Module P9.2 Radioactive decay

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1 Opening items

1.1 Module introduction

Towards the end of the 19th century, it was found that some nuclei are unstable —they spontaneously emit energetic radiation and sometimes change their chemical nature in the process. This phenomenon is known as *radioactive decay* or *radioactivity*, and is the subject of this module. In radioactive decay a *radioactive nucleus* or *radionuclide* emits *alpha*, *beta* or *gamma* radiation. In Section 2 we look at the properties of these types of *ionizing radiation*, and consider the processes by which nuclei decay and the relationship between the *parent nucleus* and the resulting *daughter nucleus*. This discussion includes an introduction to the *positron* and the *neutrino*, and shows how the penetration of matter by gamma-radiation can be described in terms of an *attenuation coefficient*. We conclude this discussion by taking a look at the four naturally-occurring *decay chains*.

Next, in Section 3, we see how a sample of radioactive material can be characterized by its *activity* and by its *half-life*, and hence introduce the *radioactive decay constant* and *radioactive decay law*. We then explain the use of radioactivity as a natural 'clock', and finish by discussing one of its applications — the dating of archaeological and geological materials by radiocarbon and potassium/argon dating.

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Study comment Having read the introduction you may feel that you are already familiar with the material covered by this module and that you do not need to study it. If so, try the *Fast track questions* given in Subsection 1.2. If not, proceed directly to *Ready to study?* in Subsection 1.3.

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1.2 Fast track questions

Study comment Can you answer the following *Fast track questions*?. If you answer the questions successfully you need only glance through the module before looking at the *Module summary* (Subsection 4.1) and the *Achievements* listed in Subsection 4.2. If you are sure that you can meet each of these achievements, try the *Exit test* in Subsection 4.3. If you have difficulty with only one or two of the questions you should follow the guidance given in the answers and read the relevant parts of the module. However, *if you have difficulty with more than two of the Exit questions you are strongly advised to study the whole module*.

Question F1

Radionuclides decay by emission of α-, β- or γ-radiation. What do these radiations consist of?




Question F2

The nucleus $^{232}_{92}\mathrm{U}$ decays as follows:

$$^{232}_{92}\mathrm{U} \rightarrow {}^{228}_{90}\mathrm{Th} + \alpha + 5.4\,\mathrm{MeV}$$

What *physical principles* determine how much of the 5.4 MeV is carried away as the kinetic energy of the α-particle? (Do not attempt to calculate a numerical value.)

Question F3

Write down the equation for the β^--decay of $^{137}_{55}\mathrm{Cs}$. Which conservation principles does the equation illustrate, and how does it do so? (It doesn't matter if you can't *name* the daughter nucleus.)

Question F4

A certain radionuclide has a half-life of 8 min. How long will it take for the activity of a sample to decay to 1% of its initial value?

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Study comment Having seen the *Fast track questions* you may feel that it would be wiser to follow the normal route through the module and to proceed directly to *Ready to study?* in Subsection 1.3.

Alternatively, you may still be sufficiently comfortable with the material covered by the module to proceed directly to the *Closing items*.

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1.3 Ready to study?

Study comment In order to begin to study this module you will need to be familiar with the following terms: *atomic mass number A*, *atomic mass unit*, *atomic number Z*, *binding energy*, *charge*, *Einstein's mass–energy equation*, *electromagnetic radiation*, *electromagnetic wave*, *electron*, *ionization*, *isotope*, *neutron*, *nucleon*, *photon*, *proton*, *rest energy* and *rest mass*. You should be familiar with the energy unit eV, and with masses expressed in atomic mass units (u) and in MeV/c^2. You should be able to use conventional symbols to represent nuclides and apply the conservation principles of *energy*, *linear momentum* and *electric charge*. You will also need to be familiar with *calculus notation*, *exponential functions*, *natural logarithms* (i.e. $\log_e$) and the relationship between them. If you are uncertain about any of these terms then you can review them by reference to the *Glossary*, which will also indicate where in *FLAP* they are developed. The following *Ready to study questions* will allow you to establish whether you need to review some of the topics before embarking on this module.

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Question R1

For the radium nucleus $^{222}_{88}\mathrm{Ra}$, write down the number of protons and neutrons in the nucleus.

Question R2

The mass of the proton is 1.672×10^{-27} kg. What is the energy equivalent of this mass?

Express the mass of the proton in MeV/c^2. (Use $e = 1.602 \times 10^{-19}$ C and $c = 2.998 \times 10^{8}$ m s^{-1}.)

Question R3

If $y = \mathrm{e}^{ax}$, write down expressions for x and dy/dx in terms of y and a.

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2 Decay processes in unstable nuclei

2.1 Introduction to radioactive decay

There are many possible combinations of protons and neutrons, but only a few correspond to completely stable nuclei. Most naturally-occurring *isotopes* are stable, but many are not. ☞ Unstable nuclei spontaneously emit energetic radiation, and may change their chemical nature. This phenomenon is called **radioactive decay** or **radioactivity**. An unstable nucleus decays into another nucleus, with the emission of *alpha-*, *beta-* or *gamma-* (α-, β- or γ-) *radiation*. The names α-, etc. were given to the three distinctive types of emission from radioactive materials at the time of their discovery (the late 19th century) before their natures were known. The initial nucleus is referred to as the **parent nucleus** (or **parent isotope**), the subsequent one as the **daughter nucleus** (or **daughter isotope**) ☞. The decay of a parent nucleus, P, to a daughter nucleus, D, generally involves the emission of other decay products and can be represented symbolically by

$$\mathrm{P} \rightarrow \mathrm{D} + \text{other products} + Q$$

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The quantity Q (which is sometimes omitted) represents the energy required to balance this process; its value can be determined by applying the *conservation of (relativistic) energy* to the process. That is to say, Q can be found by requiring that the total energy immediately before the decay is equal to the total energy immediately after the decay, where the term 'total energy' is taken to include the energy equivalent of any (rest) masses involved ☞ (as given by *Einstein's mass–energy equation*, $E = mc^2$) as well as the *relativistic kinetic energy* of the particles ☞. The energy represented by Q appears as the kinetic energy shared between the daughter nucleus, D, and other products. When Q is positive the decay is energetically allowed, whereas for negative Q it is energetically forbidden. The term **nuclide** is used to distinguish between nuclei that differ in their composition and/or internal energy, and unstable nuclei (i.e. those that undergo radioactive decay) are often referred to as **radionuclides** or **radioisotopes**.

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Figure 1 is a plot of the number of neutrons, N (where $N = A - Z$), against the numbers of protons, Z, for stable nuclides with mass number A. All stable nuclei lie on, or very near to, the **stability line** which corresponds approximately to $N = Z$ for light nuclei, but has $N > Z$ for heavier nuclei.

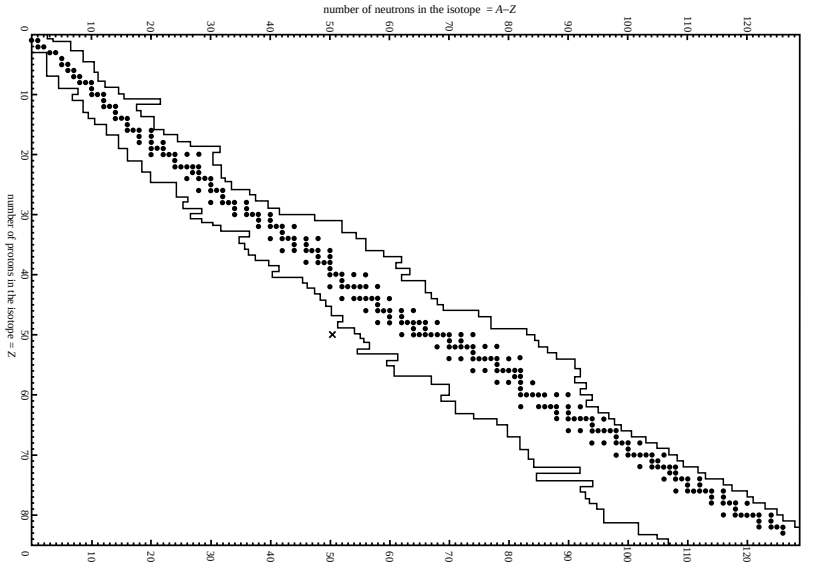

Figure 1 The number of neutrons in nuclei with $Z < 83$ plotted against the number of protons that the nuclei contain. Stable nuclei are shown as filled circles. The lines either side of the circles indicate the range within which the known unstable nuclei fall. The cross represents the latest isotope of tin to be found.

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A nucleus represented by a point some way from the stability line is unstable. This nucleus will decay into a daughter nucleus. If the point on the graph representing the daughter is near the stability line, then the daughter is likely to be stable and no further decay takes place. If however the point representing the daughter nucleus is not sufficiently near the line, then the daughter becomes the parent nucleus for a further decay, and so on until the stability line is reached.

We may therefore regard the 'purpose' of the decay as the approach to stability; each decay should bring the daughter closer to the stability line than the parent nucleus. To approach the stability line, the ratio N/Z of the neutron number to the proton number must be adjusted. Later in this section we will see how this may be achieved.

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Table 1 summarizes the properties of the three types of emission from radioactive nuclei. All three radiations interact with matter electromagnetically.

Table 1 Properties of emissions from radioactive nuclei. (See later for details.)

Type	Nature	Charge	Penetrating power
α	helium nucleus	$+2e$	low
β^-	electron	$-e$	fairly high
β^+	positron	$+e$	fairly high
γ	electromagnetic radiation	zero	very high

In α- and β-radiation the emitted particles are themselves charged and so interact powerfully with the charged particles (mainly the electrons) in matter. γ-radiation is a form of electromagnetic radiation and is therefore uncharged, though it too interacts with the electrons in matter, mainly through the electric field of the corresponding electromagnetic wave. Table 1 also indicates the radiation's penetrating power—because the charged particles interact much more powerfully than γ-radiation they lose their energy rapidly and penetrate very little. In contrast, γ-radiation interacts only feebly and is therefore more highly penetrating.

The energies of radioactive emissions are at least of the order of hundreds of keV, whereas the binding energies of electrons within atoms are of the order of tens of eV.

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- What is the likely result of the interaction between α-, β- or γ-radiation and an atomic electron?

This ability to ionize leads to a more general descriptive name for the radioactive emissions — **ionizing radiation**.

Decays are classified, according to the type of emission produced, as α-, β- or γ-decay. In each type of decay, (relativistic) energy and electric charge are conserved, as is the quantity $N + Z$ that represents the total number of nucleons (i.e. neutrons and protons taken together). In the remainder of this section, we take a look at each of these types of decay in turn.

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2.2 Alpha-decay

In **α-decay** (alpha-decay) the parent nucleus emits a helium nucleus, known as an **α-particle** in this context, which has a charge +2*e* and a mass of approximately 4 u. ☞ Thus for a parent nucleus, ${}^{A}_{Z}\mathrm{P}$:

In *α-decay*	${}^{A}_{Z}\mathrm{P} \rightarrow {}^{A-4}_{Z-2}\mathrm{D}_{\alpha} + {}^{4}_{2}\alpha + Q$	(1)

The parent nucleus is usually fairly heavy and has $N > Z$.

The daughter nucleus has two protons *and* two neutrons fewer than the parent. On the stability plot of Figure 2 the 'decay line' $\mathrm{P} \rightarrow \mathrm{D}_{\alpha}$ is therefore from right to left downwards at 45° to the horizontal.

✥ In α-decay *two* protons and *two* neutrons are emitted. How can the emission of an *equal* number of each particle change the ratio *N*/*Z* of the neutron number to the proton number and cause the daughter nucleus to be closer to the stability line than the parent nucleus?

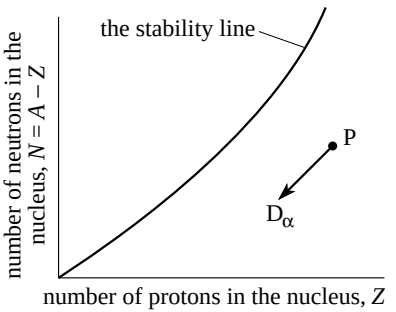

Figure 2 Alpha-decay of an unstable nucleus P.

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The emission of an α-particle from a heavy nucleus leads to an overall increase in nuclear *binding energy*, ☞ and allows some of the *rest energy* of the parent nucleus to supply the kinetic energy of the daughter nucleus and the α-particle. Typically, the energy released in an α-decay is a few MeV and this energy is divided between the daughter nucleus and the α-particle.

To see exactly how the energy is divided between the decay products we will look at a specific example, the α-decay of $^{210}_{84}$Po (polonium):

$$^{210}_{84}\text{Po} \rightarrow {}^{206}_{82}\text{Pb} + {}^{4}_{2}\text{He} + Q$$

where Q represents the total kinetic energy of the $^{206}_{82}$Pb (lead) and $^{4}_{2}$He (helium) nuclei. The decay is shown schematically in Figure 3.

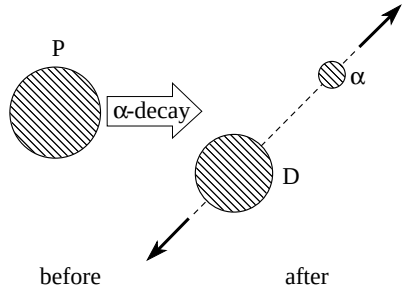

Figure 3 Schematic representation of To see exactly how the energy is divided between the decay products we will look at a specific example, the α-decay of $^{210}_{84}$Po (polonium):

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We will estimate the kinetic energy of the α-particle on the assumption that the original polonium nucleus is at rest and that in this case the kinetic energy is given by the Newtonian formula, $K = \frac{1}{2}mv^2$.
We use K_α and K_{Pb} to represent the kinetic energy of the α-particle and lead nucleus, respectively.

atomic mass of $^{210}_{84}$Po = 209.982 876 u

atomic mass of $^{206}_{82}$Pb = 205.974 455 u

atomic mass of $^{4}_{2}$He = 4.002 603 u

The difference in mass between the parent nucleus and the decay products is therefore:

209.982 876 u – (205.974 455 + 4.002 603) u = 0.005 818 u

This is the mass equivalent of the increase in binding energy and implies a Q-value of 5.42 MeV since 1 u = 931.5 MeV/c^2. To see how the kinetic energy $K_\alpha + K_{Pb}$ = 5.42 MeV is shared between the two particles, we use the fact that there is no external force acting on the decaying nucleus, so *linear momentum*, $\boldsymbol{p}$, is conserved. Since the parent nucleus is at rest, $\mathbf{0} = \boldsymbol{p}_{Pb} + \boldsymbol{p}_\alpha$. The implication of this is that the momentum vectors of the α-particle and the lead nucleus must have equal magnitude and be in opposite directions, so their magnitudes are equal, i.e. $p_{Pb} = p_\alpha$.

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To see what this implies for the kinetic energy note that:

$$K_\alpha + K_{Pb} = K_\alpha\left(1 + \frac{K_{Pb}}{K_\alpha}\right)$$

so, $$K_\alpha + K_{Pb} = K_\alpha\left(1 + \frac{m_{Pb}v_{Pb}^2}{m_\alpha v_\alpha^2}\right) = K_\alpha\left(1 + \frac{m_\alpha}{m_{Pb}}\frac{m_{Pb}^2 v_{Pb}^2}{m_\alpha^2 v_\alpha^2}\right)$$

Now, using the Newtonian formula, $p = mv$

$$K_\alpha + K_{Pb} = K_\alpha\left(1 + \frac{m_\alpha}{m_{Pb}}\frac{p_{Pb}^2}{p_\alpha^2}\right)$$

Since momentum conservation ensures that $p_{Pb} = p_\alpha$, we have, using the mass data given above:

$$K_\alpha + K_{Pb} = K_\alpha\left(1 + \frac{m_\alpha}{m_{Pb}}\right) = 1.019\,43\,K_\alpha$$

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But we know that $K_\alpha + K_{Pb} = 5.42\,\text{MeV}$, so $1.019\,43K_\alpha = 5.42\,\text{MeV}$. The kinetic energy of the α-particle is therefore 5.32 MeV and that of the lead nucleus 0.10 MeV. So, following the decay of $^{210}_{84}\text{Po}$, the $^{206}_{82}\text{Pb}$ and $^{4}_{2}\text{He}$ nuclei are emitted in opposite directions, the α-particle with an energy of 5.32 MeV and the lead nucleus with an energy of 0.10 MeV. Notice that the α-particle carries away most of the available kinetic energy; this is because its mass is much less than that of the daughter nucleus so it has to travel considerably faster to balance momentum, and its kinetic energy increases with the *square* of its speed.

This example suggests that for the general decay

$$\text{P} \rightarrow \text{D} + \alpha + Q$$

we have $$Q = K_\alpha + K_\text{D} = K_\alpha\left(1 + \frac{m_\alpha}{m_\text{D}}\right)$$

or $$Q \approx K_\alpha\left(1 + \frac{4}{A_\text{P} - 4}\right) = K_\alpha\left(\frac{A_\text{P}}{A_\text{P} - 4}\right)$$

where A_P is the mass number of the parent nucleus and we have made the approximation that $m_\alpha = 4\,\text{u}$, $m_\text{P} = A_\text{P}\,\text{u}$ and $m_\text{D} = (A_\text{P} - 4)\,\text{u}$.

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Thus the kinetic energy division of the available energy Q is given approximately by:

$$K_\alpha \approx Q\left(1-\frac{4}{A_P}\right) \qquad (2)$$

and $$K_D \approx Q\left(\frac{4}{A_P}\right) \qquad (3)$$

Notice that the kinetic energy of the α-particle depends *only* on the Q-value and on the mass of the parent nucleus. This means that *all α-particles emitted in this particular decay have the same kinetic energy*.

Question T1

The α-decay of the uranium isotope $^{238}_{92}\mathrm{U}$ is:

$$^{238}_{92}\mathrm{U} \rightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He} + 4.30\ \mathrm{MeV}$$

Calculate the kinetic energies of the $^{234}_{90}\mathrm{Th}$ and $^{4}_{2}\mathrm{He}$ nuclei. ❑

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When passing through a material, an α-particle ionizes atoms and molecules of the material by knocking out electrons, losing some of its own energy on each occasion. The mass of the α-particle is very large compared with that of the electron and so collisions with atomic electrons do not cause the α-particle to change direction substantially ☞. Since collision with a nucleus is unlikely, the α-particle usually continues in an almost straight line in the material until all its energy is lost. This loss involves a large number of collisions and is a statistical process, ensuring that the α-particle has a reasonably well defined range in any particular material. The range is the total distance travelled by the α-particle before it is brought to rest. It depends on the initial energy of the α-particle and on the medium through which it is passing. A range of a few centimetres in air at atmospheric pressure is typical. In solid materials the range is very small, of the order of tens of micrometres. ☞

Question T2

What would be the range of α-particles in a perfect vacuum? ❑

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2.3 Beta-decay

Beta-decay (β-decay) is rather more complicated than α-decay. β-decay involves the emission of a particle — named the **β-particle** — which is a high kinetic energy electron. The basic process is:

$$ {}^{A}_{Z}\mathrm{P} \rightarrow {}_{Z+1}{}^{A}\mathrm{D}_{\beta} + {}^{0}_{-1}\mathrm{e} + \ldots $$

Notice that, in order to conserve charge, β-decay must produce a daughter with a *higher Z* than the parent. In the mid 20th century it was established that this electron was also accompanied by another, neutral, particle (which we will discuss later).

A second type of β-decay can be observed:

$$ {}^{A}_{Z}\mathrm{P} \rightarrow {}_{Z-1}{}^{A}\mathrm{D}_{\beta} + {}^{0}_{+1}\mathrm{e} + \ldots $$

The particle emitted here has the same mass as the electron but it has the opposite charge of *+e* and has been named the **positron**. The β^+-particle (or positron, e^+) is the **antiparticle** of the electron. Each class of fundamental particles, protons, neutrons, electrons, etc. is thought to imply the existence of a corresponding class of antiparticles; electrons and positrons are simply one such pair of classes. Any antiparticle has the same mass as its partner particle but some of its other attributes, for example its charge, will have the *opposite* sign. In order to distinguish the two types of β-decay, emission of electrons is called β^--decay and emission of positrons β^+-decay. β^--particles are electrons, and β^+-particles are positrons. β^+-decay is rarer than β^--decay, as you will see later.

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β^--decay

There is a fundamental difference between β^--decay and α-decay. In α-decay the total number of protons (and also neutrons) before and after decay is the same. In β^--decay, however, the number of protons after decay has *increased* by one and the number of neutrons has *decreased* by one. The implication of this is that there must be some process which leads neutrons to change into protons:

$$n \rightarrow p^+ + e^- + \ldots$$

This process is fundamentally responsible for β^--decay and it must be caused by something other than the **strong interaction** that holds the protons and neutrons together in the nucleus, and the **electromagnetic interaction** that is responsible for the electrical attraction and repulsion of charged particles. We call this new interaction the **weak interaction**, ☞ since its intrinsic strength is less than that of the other two interactions. We will not discuss the weak interaction in any more detail here, except to say that because it is different from the interactions that produce the familiar 'everyday' forces of gravity and electromagnetism, and because it also differs from the strong interaction between nucleons, it is the fourth fundamental 'force' of nature.

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We said earlier that β^--decay involved the emission of another particle in addition to the electron. How do we know this? Why can we not simply have:

$$ {}^{A}_{Z}\mathrm{P} \rightarrow {}_{Z+1}{}^{A}\mathrm{D}_{\beta^-} + {}^{0}_{-1}\mathrm{e} \, ? $$

The main piece of evidence is this; when we study the kinetic energies of the β^--particles emitted in a particular β^--decay we find that, unlike α-decay, there is *not* a single unique kinetic energy for the β^--particle. All possible β^--particle (i.e. electron) kinetic energies are observed from zero up to some upper limit, which depends on the particular decay; this is shown in Figure 4.

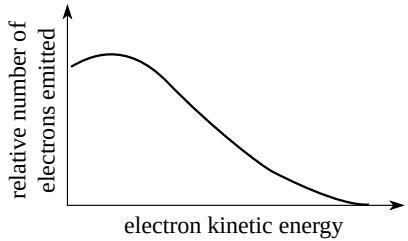

Figure 4 A typical spectrum of β^--particle kinetic energies emitted in β^--decay.

◆ Why does this observation suggest that in β^--decay there is an *additional particle* produced in the decay, to accompany the daughter nucleus and the β^--particle (the electron)?

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The observation of a continuous range of β^--particle kinetic energies in a given β^--decay process is now regarded as direct evidence of the involvement of a third particle. We say that β^--decay is a *three-body process*. However, when β^--decay was first observed this third particle was unknown and the continuous range of β^- energies a complete mystery. Indeed, so profound was the mystery that it was even suggested that the principles of energy conservation and momentum conservation might not apply to β^--decay. The position changed however in 1930 when Wolfgang Pauli (1900–1958) proposed the existence of a hitherto unobserved class of particles which he called **neutrinos**. If every β^--decay involved a particle belonging to this class, the mystery could be resolved. Subsequent studies not only confirmed the existence of Pauli's particles, they also established the existence of a *family* of neutrino-type particles. The current view is that there are *six* members of this family. The neutrino involved in β^--decay is now called the **electron antineutrino** and is represented by the symbol $\bar{\nu}_e$, ☞ so the full statement of β^--decay becomes:

β^--decay involves	$n \rightarrow p^+ + \underbrace{e^- + \bar{\nu}_e}_{\text{ejected}}$	(4)
and β^--decay itself is	${}^{A}_{Z}P \rightarrow {}^{A}_{Z+1}D_{\beta^-} + e^- + \bar{\nu}_e$	(5)

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If the electron carries off less than its maximum energy, the energy of the antineutrino accounts for the balance. The β^--decay is represented schematically in Figure 5. The energy released in β^--decay is of the order of MeV in magnitude. It is carried away predominantly by the electron and antineutrino. The electrons emitted in β^--decay have energies ranging from about 0.02 MeV to about 3 MeV.

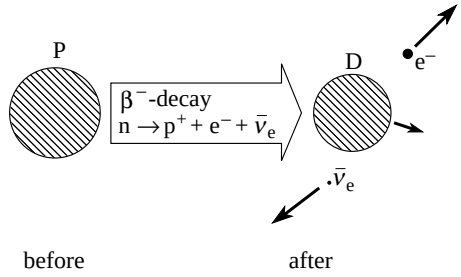

Figure 5 Schematic representation of β^--decay.

What properties must the antineutrino be postulated to have? It has no charge (if it did, charge would not be conserved in β^--decay). Energy conservation requires that the new particle, like all other neutrinos, has little or no rest mass. Without charge or rest mass and with no tendency to interact via the strong or electromagnetic interactions, the interaction of all neutrinos with matter is exceedingly feeble — in fact they can interact appreciably only via the weak nuclear interaction and cannot ionize matter. Neutrinos pass through matter with hardly any effect. They are able to pass entirely through the Earth with only a small chance of interaction! Trillions of them, coming from the Sun, pass through you every second! Their extraordinary powers of penetration make electron antineutrinos very difficult to detect, indeed, it was not until 1956 that particle detection technology was adequate for such a weakly interacting particle. Subsequently the other five neutrino family members have also been detected.

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Let us now return to the β^--particle itself (i.e. the electron), and investigate how it interacts with matter.

A β^--particle is *much* lighter than an α-particle, and so it is affected by its collisions with the atomic electrons in matter rather differently. In an interaction with an atomic electron it is deflected considerably and loses a substantial fraction of its kinetic energy. This means that an individual β^--particle does not travel in a straight line with a well-defined range, although it is carried in a generally 'forward' direction by its initial momentum. Despite this, the many β^--particles coming from a radioactive sample can be assigned a characteristic 'range' although it is not as well-defined as that of α-particles. Thus we can say that β^--particles are more penetrating than α-particles — for instance 100 keV β^--particles have a range of the order of 30 μm in aluminium.

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β^+-decay

We mentioned earlier that β^+-decay had been observed to have the general form:

$$ {}^{A}_{Z}\mathrm{P} \rightarrow {}_{Z-1}{}^{A}\mathrm{D}_{\beta^+} + \mathrm{e}^+ + \ldots $$

In this case the daughter nucleus has one proton fewer than the parent although the total number of nucleons remains the same. There must be some process involved which makes protons change into neutrons. Such a process is:

$$ \mathrm{p}^+ \rightarrow \mathrm{n} + \mathrm{e}^+ + \ldots $$

In β^+-decay the positron is found to have a continuous range of kinetic energies up to some maximum value for a given decay and so we know that a third particle is involved. Once again the new particle is a neutrino, in this case the **electron neutrino**. The electron neutrino and the electron antineutrino are particle and antiparticle and the electron neutrino is our second member of the family of six neutrino types.

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The full description of β^+-decay is then

β^+-decay involves	$p^+ \rightarrow n + \underbrace{e^+ + \nu_e}_{\text{ejected}}$	(6)
and β^+-decay itself is	${}^{A}_{Z}P \rightarrow {}^{A}_{Z-1}D_{\beta^+} + e^+ + \nu_e$	(7)

If we compare Equations 4 and 6 (for β^--decay and β^+-decay)

β^--decay involves $n \rightarrow p^+ + \underbrace{e^- + \bar{\nu}_e}_{\text{ejected}}$ (Eqn 4)

and remember that the neutron mass (1.008 665 u) exceeds the proton mass (1.007 825 u) it is apparent that it is energetically possible for a neutron to spontaneously decay into a proton, but not vice versa, i.e. the *Q*-value for Equation 4 may be positive but that for Equation 6 must be negative. For this reason an isolated neutron (outside the nucleus) will decay into a proton (the neutron *half-life* is about 10.6 min) but an isolated proton will *not* decay into a neutron — it is a stable particle.

❖ Why do you think β^+-decay, a process which is energetically unfavoured, can nonetheless occur *inside the nucleus*?

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The β^+-decay is represented schematically in Figure 6. The energy released in β^+-decay is of the order of MeV in magnitude. It is carried away predominantly by the positron and the neutrino.

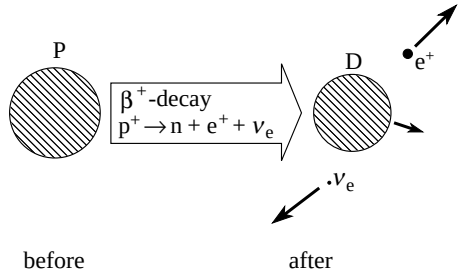

Figure 6 Schematic representation of β^+-decay.

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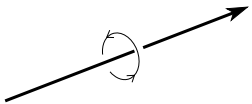
Aside

Since neither the electron neutrino nor the electron antineutrino has any electric charge you may wonder what it means to say they are particle and antiparticle. One fundamental difference between them involves a property known as **spin**. Many particles, including electrons, protons and all types of neutrinos have spin. ☞ It is actually a subtle quantum phenomenon, but for our purposes it can be crudely thought of as something like the spin of a ball about an axis. If we take the direction of motion of a neutrino or antineutrino as a suitable axis about which to measure the spin an interesting result emerges. An electron antineutrino approaching an observer always appears to spin in an anticlockwise sense, while an approaching electron neutrino always appears to spin in a clockwise sense (see Figure 7). This difference is symptomatic of the deeper difference that exists between the neutrino and the antineutrino.

antineutrino spin and direction of motion



neutrino spin and direction of motion

Figure 7 The spin directions of neutrinos and antineutrinos.

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Positrons can interact destructively with electrons by what is called **pair annihilation**. This is a common property of a particle and its partner antiparticle. The interaction is:

$$e^+ + e^- \rightarrow 2\gamma$$

The total energy (rest energy + kinetic energy) of the electron–positron pair is converted into the electromagnetic energy of the **γ-radiation**. Since electrons are plentiful in matter, positrons are absorbed very rapidly in matter via this process.
The parent nucleus is usually fairly heavy and has $N > Z$.

Question T3

On a sketch of Figure 2, indicate the position of the daughter nuclei produced if P undergoes β^+- or β^--decay. Which side of the stability line must P lie if the nucleus is to achieve greater stability by undergoing β^+-decay? ❑

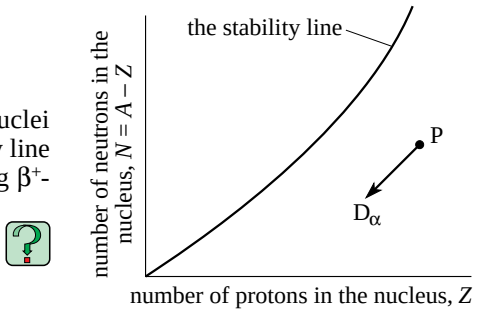

Figure 2 Alpha-decay of an unstable nucleus P.

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2.4 *Q*-values for β-decay

In Subsection 2.2 we considered the energy balance in the α-decay of $^{210}_{84}\mathrm{Po}$ and we were able to calculate the *Q*-value for the decay from the *atomic* masses of the participants. If we now extend this idea to the calculation of *Q*-values in β-decay we must take care to remember that the *atomic* masses used in the calculations include the masses of the atomic electrons and we are therefore effectively overestimating the mass of each proton by an amount equal to the mass of an electron. For α-decay we need not be concerned about this since the number of protons present before and after the decay is the same; so when we subtract the initial and final mass energies to find *Q*, the overestimate cancels out. We may therefore safely use *atomic* masses and still obtain the correct energy balance.

The situation in β-decay is different since electrons or positrons (which have the same mass as electrons) are produced in the process and the number of protons *does* change. We must take account of both these effects when we use *atomic* masses to calculate *Q*. We can however ignore the neutrinos since their masses are negligible or zero.

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In β^--decay the daughter nucleus has one additional proton and the atomic mass will assume the presence of one *additional* electron — which is equivalent to the rest mass of the emitted β^--particle. Thus, in using atomic masses in the calculation we need not worry about the rest energy of the β^--particle, and the Q-value found is the total energy which may appear distributed as the kinetic energy of the products.

In β^+-decay the daughter nucleus has one fewer proton and the atomic mass will assume the presence of one *fewer* electron. In fact the decay must still produce *one additional electron rest energy* (the rest energy of the emitted positron) and so our calculation using atomic masses will *overestimate* the available Q-value by an amount equal to twice the rest energy of an electron, which is $2m_e c^2$.

Question T4

Calculate the rest mass of an electron in atomic mass units and in MeV/c^2 given its rest mass is 9.11×10^{-31} kg, 1 eV = 1.60×10^{-19} J, the speed of light in vacuum is 3.00×10^8 m s^{-1} and 1 u = 1.66×10^{-27} kg. ❑

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Question T5

For the β^-- and β^+-decays below, calculate the energy balances Q_1 and Q_2

β^--decay: ${}^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N}_{\beta^-} + \mathrm{e}^- + \overline{\nu}_\mathrm{e} + Q_1$

β^+-decay: ${}^{13}_{7}\mathrm{N} \rightarrow {}^{13}_{6}\mathrm{C}_{\beta^+} + \mathrm{e}^+ + \nu_\mathrm{e} + Q_2$

given the following atomic masses: ${}^{13}_{6}\mathrm{C}$ = 13.003 355 u, ${}^{14}_{6}\mathrm{C}$ = 14.003 242 u, ${}^{13}_{7}\mathrm{N}$ = 13.005 739 u, ${}^{14}_{7}\mathrm{N}$ =14.003 074 u and 1 u = 931.5 MeV/c^2. ❑

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2.5 Gamma-decay

The third sort of radioactive decay, **γ-decay**, produces γ-radiation, which is electromagnetic radiation of very short wavelength ($\lambda < 50$ pm) ☞. It is emitted when a nucleus in an excited (higher energy) state loses energy. ☞ The process is a nuclear analogue to the emission of light by an excited atom, but in the nuclei the energy changes are typically a million times greater than in atoms (roughly of order MeV in nuclei compared with a few eV in atoms) and this causes γ-radiation to be of much shorter wavelength than visible light. The excited nucleus is often the outcome of a previous α- or β-decay which has resulted in the daughter nucleus being left in an excited state rather than in its ground state. γ-radiation is uncharged and its emission leaves the number of nucleons unchanged, so the initial and final nuclides differ *only* in their energy and hence must have the same chemical symbol. The excited state nucleus is denoted by an asterisk (*); so the full process is:

In *γ-decay* $\quad {}^{A}_{Z}\mathrm{D}^{*}_{\gamma} \rightarrow {}^{A}_{Z}\mathrm{D}_{\gamma} + \gamma$

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γ-decay does not change the position of the point representing the nucleus on Figure 13 (see Answer T3).

γ-radiation has the same fundamental nature as any other form of electromagnetic radiation, such as light or X-rays. It can be thought of either as a stream of particles called *photons* ☞ or as a train of *electromagnetic waves*. The photons are characterized by their energy E; the waves by their wavelength λ or their frequency f (in a vacuum the γ-rays travel at the speed of light $c = f\lambda$).

Regarded as electromagnetic waves, the very short wavelength of γ-rays ($\lambda < 50\,\text{pm} = 0.05\,\text{nm}$) and correspondingly high frequency ($f > 6 \times 10^{18}\,\text{Hz}$) contrast strongly with the properties of visible light which has much longer wavelengths (roughly 400 nm to 700 nm) and much lower frequencies (roughly $4.3 \times 10^{14}\,\text{Hz}$ to $7.5 \times 10^{14}\,\text{Hz}$). Indeed, γ-rays are generally much closer in character to X-rays which are typically regarded as having wavelengths in the range 0.03 nm to 3 nm and frequencies in the range $10^{17}\,\text{Hz}$ to $10^{19}\,\text{Hz}$.

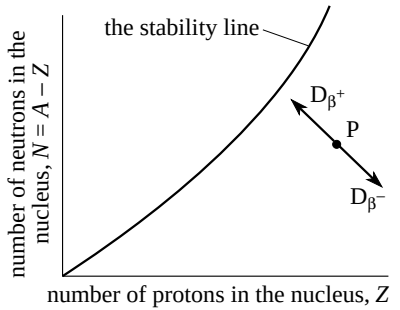

Figure 13 See Answer T3.

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When electromagnetic radiation, characterized by a frequency f, is regarded as a stream of particle-like photons, the energy E of each individual photon is given by:

Planck's formula: $E = hf$ (8)

where Planck's constant $h = 6.626 \times 10^{-34}\,\text{J s} = 4.136 \times 10^{-15}\,\text{eV s}$ ☞. The range of energies for visible light photons is from about 1.8 eV to 3.1 eV, while γ-ray photons have typical energies above 25 keV. It is customary to talk about γ-ray emissions in terms of the energies of the photons rather than in terms of the wavelength or frequency of the radiation.

Question T6

A photon of γ-radiation has an energy of 1 MeV. What is the frequency corresponding to this energy? ❑

When a nucleus reduces its energy by an amount ΔE, by making a transition from an excited state to a state of lower energy, the emitted γ-ray photon carries away this energy ΔE and so the photon frequency is given by Planck's formula (Equation 8) as:

$$f = \frac{\Delta E}{h} \qquad (9)$$

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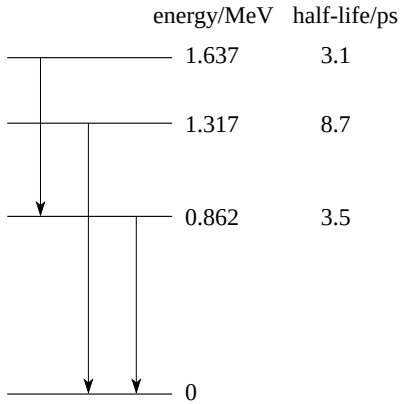
By observing such photons it is possible to determine the energies of transitions that gave rise to them. An **energy level diagram** (for $^{72}_{34}$Se) showing the energy associated with various states of the nucleus is given in Figure 8. The strongest γ-ray emitting transitions are also shown on the figure. As with atomic energy states, the lowest nuclear energy state is called the *ground state*. De-excitation from a higher energy state may result in the emission of one or more γ-rays. For example, the decay of the state at 1.637 MeV to the ground state can proceed via two γ-rays through the state at 0.862 MeV.



Question T7

Use the information in Figure 8 to determine the energies (in MeV) and frequencies of the γ-ray photons for the transitions depicted by the vertical arrows. ❑

Figure 8 Part of the energy level diagram of $^{72}_{34}$Se showing possible γ-decays and the half-lives of individual states. Each vertical arrow represents a strong transition leading to the emission of a γ-ray. The *half-life* here is equal to the time for half a sample of excited state nuclei to leave the excited state by decay to a lower state. The meaning of half-life will be discussed more fully in Section 3 of this module.

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We mentioned earlier that a radioactive decay sometimes produces a daughter nucleus in an excited state which then decays with the emission of a γ-ray. Figure 9 shows part of the energy level diagrams for $^{137}_{55}Cs$ and $^{137}_{56}Ba$ which indicates that $^{137}_{55}Cs$ may decay to an excited state of $^{137}_{56}Ba$ according to:

$$^{137}_{55}Cs \rightarrow {}^{137}_{56}Ba^{*} + e^{-} + \bar{\nu}_e \qquad \beta^{-}\text{-decay}$$

followed, almost immediately, by the γ-transition

$$^{137}_{56}Ba^{*} \rightarrow {}^{137}_{56}Ba + \gamma \qquad \gamma\text{-decay}$$

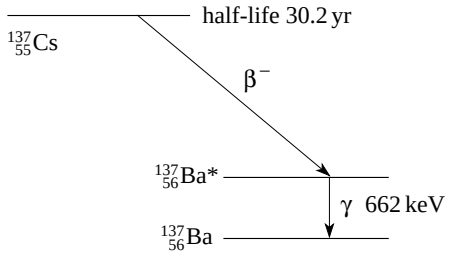

Figure 9 The decay sequence for a $^{137}_{55}Cs$ γ-ray.

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This illustrates another common feature of radioactive decay — the presence of γ-emission accompanying other emissions such as β- or α-decay. Figure 8 gives the fairly typical γ-decay half-lives of picoseconds whereas β-decay half-lives may be much longer — from seconds to many years.

This difference is due to the different interactions responsible for the two decay processes. γ-decay is electromagnetic in origin (with photons emitted) whereas β-decay is driven by the nuclear weak interaction (as discussed earlier). The different timescales involved give rise to the convention for the *naming* of the γ-rays from various nuclides.

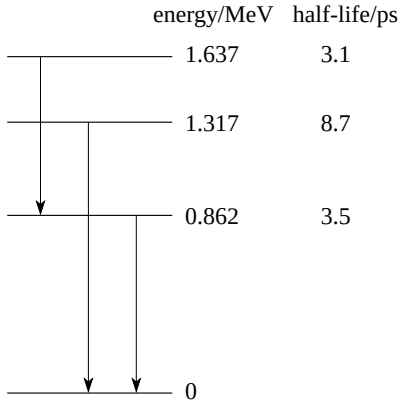

Figure 8 Part of the energy level diagram of $^{72}_{34}$Se showing possible γ-decays and the half-lives of individual states. Each vertical arrow represents a strong transition leading to the emission of a γ-ray. The *half-life* here is equal to the time for half a sample of excited state nuclei to leave the excited state by decay to a lower state. The meaning of half-life will be discussed more fully in Section 3 of this module.

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To produce a source of the γ-rays shown in Figure 9 we would first need to produce radioactive caesium $\left({}^{137}_{55}\mathrm{Cs}\right)$ (with a half-life of 30.2 yr) so that in the decay sequence the excited state of barium $\left({}^{137}_{56}\mathrm{Ba}^{*}\right)$ is produced which will then emit the required γ-ray. (If we tried to produce the excited barium directly we would have only a few picoseconds to deliver it to the user!)

For this reason the γ-ray in Figure 9 is usually (but rather loosely) described as a ${}^{137}_{55}\mathrm{Cs}$ γ-ray, even though it is produced by ${}^{137}_{56}\mathrm{Ba}$. This convention for the naming of γ-rays is in common use.

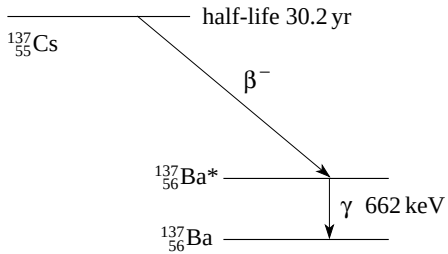

Figure 9 The decay sequence for a ${}^{137}_{55}\mathrm{Cs}$ γ-ray.

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γ-rays are much more penetrating than α- or β-particles. They are absorbed via processes which lead to the ionization of atoms in the material through which the γ-rays are passing. For the most part, the absorption of γ-rays in matter is attributable to their interaction with the electrons in the atoms of the material. However, it makes little difference which electrons are involved or to which atoms they are bound. This is because the energy of typical γ-rays (~MeV) far exceeds the binding energies of electrons in atoms (~eV) and so the electrons act effectively as free particles. The effectiveness of a material as an absorber of γ-rays is determined simply by how many electrons are contained per unit volume. Since in neutral matter each electron is accompanied by a much more massive proton, the *density* of a material is a good indicator of the electron content — i.e. good γ-ray absorbers are dense materials. It is this property that makes lead a common choice as an absorber on a small scale, but due to cost, dense concrete shielding is used on a large scale.

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The energy carried per second per unit area of cross section of a γ-ray beam (i.e. its intensity I) falls exponentially ☞ with the thickness x of the absorber traversed according to the following expression, where I_0 is the initial intensity of the γ-ray beam before it entered the material:

$$I(x) = I_0 e^{-\mu x} \qquad (10)$$

The coefficient μ, called the **attenuation coefficient**, describes how effectively the γ-ray beam is absorbed per unit thickness. The value of this coefficient depends both on the material and on the energy of the γ-ray photons. The product μx must be a dimensionless number, so the dimensions of μ are the reciprocal of length and will be related to the units of x. For instance, μ for lead with 10 MeV γ-ray photons is about 0.060 mm^{-1} but for 2 MeV γ-ray photons it is about 0.055 mm^{-1} — a small but significant difference. μ for water with 2 MeV γ-ray photons is about 5 m^{-1} (i.e. 0.005 mm^{-1}).☞) γ-rays are able to penetrate significantly through several centimetres of lead.

Question T8

For the 2 MeV γ-ray photons referred to above, what thickness of lead is needed to reduce the intensity to (a) 10% and (b) 1% of its initial value? (c) What thickness of water would give the reduction to 10% of the initial value? ❑

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2.6 Decay chains

Most unstable nuclei do not achieve stability in a single decay. For any decay, the daughter nucleus itself may be unstable — it becomes the parent for a further radioactive decay, and so on …

For example, consider the decay of the uranium nucleus $^{238}_{92}\mathrm{U}$. It decays through a series of steps, in a **nuclear decay chain**, until a stable nucleus is reached — namely the lead isotope $^{206}_{82}\mathrm{Pb}$, as shown in Figure 10 ☞. Have a look at the various links in the chain. You will see for instance that the $^{238}_{92}\mathrm{U}$ nucleus decays first by α-decay to the nucleus $^{234}_{90}\mathrm{Th}$, which then undergoes β^--decay to a $^{234}_{91}\mathrm{Pa}$ nucleus.

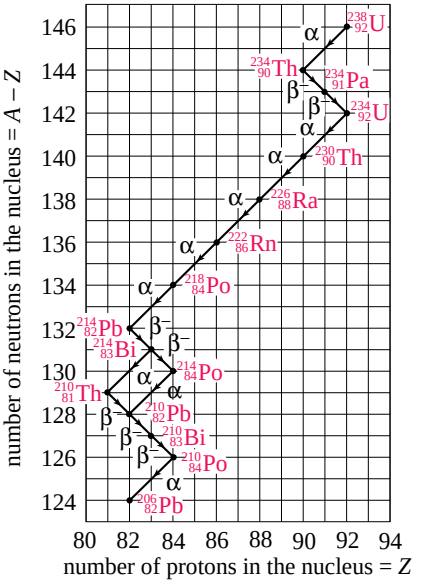

Figure 10 The decay chain $^{238}_{92}\mathrm{U} \rightarrow {}^{206}_{82}\mathrm{Pb}$.

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chain continues until eventually it ends with $^{206}_{82}\mathrm{Pb}$, which is stable.

One interesting feature of this nuclear decay chain occurs at $^{214}_{83}\mathrm{Bi}$. Here the chain forks. This bismuth isotope may decay *either* into $^{210}_{81}\mathrm{Tl}$ by α-decay *or* into $^{214}_{84}\mathrm{Po}$ by β^--decay. $^{214}_{83}\mathrm{Bi}$ is said to have two **nuclear decay channels**.

There are *four* naturally-occurring nuclear decay chains — also called the **natural radioactive series**. They are called the thorium, neptunium, uranium and actinium series.

Why are there only four series? The reason is related to the mass number *A* of the nuclei in a series. As with the uranium series shown in Figure 10, the decays may be by α-decay, by β^--decay and, additionally but less commonly, by β^+-decay. (γ-decay may also occur but this does not involve a change of isotope and so is not usually shown.)

An α-decay reduces *A* by 4; a β^-- or β^+-decay leads to no change in *A* between the parent and daughter nucleus. Therefore when *A* reduces in a series it must do so in units of four; there can therefore only be four independent series to include all values of *A*.

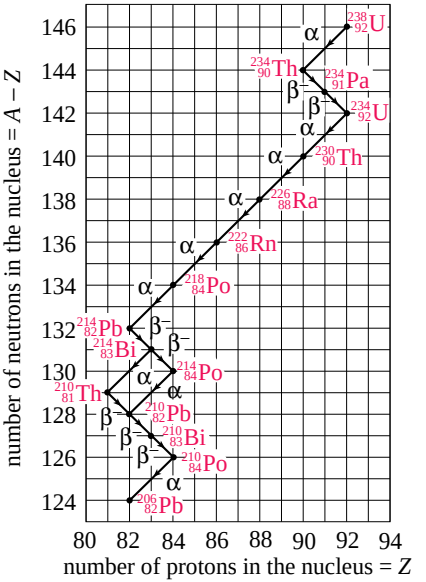

Figure 10 The decay chain $^{238}_{92}\mathrm{U} \rightarrow {}^{206}_{82}\mathrm{Pb}$.

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Suppose, for instance, that a nucleus in a series has a mass number, A, that is a multiple of four; $A_{\text{Parent}} = 4n$, where n is an integer. This nucleus, if it decays by β^-- or β^+-decay gives $A_{\text{Daughter}} = 4n$; if it decays by α-decay it leads to $A_{\text{Daughter}} = 4(n-1)$. In both cases both parent and daughter nuclei have a mass number that is a multiple of four, as will *all* nuclei in this series. Other series will also link nuclei with mass numbers changing by four or zero but with a difference of 1, 2 or 3 from the multiples of 4 we have just discussed. There are therefore just four series as is illustrated in Table 2, each of which consists of decays that either leave A unchanged or reduce its value by 4.

Table 2 The four natural radioactive series.

Name of series	A (n = an integer)	Final stable nucleus	Longest lived nucleus	Half-life of longest lived intermediate nucleus/Gyr
thorium	$4n$	$^{208}_{82}\text{Pb}$	$^{232}_{90}\text{Th}$	14
neptunium	$4n + 1$	$^{209}_{83}\text{Bi}$	$^{237}_{93}\text{Np}$	0.0021
uranium	$4n + 2$	$^{206}_{82}\text{Pb}$	$^{238}_{92}\text{U}$	4.5
actinium	$4n + 3$	$^{207}_{82}\text{Pb}$	$^{235}_{92}\text{U}$	0.70

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Question T9

$^{217}_{85}$At (astatine) decays to $^{209}_{83}$Bi (bismuth) by a succession of α- and β^--decays. (a) To which natural series does this decay belong? (b) How many steps of each type must there be in this part of the decay chain?
(Note that you are not asked for the *order* of the various steps.) ❑

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3 Radioactive half-life and radioactive dating

3.1 The properties of radioactive materials

So far we have been concerned with the particles emitted as a result of radioactive decay and the changes to individual nuclei. We now turn to the large-scale properties of radioactive materials and how they change with time. The **activity** of a sample is defined as the *average* number of nuclear decays per second. The SI unit of activity is named after the French physicist Anton Henri Becquerel (1852–1908), the first person to observe radioactivity:

> 1 **becquerel** = 1 Bq = 1 nuclear decay per second

For commonly occurring radioactive samples, activities are often measured in gigabecquerels (GBq, 1 GBq = 10^9 Bq) because the Bq is a *very* small unit. The historical unit, the curie (Ci) was originally defined as the activity of 1 g of the radium isotope $^{226}_{88}\mathrm{Ra}$ ☞. Improved methods of measurement caused the numerical value of this unit to be changed from time to time as more accurate determinations were made. To avoid such changes in the future the curie is now *defined* to have *precisely* the value:

$$1\ \mathrm{Ci} = 3.70 \times 10^{10}\ \text{decays per second} = 37.0\ \mathrm{GBq}$$

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Using this modern definition, the *measured* activity of 1 g of $^{226}_{88}\mathrm{Ra}$ is a few per cent smaller than 1 Ci.

It is important to realize that radioactive decay is a *random* process — it is impossible to predict exactly when a particular nucleus will decay. Only the *probability* that it will have decayed after a certain time can be predicted. ☞ We work in terms of the *average* (mean) number of decays per second because measurements of the actual number of decays in any one second will fluctuate randomly about the average. If the activity is large, these fluctuations will be only a small percentage of the average, but for low activities, the fluctuations become significant in comparison with the average.

For a sample containing a single radionuclide, the activity is proportional to the number of unstable nuclei present. ☞ It depends *only* on the identity and number of nuclei, and is *not* affected by any external factors such as temperature or pressure, nor does it depend on whether the radioactive nuclei are in single atoms or combined in a chemical compound. These results are not surprising when we remember that radioactivity is a *nuclear* process, not an atomic (electron) process, and the conditions within a nucleus are not affected by external influences or by chemical processes.

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If $N(t)$ is the number of unstable nuclei present in the sample, and $R(t)$ the activity of the sample at time t, then

$$R(t) = \lambda N(t) \qquad (11)$$

where λ, the **decay constant**, is a characteristic property of the particular radionuclide.

- What are the SI units of λ?

One way of interpreting $\lambda = R(t)/N(t)$ is to say that at any time t, when $N(t)$ nuclei are present, λ is numerically equal to the *fraction* of nuclei that will decay in the next second. Alternatively, λ can be viewed as the *probability* that any given remaining nucleus will decay during the next second. However, both these interpretations of λ are strictly only applicable when $N(t)$ can be taken as a constant over the one-second period, i.e. when $\lambda << 1\ s^{-1}$.

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More rigorously, if over a short time interval Δt the average activity is $\langle R(t) \rangle$ and the average number of parent nuclei present is $\langle N(t) \rangle$ then the small change in $N(t)$ over the period is $\Delta N(t)$ where

$$\Delta N(t) = -\langle R(t) \rangle \Delta t = -\lambda \langle N(t) \rangle \Delta t$$

or $\langle R(T) \rangle = \dfrac{-\Delta N(t)}{\Delta t}$ and $\dfrac{\Delta N(t)}{\Delta t} = -\lambda \langle N(t) \rangle$

If we take the limit of these expressions as $\Delta t \to 0$, using the standard calculus notation ☞

$\lim_{\Delta t \to 0} \left[\dfrac{\Delta N(t)}{\Delta t} \right] = \dfrac{dN(t)}{dt}$, we find

$$R(t) = -\frac{dN(t)}{dt} \qquad (12)$$

and $$\frac{dN(t)}{dt} = -\lambda N(t) \qquad (13)$$

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We now see that λ is the constant of proportionality relating $N(t)$ to the rate of change of $N(t)$. Since we are using the *instantaneous* rate of change, this interpretation includes situations where $N(t)$ (and hence the rate of decay $R(t)$) change significantly during one second.

Now let us look more closely at how the activity of a sample of radioactive material changes with time.

Figure 11 (next page) shows a plot of activity, $R(t)$, against time for a typical case. The time axis is marked off in equal intervals, each of duration τ, during which the activity decreases by a factor of two, i.e. τ is the time taken for half the radioactive sample to decay.

> The time τ after which the activity has diminished to one-half of its initial value is called the **half-life** of the radionuclide.

The value of τ is a characteristic of the radionuclide, and is not affected by external conditions ☞. It does not depend on the initial activity of the sample. You can check this by choosing *any* point on Figure 11 (next page) and finding the time for the activity to fall to half your chosen value.

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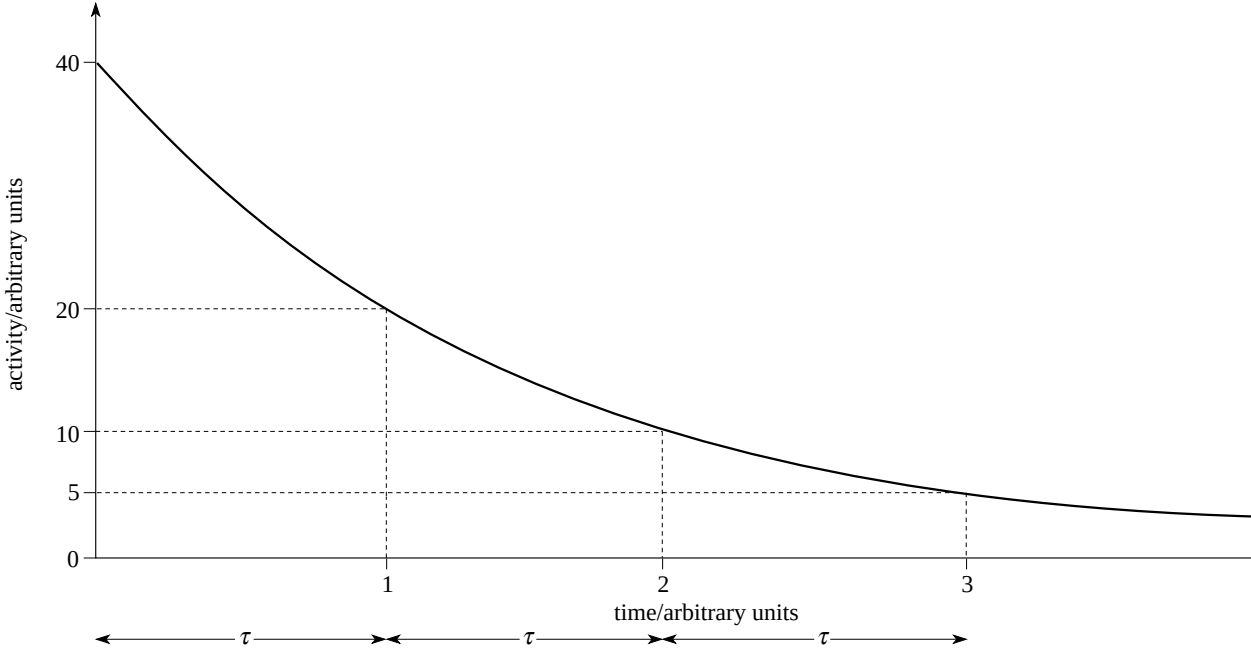

Figure 11 A graph of the activity of a radioactive sample against time.

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- What would be the shape of the graph of $N(t)$ plotted against time for the same sample?

- How does $N(t)$ change during a time interval τ?

We could also define the half-life as the time τ over which the number of radioactive nuclei present falls to half its initial value.

Equation 13 is an example of a *differential equation*.

$$\frac{dN(t)}{dt} = -\lambda N(t) \qquad \text{(Eqn 13)}$$

To solve this equation we need to find $N(t)$ such that its *first derivative* with respect to t reproduces the original function $N(t)$ with a numerical multiplier $(-\lambda)$. It is shown elsewhere in *FLAP* that the solution involves an *exponential function* of time ☞.

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In fact it has the following form:

The *radioactive decay law* $N(t) = N_0 e^{-\lambda t}$ (14)

since we then have

$$\frac{dN(t)}{dt} = -\lambda N_0 e^{-\lambda t} = -\lambda N(t)$$

as required by Equation 13.

$$\frac{dN(t)}{dt} = -\lambda N(t) \qquad \text{(Eqn 13)}$$

Also at $t = 0$ the value of $N(0)$ becomes N_0 according to Equation 14, so N_0 *is the number of radioactive nuclei present at* $t = 0$.

$$R(t) = -\frac{dN(t)}{dt} \qquad \text{(Eqn 12)}$$

From Equations 12, 13 and 14 the activity $R(t) = -\dfrac{dN(t)}{dt} = \lambda N(t) = \lambda N_0 e^{-\lambda t}$

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We can therefore write:

The *activity law*	$R(t) = R_0 e^{-\lambda t}$	(15)

where $R_0 = \lambda N_0$, the initial activity at $t = 0$.

Equation 13

$$\frac{dN(t)}{dt} = -\lambda N(t) \qquad \text{(Eqn 13)}$$

asserts the *independence* of each radioactive nucleus — each nucleus of a given type has the same probability of decay in a given time, and so the total decay rate must be proportional to the total number of participating nuclei. *This aspect of independence is at the heart of the many exponential processes in physics* — including those in Equations 14 and 15.

The *radioactive decay law* $N(t) = N_0 e^{-\lambda t}$ (Eqn 14)

Equation 14 (the **radioactive decay law**) and Equation 15 (the **activity law**) express the fact that in a sample composed of nuclei of a given type both the activity and the number of radioactive nuclei fall exponentially with time, with the same decay constant. Of course the activity can be readily observed and so this is the usual method of measuring the decay constant.

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Radioactive decay can be described either in terms of the decay constant λ (with units of s^{-1}) or the half-life τ (with units of s). We can see how these two are related if we substitute $t = \tau$ in Equation 15,

The *activity law* $R(t) = R_0 e^{-\lambda t}$ (Eqn 15)

remembering that $R(\tau) = \frac{1}{2} R_0$, we find

$$\tfrac{1}{2} R_0 = R_0 e^{-\lambda \tau}$$

If we divide both sides of this equation by R_0, it cancels from the equation. This confirms that τ *is* independent of the initial activity R_0. If we then take the reciprocal of each side of the equation, we find

$$e^{\lambda t} = 2$$

Taking natural logarithms of each side gives us

$$\lambda \tau = \log_e 2$$

This leads us to the following relationship between decay constant and half-life:

$$\lambda = \frac{\log_e 2}{\tau} = \frac{0.693}{\tau} \qquad (16)$$

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Therefore *either* the half-life *or* the decay constant gives a complete description of how the activity of a sample of radionuclide changes with time. It is in fact more usual to quote the half-life (see, for example, Table 2). Some half-lives are very long, so it is customary to quote their values in suitable units of time — minutes, days, years, or even multiples of years.

Table 2 The four natural radioactive series.

Name of series	A (n = an integer)	Final stable nucleus	Longest lived nucleus	Half-life of longest lived intermediate nucleus/Gyr
thorium	$4n$	$^{208}_{82}\mathrm{Pb}$	$^{232}_{90}\mathrm{Th}$	14
neptunium	$4n + 1$	$^{209}_{83}\mathrm{Bi}$	$^{237}_{93}\mathrm{Np}$	0.0021
uranium	$4n + 2$	$^{206}_{82}\mathrm{Pb}$	$^{238}_{92}\mathrm{U}$	4.5
actinium	$4n + 3$	$^{207}_{82}\mathrm{Pb}$	$^{235}_{92}\mathrm{U}$	0.70

It is then usual to give λ in the equivalent reciprocal unit, using Equation 16.

$$\lambda = \frac{\log_e 2}{\tau} = \frac{0.693}{\tau} \qquad \text{(Eqn 16)}$$

For example, if $\tau = 2.5$ min, then $\lambda = 0.693/(2.5\ \text{min}) = 0.28\ \text{min}^{-1}$.

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Question T10

The half-life of $^{131}_{53}\mathrm{I}$ is 8.0 days. Calculate, (a) the decay constant of this isotope, and (b) the time for the activity of a sample to decrease by a factor of 10. ❑

Note that the time for the activity to decrease by a factor of 10 also is independent of the actual activities. The time taken to change by *any* chosen factor depends only on the particular factor chosen—this is a characteristic of *all* exponential changes.

We are now able to calculate the activity of any sample of radionuclide. For example, suppose we wish to calculate the initial activity of a 1 μg sample of $^{60}_{27}\mathrm{Co}$, with a half-life of 5.3 years. We can find this by using $R_0 = \lambda N_0$, once we know λ and N_0.

The decay constant λ of $^{60}_{27}\mathrm{Co}$ (from Equation 16) is:

$$\lambda = \frac{0.693}{\tau} = \frac{0.693}{5.3 \times 365 \times 24 \times 60 \times 60\,\mathrm{s}} = 4.15 \times 10^{-9}\ \mathrm{s}^{-1}$$

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The number of nuclei N_0 in 1 μg of $^{60}_{27}\mathrm{Co}$ is approximately given by:

$$N_0 = \frac{1.00 \times 10^{-9}\ \mathrm{kg}}{(60\ \mathrm{u}) \times (1.66 \times 10^{-27}\ \mathrm{kg\,u^{-1}})} = 1.00 \times 10^{16}$$

So, the activity is

$$R_0 = \lambda N_0 = (4.15 \times 10^{-9}\ \mathrm{s^{-1}}) \times 1.00 \times 10^{16} = 4.15 \times 10^{7}\ \mathrm{s^{-1}} = 41.5\ \mathrm{MBq}$$

Question T11

The half-life of $^{131}_{53}\mathrm{I}$ is 8.0 days. Calculate the activity of 1 μg of the isotope.

(*Hint*: Use Answer T10(a).) ❑

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3.2 Radiocarbon dating

We have already mentioned that the decay constant (and half-life) of a nuclide is unaffected by temperature and pressure, and is independent of the chemical environment, for example, whether the nucleus is part of the atom in a molecule or not. Radioactive nuclei can therefore be used as a 'clock' that is unaffected by changes in conditions. This is most useful when attempting to measure very long periods of time, for example in dating archaeological samples. One isotope that can be used in this way is $^{14}_{6}\mathrm{C}$ (known as carbon-14).

Carbon-14 decays by β^--emission to nitrogen-14, with a half-life of 5730 years. As it is a light element, it is not produced in any of the natural radioactive decay chains. You might expect therefore that the amount of $^{14}_{6}\mathrm{C}$ on Earth would long ago have been reduced to zero. However, it is found that the proportion of $^{14}_{6}\mathrm{C}$ to the stable isotope $^{12}_{6}\mathrm{C}$ is remarkably constant at about $1:10^{12}$. This is because $^{14}_{6}\mathrm{C}$ is constantly being produced in the upper atmosphere due to bombardment by *cosmic rays*. Plants, for example, take in carbon (as carbon dioxide) from the atmosphere and incorporate it in their living tissue. The tissue will have the same ratio of $^{14}_{6}\mathrm{C}$ to $^{12}_{6}\mathrm{C}$ ratio as the atmosphere while carbon dioxide is being absorbed. However, when the plant dies it ceases to take in carbon from the atmosphere. From that moment the proportion of $^{14}_{6}\mathrm{C}$ will start to decrease with the characteristic half-life of the unstable $^{14}_{6}\mathrm{C}$ (the amount of the stable isotope $^{12}_{6}\mathrm{C}$ will, of course, remain constant).

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The proportion of $^{14}_{6}C$ in once-living material is therefore a measure of its age. This is the principle behind **radiocarbon dating**, summarized in Figure 12.

The activity per unit mass of a specimen is called the **specific activity** of the specimen. The strict SI unit for this quantity would be Bq kg^{-1} but other units, such as Bq g^{-1} or mCi g^{-1}, are often used.

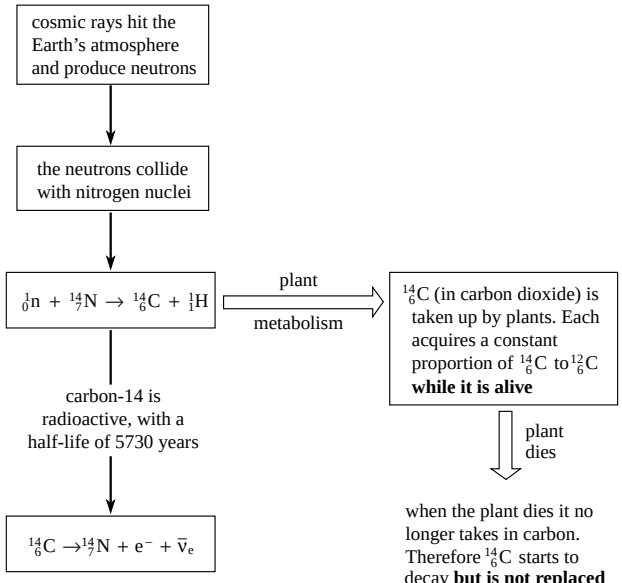

Figure 12 The basic principles behind radiocarbon dating.

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✥ A live piece of wood has an average specific activity of $15\,\mathrm{Bq\,g^{-1}}$ due to the β^--decay of $^{14}_{6}\mathrm{C}$. Assuming that the average specific activity of live wood has been constant for at least the past 6000 years, what average specific activity would you expect from a piece of wood 'harvested' 5730 years ago?

So, a measurement of the specific activity R ☞) of a plant specimen indicates the time since its death providing we know its specific activity R_0 when it was alive. It is useful to have a general formula relating the present specific activity of a sample to its age in terms of the half-life. From Equation 15,

The *activity law* $R(t) = R_0 e^{-\lambda t}$ (Eqn 15)

$R_0/R(t) = e^{\lambda t}$, so

$\lambda t = \log_e(R_0/R)$ and hence:

$$t = \frac{\log_e (R_0/R)}{\lambda} \qquad (17)$$

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Using Equation 16 to eliminate λ

$$\lambda = \frac{\log_e 2}{\tau} = \frac{0.693}{\tau} \qquad \text{(Eqn 16)}$$

we obtain

$$t = \tau\left[\frac{\log_e (R_0/R)}{\log_e 2}\right] \qquad (18)$$

where τ is the half-life.

Question T12

A sample of cedar wood taken from the Egyptian pyramid of Sneferu has a specific activity from $^{14}_{6}C$ decay corresponding to 55% of the present natural level. How old is the sample?

(You should assume that the natural level has been constant over the time concerned.) ❑

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We should always be concerned about the reliability of any physical measurement. One limiting factor in this case is our ability to measure the very low activities that the very old specimens show. As a rough measure, after about ten half-lives have elapsed the activity will be too weak to measure accurately. This gives an upper time limit of the order of 50 thousand years beyond which it would be unwise to rely on radiocarbon dating.

The primary limitation, though, concerns our knowledge of R_0 and relates to the assumption that the proportion of ${}^{14}_{6}\mathrm{C}$ in the atmosphere has been constant over a very long period. We can check this by comparing the radiocarbon dates of suitable objects with dates found by other, independent, means. For example, the objects being dated may be paper or papyrus documents whose dates are known, or there may be documentary evidence for the dates of other artefacts. Broadly speaking, such checks support the acceptance of a fairly uniform level of ${}^{14}_{6}\mathrm{C}$ and thus indicate a fairly constant flux of cosmic rays.

There is, however, a lower age limit to the use of carbon dating because of human intervention. The burning of fossil fuels in much larger amounts since about the middle of the 18th century has significantly reduced the proportion of ${}^{14}_{6}\mathrm{C}$ in the atmosphere. This is because these fuels contain negligible amounts of ${}^{14}_{6}\mathrm{C}$, containing ${}^{12}_{6}\mathrm{C}$ almost exclusively. Also the testing of nuclear weapons in the atmosphere in the 1950s and 1960s approximately doubled the rate of production of ${}^{14}_{6}\mathrm{C}$ during these decades.

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3.3 Potassium/argon dating

As we noted above, there is an upper age limit of about 50 thousand years for radiocarbon dating. To a geologist wishing to date a rock sample this is a *very* short time! Fortunately there are other decays with much longer half-lives that may be used for dating geological samples. One is the β^+-decay of potassium-40 to argon-40:

$$^{40}_{19}\mathrm{K} \rightarrow {}^{40}_{18}\mathrm{Ar} + \mathrm{e}^+ + \nu_\mathrm{e}$$

which has a half-life of 1.3×10^9 years. Another is the β^--decay of rubidium-87 to strontium-87:

$$^{87}_{37}\mathrm{Rb} \rightarrow {}^{87}_{38}\mathrm{Sr} + \mathrm{e}^- + \overline{\nu}_\mathrm{e}$$

which has a half-life of 4.8×10^{10} years. In either case, the ratio of daughter nuclei to parent nuclei in a sample of rock gives a measure of its age. The method relies on four assumptions:

- when the rock solidified (which we shall call time $t = 0$) a certain number of parent nuclei, $N_\mathrm{P}(0)$, were trapped;
- at that time there were no daughter nuclei present, i.e. $N_\mathrm{D}(0) = 0$;
- no parent or daughter nuclei have escaped from the sample since;
- daughter nuclei have been created within the sample *only* by the one decay process and not as a result of some other radioactive decay.

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If all the above conditions are satisfied, then for times $t > 0$,

$$N_P(0) = N_P(t) + N_D(t)$$

since the decay of a parent nucleus (${}^{40}_{19}\mathrm{K}$, for instance) leads to the creation of a daughter nucleus (${}^{40}_{18}\mathrm{Ar}$ in this case), i.e. the number of parent plus daughter nuclei is constant. We also know, from Equation 14, that $N_P(t) = N_P(0)e^{-\lambda t}$. This allows us to find a relationship between time t and the ratio N_D/N_P. Combining our two expressions we find:

$$N_P(t) = N_P(0)e^{-\lambda t} = [N_P(t) + N_D(t)]e^{-\lambda t}$$

i.e. $N_P(t)e^{\lambda t} = [N_P(t) + N_D(t)]$

and hence

$$e^{\lambda t} = \left[1 + \frac{N_D(t)}{N_P(t)}\right]$$

If we take natural logarithms of both sides and then divide by λ, we find

$$t = \frac{1}{\lambda}\log_e\left[1 + \frac{N_D(t)}{N_P(t)}\right]$$

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Using Equation 16

$$\lambda = \frac{\log_e 2}{\tau} = \frac{0.693}{\tau} \qquad \text{(Eqn 16)}$$

to substitute for λ we obtain the age in terms of the half-life of the parent nucleus:

$$t = \frac{\tau}{\log_e 2} \log_e \left[1 + \frac{N_D(t)}{N_P(t)} \right] \qquad (19)$$

Question T13

A sample of rock is thought by geologists to be 3.5 billion (3.5×10^9) years old. What argon/potassium ratio is expected for the sample? ❑

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4 Closing items

4.1 Module summary

1 Unstable (radioactive) nuclei undergo *α*-, *β-* or *γ-decay*. *α-particles* are $^4_2$He nuclei; *β–-particles* are electrons, *β^+-particles* are *positrons*; *γ-rays* are energetic photons of electromagnetic radiation.

2 The position of an unstable nucleus with respect to the nuclear *stability line* determines its likely mode(s) of decay.

3 Any given β-decay produces β-particles with a continuous range of energies up to a maximum. This implies that another particle is emitted during β-decay. The *neutrino* was postulated (and later confirmed experimentally) to fulfil this role.

4 β-decay arises from the 'decay' of a neutron to a proton (β^--decay) or vice versa (β^+-decay) within a nucleus. In addition to the *daughter nucleus* and the β-particle a *neutrino* (β^+-decay) or an *antineutrino* (β^--decay) is emitted during the decay of the *parent nucleus*.

5 An *antiparticle* has the same mass as its corresponding particle, but is opposite in other respects (e.g. its charge has the same magnitude but opposite sign).

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6 An excited nucleus can exist in a number of different states, each characterized by its own particular energy. These energies can be displayed on an *energy level diagram*. γ-decay is the result of a transition from an excited state of the nucleus to a state of lower energy. The energy of a γ-ray emitted in such a transition is given by the energy difference between the two nuclear states involved and is related to the frequency of the γ-radiation through the Planck formula $E = hf$.

7 α-, β- and γ-rays are all *ionizing radiations*. α-radiation most readily loses energy via ionization so it has the shortest range in matter, γ-radiation is absorbed most weakly and has the greatest range in matter, with the intensity in a beam falling exponentially with distance travelled.

8 The daughter nuclei arising from the decay of heavy parent nuclei are usually themselves unstable. A *decay chain* continues until a stable daughter nucleus is produced. There are four naturally-occurring decay chains.

9 Both the *activity* and the number of parent nuclei in a sample of a radioactive isotope decay exponentially with time:

$$N(t) = N_0 e^{-\lambda t} \qquad \text{(Eqn 14)}$$

$$R(t) = R_0 e^{-\lambda t} \qquad \text{(Eqn 15)}$$

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10 The time τ for activity or number of radionuclei to fall to one-half of the initial value is called the *half-life* of the radionuclide. The half-life is independent of the initial value of activity or number of nuclei:

$$\tau = \frac{\log_e 2}{\lambda}$$

11 Any radioactive decay can be characterized either by its half-life or its *decay constant*. These two values are related to each other and are unaffected by the physical or chemical environment of the radionuclide.

12 Radioactive decay can be used to date objects of organic origin (radiocarbon dating) or geological specimens (potassium/argon dating).

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4.2 Achievements

Having completed this module, you should be able to:

A1 Define the terms that are emboldened and flagged in the margins of this module.

A2 State the properties of α-, β- and γ-radiation and state how α-, β- and γ-decay changes the mass number and atomic number of a nucleus.

A3 Calculate the kinetic energies of the products of α-decay from a stationary nucleus and describe the physical principles underlying the calculation.

A4 Describe the underlying β^-- and β^+-decay processes and explain why the neutrino (or antineutrino) was postulated.

A5 State how a particle differs from its partner antiparticle.

A6 Calculate the energy of a γ-ray photon given its frequency, or vice versa.

A7 Give a general account of the absorption of α-, β- and γ-radiation in matter.

A8 Calculate the reduction in intensity of a γ-ray beam by a specified thickness of a material, given the attenuation coefficient.

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A9 Use the activity law to calculate the activity of a radioactive sample after given times.

A10 Calculate the activity of a given mass of a radionuclide of known half-life.

A11 State the basic principles underlying the techniques of radiocarbon dating and potassium/argon dating, and state the main limitations of each technique.

A12 Calculate sample ages using radiocarbon and potassium/argon dating information.

Study comment You may now wish to take the *Exit test* for this module which tests these Achievements. If you prefer to study the module further before taking this test then return to the *Module contents* to review some of the topics.

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4.3 Exit test

Study comment Having completed this module, you should be able to answer the following questions each of which tests one or more of the Achievements.

Question E1

(*A2*) State the possible types of decay of unstable nuclei. For each, say what is produced in addition to the daughter nucleus.

Question E2

(*A3*) For the α-decay of $^{216}_{86}\mathrm{Rn}$ the Q-value is 8.16 MeV. What is the initial kinetic energy of the daughter polonium nucleus?

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Question E3

(*A4*) Write down the processes underlying both types of β-decay. What features of the decay lead to the postulation of the neutrino (or antineutrino)?

Question E4

(*A5*) A certain particle called a pion is denoted π^+. Suggest a suitable symbol for its antiparticle (other than $\overline{\pi^+}$). Give a reason for your suggestion. What will be the relationship between the masses of the π^+ and its antiparticle?

Question E5

(*A6*) What is the frequency corresponding to a 10 MeV γ-ray photon?

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Question E6

(*A2* and *A7*) How is energy transferred from α-, β^-- and γ-radiation to matter? What, in general terms, determines the rate of absorption?

Question E7

(*A8*) What thickness of lead will halve the intensity of a 10 MeV beam of γ-rays? The attenuation coefficient is $0.06\ \text{mm}^{-1}$.

Question E8

(*A9*) The half-life of $^{212}_{83}\text{Bi}$ is 60.6 min. How long will it be before the activity of a 3.0 GBq source has decreased to 0.30 MBq?

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Question E9

(*A10*) Estimate the activity, in Bq and in Ci, of 0.1 μg of $^{216}_{84}$Po. The half-life is 0.15 s.
(Use 1 u = 1.66×10^{-27} kg and 1 Ci = 37.0 GBq.)

Question E10

(*A11* and *A12*) The activity of a sample of charcoal from an ancient fire is found to be 15% of that from freshly made charcoal. Estimate the age of the sample. On what assumptions does the estimate depend?
(The half-life of $^{14}_{6}$C is 5730 years.)

Question E11

(*A12*) In a specimen of rock the argon/potassium ratio is 2.70. Estimate the age of the rock.
(The half-life of $^{40}_{19}$K is 1.3×10^{9} years.)

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Study comment This is the final *Exit test* question. When you have completed the *Exit test* go back to Subsection 1.2 and try the *Fast track questions* if you have not already done so.

If you have completed **both** the *Fast track questions* and the *Exit test*, then you have finished the module and may leave it here.

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