}\\ \text{nucleus}\end{pmatrix}$$

So, upon dividing both sides by c^2 we get

mass of free particles = mass of nucleus + (binding energy of nucleus)/c^2

Since mass defect = mass of free particles – mass of nucleus, it follows that

mass defect = (binding energy of nucleus)/c^2 (2)

The above expressions tell us that any stable nucleus (i.e. one that requires an input of energy in order to separate its nucleons) *must* have less mass than the sum of the masses of its free constituent nucleons. So the mass defect is an inevitable consequence of special relativity as expressed in Einstein's equation.

Question T8

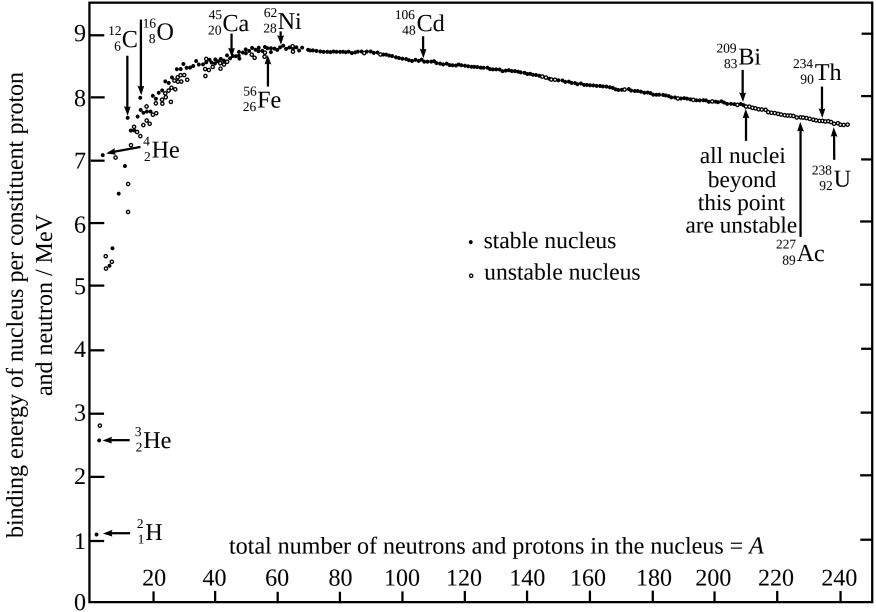
Calculate the mass defect for ${}^{12}_{6}$C. Express this in units of u and in MeV/c^2 . What is the binding energy of ${}^{12}_{6}$C? ❏

3.3 The nuclear binding energy graph

The calculation in Question T8 can of course be repeated using the mass measurements of all stable nuclei. As mentioned in Subsection 3.1, the values of the binding energies vary from 2 MeV for the lightest nucleus to almost 2000 MeV for the heaviest. More enlightening information on the stability of nuclei and the nature of the strong interaction itself comes from a plot of the binding energy *per nucleon* against the mass number *A*. This graph is usually *called* the **nuclear binding energy graph** and it is shown in Figure 4. As the binding energy of a nucleus is the minimum energy required to 'unbind' the nucleus into free protons and neutrons, the binding energy per nucleon is simply the binding energy divided by the mass number *A*.

Figure 4 shows all stable and some unstable nuclei. In Section 4 we will look at unstable nuclei, but let us first concentrate on the stable nuclei — shown as filled dots in Figure 4.

Figure 4 The nuclear binding energy graph.



The graph may be analysed in three parts.

- In the first part of the graph (from A = 1 to about 30) the nucleus is becoming established as a stable unit, the mean binding energy per nucleon is rising as each nucleon surrounds itself with a full complement of other nucleons as near neighbours with which it can interact attractively. In this region protons and neutrons can be added equally easily since the effect of the weak electrostatic repulsion is small. Notice, however, that even within this steeply rising part of the curve there are one or two striking anomalies in the relatively smooth curve. Most significant is the nucleus of helium ${}^{4}_{2}\mathrm{He}$, which appears to be much more stable than its general position at A = 4 might suggest. (${}^{12}_{6}\mathrm{C}$ and ${}^{16}_{8}\mathrm{O}$ are similar, but less striking anomalies.) It is no surprise then to find that the nucleus of helium often emerges as a separate entity from the decay of other *unstable nuclei* — it is the **α-particle** (alpha-particle), of which more will be said later in this module.

- In the second part of the graph (from $A = 30$ to about 100) the mean binding energy stays about constant at around 8.5 MeV/nucleon. This is the region where the nuclear binding interaction is in competition with the unbinding influence of the electrostatic repulsion between the protons. Since the latter is increasing as Z^2 while the attraction is only increasing as A, the repulsion must eventually 'win out'. In the first part of the middle region the attraction is a slight winner and in the final part it is a slight loser.
- In the third part of the graph ($A > 100$) the electrostatic repulsion, rising as Z^2 due to its long-range nature, begins to dominate the strong interaction, which is only rising as A due to its short-range nature. The mean binding energy per nucleon falls gradually and the nuclei become less and less stable, until at $Z = 83$, $A = 209$ it is the 'end of the road' for long-term nuclear stability.

The constant binding energy per nucleon seen in the middle of Figure 4 is reminiscent of one of the properties of a liquid. For a liquid, such as water, we define the *latent heat of vaporization* to be the energy required to turn a given *mass* of liquid water into steam — that is, to dismantle the structure of the liquid and disperse the molecules as separate units. This is clearly analogous to the unbinding of the nucleus into separate nucleons. Latent heat is usually defined *per unit mass* (1 kg) but we could equally well give its value *per molecule* (although this would be inconvenient for the liquid case because of the very large numbers of molecules involved in a unit mass of 1 kg). Nevertheless, the important point is that in both the nucleus and the liquid the required investment of energy to dismantle the structure rises in proportion to the number of 'particles' (nucleons or molecules) involved. Moreover, the underlying cause of this similarity is the same in both cases; the attractive forces that hold the structure together are effectively *short range*. The force that attracts molecules together to allow liquefaction is called the *van der Waals force* and is effective only over separations of about 10^{-9} m. ☞ The strong nuclear interaction is also short-range, though on a nuclear scale (~10^{-14} m). The similarity between the liquid and the nucleus is quite striking and can be pursued further than we are able to do here. ☞

4 Unstable nuclei

We will look briefly in this section at the properties of *unstable nuclei*. By this we mean nuclei which, after a characteristic time, split up into fragments so that the original nucleus no longer exists. This process is called **radioactive decay** and any nucleus that exhibits radioactive decay is called a **radioactive nucleus**; some of these nuclei are represented as open circles on Figure 4. We will apply some of the results and concepts discussed in Sections 2 and 3 to this process, in particular Einstein's equation.

4.1 A second look at Einstein's equation

When we introduced Einstein's equation $E = mc^2$ in Subsection 3.2 we were dealing with particles that were at rest relative to their observer. The binding energy was defined hypothetically as the energy necessary to *just* break up (i.e. unbind) the stationary nucleus into its free constituents (protons and neutrons), which meant that the resultant constituents would have *no* *kinetic energy*. But when a radioactive nucleus at rest decays, the fragments always *do* have kinetic energy relative to the original nucleus. What difference does this make to the use of Einstein's equation?

Einstein's equation remains true, even when a particle has kinetic energy, but great care is needed in using it. It is certainly the case, confirmed by many experiments, that as the energy of a particle increases, so does its mass. For example, it becomes harder to accelerate a particle as its energy grows, just as you would expect for a particle of growing mass. However, nuclear physics would become almost impossible to discuss if we had to keep changing the masses of particles according to their speed. In order to avoid this, the masses that nuclear physicists refer to and use in their calculations are defined in a very precise way.

When we look up tables of masses, for instance Table 1, we are looking at what is known as the **rest mass** m_0 of the atom or nucleus. This is defined to be the mass determined when the particle is *at rest relative to the observer who measures it*. So, no matter how fast a particle may be moving, the rest mass used to characterize it remains constant. It is *implicit* in all mass tables that it is the rest mass that is tabulated. The energy of a particle at rest or its **rest mass energy** is often called its **rest energy** E_0 — defined by Einstein's equation as $E_0 = m_0c^2$. The energies we considered in the previous section were all rest energies. Thus, in the absence of any potential energy, the *total* energy, E, of any nucleus will be the sum of its rest energy E_0 and its kinetic energy $E_{\rm kin}$:

total energy	$E = E_0 + E_{\rm kin} = m_0c^2 + E_{\rm kin}$	
and	$E_{\rm kin} = E - E_0$	(3)

Now, Einstein showed, as part of his special theory of relativity, that when a particle of rest mass m_0 travels at speed υ its total energy is

$$E = \frac{m_0c^2}{\sqrt{1 - \upsilon^2/c^2}}$$

So, it follows from Equation 3 that the kinetic energy of such a particle is given by

$$E_{\text{kin}} = E - E_0 = m_0c^2\left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right)$$

In nuclear physics we *must* adopt this way of calculating kinetic energy. The familiar expression, $E_{\text{kin}} = \frac{1}{2}m_0v^2$, is true as an approximation *only* for particles with a kinetic energy very much smaller than their rest energy. (This is equivalent to saying that their speeds v must be much less than c.) ☞ Einstein's equation and the above boxed expressions apply to *all* particles, regardless of their energy. This method of finding kinetic energy has the added advantage of being very straightforward. For example, if we are told that a proton has a total energy of 1 GeV (i.e. 1000 MeV) ☞ then, since we know (from Subsection 3.2) that its rest energy is 938 MeV, it must have a kinetic energy of 62 MeV.

4.2 Some examples of unstable nuclei

The discovery of the existence of the first unstable nucleus is an interesting story. In 1896 Henri Becquerel (1852–1908) was studying the properties of the element uranium. He found that photographic plates in the vicinity of his experiments became fogged, even though they were carefully protected from light. He might have thrown the plates away as defective and thought no more about it. But he was a careful investigator who followed up this unexplained result and identified the source of the fogging as some emission from the uranium.

Since Becquerel's time many unstable nuclei have been discovered and many have been manufactured. Table 2 shows a list of some unstable nuclides, in a format similar to that of Table 1 for the stable nuclides. Table 2 differs from Table 1 in the addition of the right-hand column, headed **half-life**. Half-life is a measure of how quickly a radioactive material decays. For example, if a sample has a half-life of 10 minutes, after this time half of the initial number of radioactive nuclei will have decayed. ☞

You will see, even from the few examples in Table 2, that the range of values of half-life is enormous: from milliseconds for ${}^{218}_{86}\text{Rn}$ to 2×10^{15} yr for ${}^{186}_{76}\text{Os}$. The *very* large half-life of ${}^{186}_{76}\text{Os}$ or ${}^{238}_{92}\text{U}$ shows why we have used the term *completely stable* in the context of Figure 3. Even over a span of years a sample of osmium or uranium might *appear* to be stable because the decay rate is so low.

Table 2 Properties of some unstable nuclides.

Z	Element	Symbol	A	Atomic mass/ u	Relative abundance/%	Half-life
1	hydrogen	H	3	3.016 050		12.3 yr
4	beryllium	Be	8	8.0053		$\sim 3 \times 10^{-16}$ s
6	carbon	C	14	14.003 242		5730 yr
19	potassium	K	40	39.963 999	0.01	1.28×10^{9} yr
53	iodine	I	131	130.906 119		8.04 days
76	osmium	Os	186	185.953 852	1.6	2×10^{15} yr
78	platinum	Pt	190	189.959 937	0.013	6×10^{11} yr
84	polonium	Po	218	218.008 930		3.05 min
86	radon	Rn	218	218.005 580		0.035 s
			222	222.017 574		3.82 days
87	francium	Fr	221	221.014 183		4.8 min

A neutron has an atomic mass of 1.00816651u and, as a free particle, is unstable with a half-life of 10.61min.

Nevertheless, it was the emissions from such decays from uranium that fogged Becquerel's photographic plates and without this *experimental evidence* of decay, its occurrence might have gone unnoticed. The half-lives of the two isotopes of uranium, ${}^{235}_{92}U$ and ${}^{238}_{92}U$, are so long that they have quoted relative abundances, just as if they were stable isotopes.

Table 2 Properties of some unstable nuclides.

Z	Element	Symbol	*A*	Atomic mass/ u	Relative abundance/%	Half-life
88	radium	Ra	222	222.015 353		38 s
			226	226.025 406		1.60×10^3 yr
90	thorium	Th	230	230.033 131		8.0×10^4 yr
			233	233.041 579		22.3 min
92	uranium	U	235	235.043 925	0.72	7.04×10^8 yr
			238	238.050 786	99.28	4.47×10^9 yr

4.3 Conservation principles in decays

We shall now consider the decay of an unstable nucleus. ☞ What will it decay into? Will any of the mass energy locked up in the nucleus be released? How much will be released? We will aim to solve these questions using only *measured* (rest) masses, Einstein's equation and *three* basic **conservation principles**:

1. The *principle of charge conservation*: The total electric charge after a decay is the same as the total charge beforehand.
2. The *principle of nucleon number conservation*: The total number of nucleons remains the same throughout the decay. ☞
3. The *principle of (relativistic) energy conservation*: The energy of a nucleus before it decays is equal to the energy shared by the particles that it decays into. If the nucleus is at rest before the decay its energy will consist entirely of rest energy, but its energy after the decay will include the kinetic energy of the decay products as well as their rest energies.

Let us illustrate the first two principles with an example.

The unstable nucleus of radium, ${}^{222}_{88}\text{Ra}$, is observed to decay into a radon nucleus, ${}^{218}_{86}\text{Rn}$, and a helium nucleus, ${}^{4}_{2}\text{He}$. For historical reasons a ${}^{4}_{2}\text{He}$ nucleus ejected in radioactive decay is called an *α-particle*. The decay reaction will be written:

$${}^{222}_{88}\text{Ra} \rightarrow {}^{218}_{86}\text{Rn} + {}^{4}_{2}\text{He}$$

The number of protons in the radium nucleus is equal to its atomic number, 88. The right-hand side of the decay equation shows that these protons are shared by the two product nuclei, 86 in the radon nucleus and 2 in the helium nucleus. Therefore the number of protons is conserved in this process, and since protons are the only charged particles involved there is a total charge of 88e both before and after the decay. It follows that *charge is conserved* in the decay. The equation also shows that the mass number is conserved: 222 = 218 + 4. The mass number is equal to the total number of nucleons (i.e. neutrons *and* protons) in the nucleus. So *nucleon number is also conserved* in the decay. So you can see the first two principles of conservation are observed by this decay process. ☞ Question T9 asks you to use these two conservation principles to deduce the products of a radioactive decay.

Question T9

A stationary nucleus of thorium ${}^{230}_{90}\mathrm{Th}$ undergoes decay into a nucleus of radium Ra and an α-particle. What is the equation of the decay? ❏

Table 2 Properties of some unstable nuclides.

Z	Element	Symbol	A	Atomic mass/ u	Relative abundance/%	Half-life
88	radium	Ra	222	222.015 353		38 s
			226	226.025 406		1.60×10^3 yr
90	thorium	Th	230	230.033 131		8.0×10^4 yr
			233	233.041 579		22.3 min
92	uranium	U	235	235.043 925	0.72	7.04×10^8 yr
			238	238.050 786	99.28	4.47×10^9 yr

To see how the third principle, the conservation of relativistic energy is applied, we return to our earlier example and look at the energy associated with the ${}^{222}_{88}\mathrm{Ra}$ decay. Tables 1 and 2 give the following atomic *rest* masses:

mass of ${}^{222}_{88}\mathrm{Ra} = 222.015\,353\,\mathrm{u}$ mass of ${}^{4}_{2}\mathrm{He} = 4.002\,603\,\mathrm{u}$

mass of ${}^{218}_{86}\mathrm{Rn} = 218.005\,580\,\mathrm{u}$

The total rest mass of the decay fragments is therefore $218.005\,580\,\mathrm{u} + 4.002\,603\,\mathrm{u} = 222.008\,183\,\mathrm{u}$.

This is less than the rest mass of the radium nucleus. So the energy that this represents, using Einstein's equation, is only part of the energy available from the rest energy of the radium nucleus. The energy left over — the equivalent of 0.007 170 u — appears as *kinetic* energy shared between the two product nuclei.

We express the energy balance by writing the equation:

$$^{222}_{88}\mathrm{Ra} \rightarrow {}^{218}_{86}\mathrm{Ra} + {}^{4}_{2}\mathrm{He} + Q$$

where the symbol Q stands for the kinetic energy available to the products.

The kinetic energy available in a decay is commonly called the ***Q*-value** of the decay. It is given (using rest masses) as:

$$Q = (\text{mass of nucleus} - \text{sum of masses of decay fragments}) \times c^2 \qquad (4)$$

If we use masses in units of MeV/c^2 then Q may readily be expressed in units of MeV. So in our particular example, using the answer to Question T7 that 1 u = 931.502 MeV/c^2, we have $Q = 0.007\,170 \times 931.502\,(\mathrm{MeV}/c^2) \times c^2 = 6.679\,\mathrm{MeV}$. The full equation of the decay is:

$$^{222}_{88}\mathrm{Ra} \rightarrow {}^{218}_{86}\mathrm{Rn} + {}^{4}_{2}\mathrm{He} + 6.679\,\mathrm{MeV}$$

Question T10

Using Table 2, work out the Q-value and hence write down a full equation for the decay of thorium $^{230}_{90}\mathrm{Th}$, that was the subject of Question T9

$$^{230}_{90}\mathrm{Th} \rightarrow {}^{226}_{88}\mathrm{Ra} + {}^{4}_{2}\mathrm{He}$$ ❑

Table 2 Properties of some unstable nuclides.

Z	Element	Symbol	A	Atomic mass/ u	Relative abundance/%	Half-life
1	hydrogen	H	1	1.007 825	99.985	
			2	2.014 102	0.015	
2	helium	He	4	4.002 603	99.999	
88	radium	Ra	222	222.015 353		38 s
			226	226.025 406		1.60×10^3 yr
90	thorium	Th	230	230.033 131		8.0×10^4 yr
			233	233.041 579		22.3 min
92	uranium	U	235	235.043 925	0.72	7.04×10^8 yr
			238	238.050 786	99.28	4.47×10^9 yr

Forbidden decay

Finally we ask, are all decays possible which conserve the number of protons and neutrons? Or can we exclude some? Let us take our ${}^{222}_{88}\text{Ra}$ nucleus as an example again.

✦ Does the proposed decay ${}^{222}_{88}\text{Ra} \rightarrow {}^{221}_{87}\text{Fr} + {}^{1}_{1}\text{H}$ conserve the charge and the number of nucleons?

This new decay mode corresponds to the decay of ${}^{222}_{88}\text{Ra}$ with the loss of a proton (i.e. a hydrogen nucleus ${}^{1}_{1}\text{H}$). This might be an apparently reasonable decay to expect. But the energy balance tells a different story. The rest mass of ${}^{222}_{88}\text{Ra}$ is still 222.015 353 u but the rest mass of ${}^{221}_{87}\text{Fr}$ is 221.014 183 u and that of ${}^{1}_{1}\text{H}$ is 1.007 825 u. Therefore, the Q-value (kinetic energy), as in Equation 4,

$$Q = (\text{mass of nucleus} - \text{sum of masses of decay fragments}) \times c^2 \qquad \text{(Eqn 4)}$$

would be:

$$[222.015\,353 - (221.014\,183 + 1.007\,825)] \times 931.502\,(\text{MeV}/c^2) \times c^2$$

$$= -0.006\,653 \times 931.502\,(\text{MeV}/c^2) \times c^2 = -6.20\,\text{MeV}$$

The answer is *negative*. There is not enough rest energy in the nucleus to provide the *rest* energies of the fragments *let alone some extra kinetic energy*. By the third principle stated above, this reaction is not permitted.

> If $Q < 0$, the decay reaction cannot take place spontaneously.

We can now understand why α-decay is a favoured decay mode for a large nucleus when we remember the point made in Subsection 3.3, that the α-particle is itself an unusually stable entity with a relatively low energy. Its emission tends to leave energy available to produce a positive Q-value for the decay process.

Question T11

From the data in the table determine whether the decay $^{190}_{78}\text{Pt} \rightarrow {}^{186}_{76}\text{Os} + {}^{4}_{2}\text{He}$ is possible. ❏

Properties of some unstable nuclides.

Z	Element	Symbol	A	Atomic mass/ u	Relative abundance/%	Half-life
1	hydrogen	H	3	3.016 050		12.3 yr
2	helium	He	4	4.002 603	99.999	
4	beryllium	Be	8	8.0053		$\sim 3 \times 10^{-16}$ s
6	carbon	C	14	14.003 242		5730 yr
19	potassium	K	40	39.963 999	0.01	1.28×10^{9} yr
53	iodine	I	131	130.906 119		8.04 days
76	osmium	Os	186	185.953 852	1.6	2×10^{15} yr
78	platinum	Pt	190	189.959 937	0.013	6×10^{11} yr
84	polonium	Po	218	218.008 930		3.05 min
86	radon	Rn	218	218.005 580		0.035 s
			222	222.017 574		3.82 days
87	francium	Fr	221	221.014 183		4.8 min

5 Closing items

5.1 Module summary

1 Each atom has a nucleus — a positively charged ‘core’ that has a diameter of approximately 10^{-14} m. Apart from the hydrogen nucleus ${}^{1}_{1}H$, which is a *proton*, nuclei in general consist of combinations of protons and *neutrons*.

2 Each nucleus is characterized by its *atomic number* Z which gives the number of protons in the nucleus and its *mass number* A which gives the total number of *nucleons* (neutrons and protons) in the nucleus. Different *isotopes* of an element have different mass numbers but the same atomic number.

3 Nucleons are bound together in the nucleus by the *strong interaction*, which acts between all nucleons, is attractive, and has a range of about 10^{-14} m.

4 Energy has a mass equivalent. The mass m that is equivalent to energy E is given by *Einstein’s equation*

$$E = mc^2 \qquad \text{(Eqn 1)}$$

where c is the speed of light.

5 Nuclear energies are conveniently measured in units of MeV (megaelectronvolts)

6 Nuclear masses are conveniently measured in *atomic mass units* (u) or in MeV/c^2, where c is the speed of light.

7 The mass of a stable nucleus that contains more than one constituent is less than the sum of the masses of its free constituents. The difference is called the *nuclear binding energy*, and this may be calculated from measured values of nuclear masses.

8 The binding energy per constituent nucleon is, over a substantial range of nuclei, approximately constant at about 8.5 MeV. Very light and very heavy nuclei have smaller binding energies per nucleon.

9 The shape of the *nuclear binding energy graph* is consistent with the characteristics of the strong interaction between nucleons.

10 Not all nuclei are stable. Radioactive nuclei are unstable; they undergo *radioactive decay*. The half-life is the characteristic time after which half an original sample of radioactive nuclei can be expected to have decayed. There is an enormous range in values of *half-life* for *radioactive nuclei*.

11 The total (relativistic) energy of a particle is equal to its *rest energy* plus its kinetic energy. Tables of atomic masses are tables of *rest mass*.

12 Charge, nucleon number and (relativistic) energy are conserved in radioactive decay processes.

13 Mass tables can be used to calculate the kinetic energy of the decay products and to determine whether a reaction will be permitted by energy conservation.

5.2 Achievements

Having completed this module, you should be able to:

A1 Define the terms that are emboldened and flagged in the margins of the module.

A2 Describe the contents of a nucleus denoted in the standard way.

A3 Explain why nuclei generally have more neutrons than protons.

A4 Use Einstein's equation to calculate the rest energy of a particle given its mass in kg, with the answer given in units of J or MeV/c^2.

A5 Calculate the mass defect (in units of u and MeV/c^2) of a nucleus using mass tables and the conversion factor from u to MeV/c^2.

A6 Calculate the binding energy of a nucleus in MeV using mass tables and the conversion factor from u to MeV/c^2.

A7 Write down an appropriate α-decay equation, and decide whether a given α-decay equation is plausible.

A8 Calculate the Q-value of a given α-decay and hence decide whether the decay is energetically possible.

Study comment You may now wish to take the *Exit test* for this module which tests these Achievements. If you prefer to study the module further before taking this test then return to the *Module contents* to review some of the topics.

5.3 Exit test

Study comment Having completed this module, you should be able to answer the following questions each of which tests one or more of the Achievements.

Question E1

(*A2*) The nucleus $^{27}_{13}\text{Al}$ has an atomic mass of 26.981 541 u.

(a) How many protons does the nucleus contain?

(b) How many neutrons does the nucleus contain?

(c) By what factor is $^{27}_{13}\text{Al}$ more massive than $^{12}_{6}\text{C}$?

Question E2

(*A3*) All nuclei with large Z have more neutrons than protons. Explain in terms of the interactions concerned why this might be so.

Question E3

(*A4*) The (rest) mass of the neutron is $1.675 \times 10^{-27}\,\text{kg}$. Use the Einstein equation to calculate its rest energy. Give your answer in J and in MeV. (Use $c = 2.998 \times 10^{8}\,\text{m}\,\text{s}^{-1}$)

Question E4

($A5$ and $A6$) Using data from the table calculate:

(a) The mass defect of the nucleus of $^{197}_{79}\text{Au}$ in u, MeV/c^2 and GeV/c^2.

(b) The binding energy of the nucleus of $^{197}_{79}\text{Au}$ in GeV.

($1\,\text{u} = 931.502\,\text{MeV}/c^2$ and Au is the chemical symbol for gold)

Properties of some common nuclides

Z	Element	Symbol	A	Atomic mass/u	Relative abundance/%
1	hydrogen	H	1	1.007 825	99.985
30	zinc	Zn	64	63.929 145	48.6
			66	65.926 035	27.9
			67	66.927 129	4.1
			68	67.924 846	18.8
79	gold	Au	197	196.966 560	100
82	lead	Pb	204	203.973 037	1.4
			206	205.974 455	24.1
			207	206.975 885	22.1
			208	207.976 641	52.4
83	bismuth	Bi	209	208.980 388	100

* The neutron, in the free state, is not stable; it has a mean lifetime of approximately 15 min. It has a mass of 1.008 665 u.
In contrast, a free proton is stable and has a mass of 1.007 276 u.

Question E5

($A7$ and $A8$)   A nucleus of radon $^{222}_{86}Rn$ decays into a nucleus of polonium (Po) and an α-particle.

(a) Write down the equation of the decay.

(b) Calculate the Q-value for the decay, using data from the table, and hence complete the decay equation. ($1\,u = 931.502\,MeV/c^2$)

Z	Element	Symbol	A	Atomic mass/ u	Relative abundance/%	Half-life
1	hydrogen	H	3	3.016 050		12.3 yr
2	helium	He	4	4.002 603	99.999	
4	beryllium	Be	8	8.0053		$\sim 3 \times 10^{-16}$ s
6	carbon	C	14	14.003 242		5730 yr
19	potassium	K	40	39.963 999	0.01	1.28×10^{9} yr
53	iodine	I	131	130.906 119		8.04 days
76	osmium	Os	186	185.953 852	1.6	2×10^{15} yr
78	platinum	Pt	190	189.959 937	0.013	6×10^{11} yr
84	polonium	Po	218	218.008 930		3.05 min
86	radon	Rn	218	218.005 580		0.035 s
			222	222.017 574		3.82 days
87	francium	Fr	221	221.014 183		4.8 min

Question E6

(*A7* and *A8*) For each of the following decays, decide whether it is forbidden (use data from the table) and give a reason for your decision:

(a) $^{226}_{88}\text{Ra} \rightarrow {}^{223}_{87}\text{Fr} + {}^{1}_{1}\text{H}$

(b) $^{235}_{92}\text{U} \rightarrow {}^{230}_{90}\text{Th} + {}^{4}_{2}\text{He} + {}^{1}_{0}\text{n}$

Z	Element	Symbol	A	Atomic mass/ u	Relative abundance/%	Half-life
1	hydrogen	H	1	1.007 825	99.985	
			2	2.014 102	0.015	
2	helium	He	4	4.002 603	99.999	
87	francium	Fr	221	221.014 183		4.8 min
88	radium	Ra	222	222.015 353		38 s
			226	226.025 406		1.60×10^{3} yr
90	thorium	Th	230	230.033 131		8.0×10^{4} yr
			233	233.041 579		22.3 min
92	uranium	U	235	235.043 925	0.72	7.04×10^{8} yr
			238	238.050 786	99.28	4.47×10^{9} yr

* The neutron, in the free state, is not stable; it has a mean lifetime of approximately 15 min. It has a mass of 1.008 665 u. In contrast, a free proton is stable and has a mass of 1.007 276 u.

Study comment This is the final *Exit test* question. When you have completed the *Exit test* go back to Subsection 1.2 and try the *Fast track questions* if you have not already done so.

If you have completed **both** the *Fast track questions* and the *Exit test*, then you have finished the module and may leave it here.