



# Algebraic Fractions

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The aim of this document is to provide a short, self assessment programme for students who wish to acquire a basic competence in the use of algebraic fractions.

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# 1. Algebraic Fractions(Introduction)

Algebraic fractions have properties which are the same as those for numerical fractions, the only difference being that the the **numerator** (top) and **denominator** (bottom) are both algebraic expressions.

**Example 1** Simplify each of the following fractions.

$$(a) \quad \frac{2b}{7b^2}, \quad (b) \quad \frac{3x + x^2}{6x^2}.$$

**Solution**

$$(a) \quad \frac{2b}{7b^2} = \frac{2 \times \cancel{b}}{7 \times b \times \cancel{b}} = \frac{2}{7b}$$

$$(b) \quad \begin{aligned} \frac{3x + x^2}{6x^2} &= \frac{x \times (3 + x)}{x \times 6x} \\ &= \frac{\cancel{x} \times (3 + x)}{\cancel{x} \times 6x} = \frac{3 + x}{6x} \end{aligned}$$

**N.B.** The cancellation in (b) is allowed since  $x$  is a common factor of the **numerator** and the **denominator**.

Sometimes a little more work is necessary before an algebraic fraction can be reduced to a simpler form.

**Example 2** Simplify the algebraic fraction

$$\frac{x^2 - 2x + 1}{x^2 + 2x - 3}$$

**Solution**

In this case the numerator and denominator can be factored into two terms, thus

$$x^2 - 2x + 1 = (x - 1)^2, \quad \text{and} \quad x^2 + 2x - 3 = (x - 1)(x + 3).$$

( See the package on **factorising expressions**). With this established the simplification proceeds as follows:

$$\begin{aligned} \frac{x^2 - 2x + 1}{x^2 + 2x - 3} &= \frac{(x - 1) \times (x - 1)}{(x + 3) \times (x - 1)} \\ &= \frac{x - 1}{x + 3} \quad (\text{cancelling } (x - 1)) \end{aligned}$$

**EXERCISE 1.** Simplify each of the following algebraic fractions. (Click on the green letters for solution.)

(a)  $\frac{8y}{2y^3}$

(b)  $\frac{2y}{4x}$

(c)  $\frac{7a^6b^3}{14a^5b^4}$

(d)  $\frac{(2x)^2}{4x}$

(e)  $\frac{5y + 2y^2}{7y}$

(f)  $\frac{5ax}{15a + 10a^2}$

(g)  $\frac{2z^2 - 4z}{2z^2 - 10z}$

(h)  $\frac{y^2 + 7y + 10}{y^2 - 25}$

(i)  $\frac{w^2 - 5w - 14}{w^2 - 4w - 21}$

Now try this short quiz.

**Quiz** Which of the following is a simplified version of

$$\frac{t^2 + 3t - 4}{t^2 - 3t + 2}?$$

(a)  $\frac{t - 4}{t - 2}$

(b)  $\frac{t - 4}{t + 2}$

(c)  $\frac{t + 4}{t - 2}$

(d)  $\frac{t + 4}{t + 2}$

So far, simplification has been achieved by cancelling common factors from the **numerator** and **denominator**. There are fractions which can be simplified by *multiplying* the **numerator** and **denominator** by an appropriate common factor, thus obtaining an equivalent, simpler expression.

**Example 3** Simplify the following fractions.

$$(a) \quad \frac{\frac{1}{4} + y}{\frac{1}{2}} \qquad (b) \quad \frac{3x + \frac{1}{x}}{2}$$

**Solution**

(a) In this case, multiplying both the **numerator** and the **denominator** by 4 gives:

$$\frac{\frac{1}{4} + y}{\frac{1}{2}} = \frac{4\left(\frac{1}{4} + y\right)}{4\left(\frac{1}{2}\right)} = \frac{1 + 4y}{2}$$

(b) To simplify this expression, multiply the **numerator** and **denominator** by  $x$ . Thus

$$\frac{3x + \frac{1}{x}}{2} = \frac{x\left(3x + \frac{1}{x}\right)}{2x} = \frac{3x^2 + 1}{2x}$$

Now try this exercise on similar examples.

**EXERCISE 2.** Simplify each of the following algebraic fractions. (Click on the **green** letters for solution.)

(a)  $\frac{4y - \frac{3}{2}}{2}$

(b)  $\frac{2x + \frac{1}{2}}{x + \frac{1}{4}}$

(c)  $\frac{z - \frac{1}{3}}{z - \frac{1}{2}}$

(d)  $\frac{2 - \frac{1}{x}}{2}$

(e)  $\frac{3t - \frac{2}{t}}{\frac{1}{2}}$

(f)  $\frac{z - \frac{1}{2z}}{z - \frac{1}{3z}}$

For the last part of this section, try the following short quiz.

**Quiz** Which of the following is a simplified version of

$$\frac{x - \frac{1}{x+1}}{x - 1} ?$$

(a)  $\frac{x^2 - x + 1}{x^2 + x + 1}$

(b)  $\frac{x^2 - x + 1}{x^2 - 1}$

(c)  $\frac{x^2 - 1}{x^2 - x - 1}$

(d)  $\frac{x^2 + x - 1}{x^2 - 1}$

## 2. Addition of Algebraic Fractions

Addition (and subtraction) of algebraic fractions proceeds in exactly the same manner as for numerical fractions.

**Example 4** Write the following sum as a single fraction in its simplest form.

$$\frac{2}{x+1} + \frac{1}{x+2}$$

**Solution** The *least common multiple* of the denominators (see the package on **fractions**) is  $(x+1)(x+2)$ . Thus

$$\begin{aligned}\frac{2}{x+1} + \frac{1}{x+2} &= \frac{2 \times (x+2)}{(x+1) \times (x+2)} + \frac{1 \times (x+1)}{(x+2) \times (x+1)} \\ &= \frac{2x+4}{(x+1)(x+2)} + \frac{x+1}{(x+1)(x+2)} \\ &= \frac{(2x+4) + (x+1)}{(x+1)(x+2)} = \frac{3x+5}{(x+1)(x+2)}\end{aligned}$$



**EXERCISE 3.** Evaluate each of the following fractions. (Click on the green letters for solution.)

(a)  $\frac{2}{y} + \frac{3}{z}$

(b)  $\frac{1}{3y} - \frac{2}{5y}$

(c)  $\frac{3z+1}{3} - \frac{2z+1}{2}$

(d)  $\frac{3t+1}{2} + \frac{1}{t}$

(e)  $\frac{x+1}{2} + \frac{1}{x-1}$

(f)  $\frac{2}{w+3} - \frac{5}{w-1}$

**Quiz** Which of the following values of **a** is needed if

$$\frac{\mathbf{a}}{2x+1} + \frac{1}{x+2} = \frac{4x+5}{(2x+1)(x+2)}?$$

(a) **a** = 3

(b) **a** = -3

(c) **a** = 2

(d) **a** = -2

### 3. Simple Partial Fractions

The last quiz was an example of *partial fractions*, i.e. the technique of decomposing a fraction as a sum of simpler fractions. This section will consider the simpler forms of this technique.

**Example 5** Find the partial fraction decomposition of  $4/(x^2 - 4)$ .

**Solution** The denominator factorises as  $x^2 - 4 = (x - 2)(x + 2)$ . (See the package on **quadratics**.) The partial fractions will, therefore, be of the form  $a/(x - 2)$  and  $b/(x + 2)$ . Thus

$$\begin{aligned}\frac{a}{x - 2} + \frac{b}{x + 2} &= \frac{4}{(x - 2)(x + 2)} \\ \frac{a(x + 2)}{(x - 2)(x + 2)} + \frac{b(x - 2)}{(x + 2)(x - 2)} &= \frac{4}{(x - 2)(x + 2)} \\ \frac{(a + b)x + 2(a - b)}{(x - 2)(x + 2)} &= \frac{4}{(x - 2)(x + 2)} \\ \text{so that } (a + b)x + 2(a - b) &= 4\end{aligned}$$

The last line is

$$(a + b)x + 2(a - b) = 4,$$

and this enables  $a$  and  $b$  to be found. For the equation to be true *for all* values of  $x$  the coefficients must match, i.e.

$$a + b = 0 \quad (\text{coefficients of } x)$$

$$2a - 2b = 4 \quad (\text{constant terms})$$

where the first equation holds since there is no  $x$  term in  $4/(x^2 - 4)$ . This set of simultaneous equations may be solved to give  $a = 1$  and  $b = -1$ . (See the package on **simultaneous equations** for a method of finding these solutions.)

Thus

$$\frac{4}{(x - 2)(x + 2)} = \frac{1}{x - 2} - \frac{1}{(x + 2)}$$

**EXERCISE 4.** For each of the following, find  $a$  and  $b$ . (Click on the green letters for solution.)

$$(a) \quad \frac{a}{x-2} + \frac{b}{x+2} = \frac{4x}{(x-2)(x+2)}$$

$$(b) \quad \frac{a}{z-3} + \frac{b}{z+2} = \frac{z+7}{(z-3)(z+2)}$$

$$(c) \quad \frac{a}{w-4} + \frac{b}{w+1} = \frac{3w-2}{(w-4)(w+1)}$$

Now try this final, short quiz.

**Quiz** If  $\frac{a}{2x-3} + \frac{b}{3x+4} = \frac{x+7}{(2x-3)(3x+4)}$ , which of the following is the solution to the equation?

$$(a) \quad a = 1, b = -1$$

$$(c) \quad a = 1, b = 1$$

$$(b) \quad a = -1, b = 1$$

$$(d) \quad a = -1, b = -1$$

## 4. Quiz on Algebraic Fractions

**Begin Quiz** In each of the following, choose:

1. the simplified form of  $(z^2 + 4z - 5)/(z^2 - 4z + 3)$

(a)  $(z + 5)/(z + 3)$

(b)  $(z + 5)/(z - 3)$

(c)  $(z - 5)/(z + 3)$

(d)  $(z - 5)/(z - 3)$

2. the sum  $[1/(w - 2)] - [1/(w + 7)]$

(a)  $5/(w^2 + 5w - 14)$

(b)  $5/(w^2 - 5w + 14)$

(c)  $9/(w^2 - 5w + 14)$

(d)  $9/(w^2 + 5w - 14)$

3.  $a$  and  $b$  if

$$[a/(3x + 2)] + [b/(4x - 3)] = (x - 5)/[(3x + 2)(4x - 3)]$$

(a)  $a = 1, b = 1$

(b)  $a = -1, b = 1$

(c)  $a = -1, b = -1$

(d)  $a = 1, b = -1$

**End Quiz**

## Solutions to Exercises

### Exercise 1(a)

$$\begin{aligned}\frac{8y}{2y^3} &= \frac{4 \times 2y}{y^2 \times 2y} \\ &= \frac{4 \times \cancel{2} \times \cancel{y}}{y^2 \times \cancel{2} \times \cancel{y}} \\ &= \frac{4}{y^2}\end{aligned}$$

Click on the green square to return



**Exercise 1(b)**

$$\begin{aligned}\frac{2y}{4x} &= \frac{2 \times y}{2 \times 2x} \\ &= \frac{\cancel{2} \times y}{\cancel{2} \times 2x} \\ &= \frac{y}{2x}\end{aligned}$$

Click on the green square to return



**Exercise 1(c)** The fraction is  $\frac{7a^6b^3}{14a^5b^4}$ . This time, instead of expanding the factors, it is easier to use the rule for powers

$$\frac{a^m}{a^n} = a^{m-n}.$$

(See the package on **powers**.) Thus

$$\begin{aligned}\frac{7a^6b^3}{14a^5b^4} &= \frac{7}{14} \times \frac{a^6}{a^5} \times \frac{b^3}{b^4} \\ &= \frac{1}{2} \times a^{6-5} \times b^{3-4} \\ &= \frac{1}{2} \times a^1 \times b^{-1} = \frac{a}{2b}\end{aligned}$$

Click on the green square to return





**Exercise 1(d)**

$$\begin{aligned}\frac{(2x)^2}{4x} &= \frac{2 \times x \times 2 \times x}{2 \times 2 \times x} \\ &= \frac{\cancel{2} \times \cancel{x} \times \cancel{2} \times x}{\cancel{2} \times \cancel{2} \times \cancel{x}} \\ &= x.\end{aligned}$$

Click on the green square to return



**Exercise 1(e)**

$$\begin{aligned}\frac{5y + 2y^2}{7y} &= \frac{y \times (5 + 2y)}{7 \times y} \\ &= \frac{\cancel{y} \times (5 + 2y)}{7 \times \cancel{y}} \\ &= \frac{5 + 2y}{7}.\end{aligned}$$

Click on the green square to return



**Exercise 1(f)**

$$\begin{aligned}\frac{5ax}{15a + 10a^2} &= \frac{5 \times a \times x}{5 \times a \times (3 + 2a)} \\ &= \frac{\cancel{5} \times \cancel{a} \times x}{\cancel{5} \times \cancel{a} \times (3 + 2a)} \\ &= \frac{x}{3 + 2a}\end{aligned}$$

Click on the green square to return



**Exercise 1(g)**

$$\begin{aligned}\frac{2z^2 - 4z}{2z^2 - 10z} &= \frac{2 \times z \times (z - 2)}{2 \times z \times (z - 5)} \\ &= \frac{\cancel{2} \times \cancel{z} \times (z - 2)}{\cancel{2} \times \cancel{z} \times (z - 5)} \\ &= \frac{z - 2}{z - 5}.\end{aligned}$$

Click on the green square to return



**Exercise 1(h)** In this case, some initial factorisation is needed (see the package on **factorising expressions**).

$$y^2 + 7y + 10 = (y + 5)(y + 2) \quad \text{and} \quad y^2 - 25 = (y + 5)(y - 5)$$

Thus

$$\begin{aligned} \frac{y^2 + 7y + 10}{y^2 - 25} &= \frac{(y + 5)(y + 2)}{(y + 5)(y - 5)} \\ &= \frac{y + 2}{y - 5} \end{aligned}$$

where the factor  $(y + 5)$  has been cancelled.

Click on the green square to return



**Exercise 1(i)** Again, in this case, some initial factorisation is needed (see the package on **factorising expressions**).

$$w^2 - 5w - 14 = (w - 7)(w + 2) \quad \text{and} \quad w^2 - 4w - 21 = (w - 7)(w + 3)$$

Thus

$$\begin{aligned} \frac{w^2 - 5w - 14}{w^2 - 4w - 21} &= \frac{(w - 7)(w + 2)}{(w - 7)(w + 3)} \\ &= \frac{w + 2}{w + 3}, \end{aligned}$$

where the factor  $(w - 7)$  has been cancelled.

Click on the green square to return



**Exercise 2(a)** The fraction is simplified by multiplying both the **numerator** and the **denominator** by 2.

$$\frac{4y - \frac{3}{2}}{2} = \frac{2(4y - \frac{3}{2})}{2 \times 2} = \frac{8y - 3}{4}$$

Click on the green square to return



**Exercise 2(b)** This fraction is simplified by multiplying both the **numerator** and the **denominator** by 4. Thus

$$\frac{2x + \frac{1}{2}}{x + \frac{1}{4}} = \frac{4(2x + \frac{1}{2})}{4(x + \frac{1}{4})} = \frac{8x + 2}{4x + 1}$$

Click on the green square to return





**Exercise 2(c)** In this case, since the **numerator** contains the fraction  $1/3$  and the **denominator** contains the fraction  $1/2$ , the common factor needed is  $2 \times 3 = 6$ . Thus

$$\frac{z - \frac{1}{3}}{z - \frac{1}{2}} = \frac{6(z - \frac{1}{3})}{6(z - \frac{1}{2})} = \frac{6z - 2}{6z - 3}$$

Click on the green square to return



**Exercise 2(d)** For this fraction the required multiplier is  $x$ .

$$\frac{2 - \frac{1}{x}}{2} = \frac{x \left(2 - \frac{1}{x}\right)}{2x} = \frac{2x - 1}{2x}$$

Click on the green square to return



**Exercise 2(e)** Here the **numerator** includes the fraction  $2/t$  and the **denominator** is the fraction  $1/2$ , so the required multiplier is  $2t$ .

$$\frac{3t - \frac{2}{t}}{\frac{1}{2}} = \frac{2t \left(3t - \frac{2}{t}\right)}{2t \left(\frac{1}{2}\right)} = \frac{6t^2 - 4}{t}$$

Click on the green square to return



**Exercise 2(f)** For the last part of this exercise, since the **numerator** includes the fraction  $1/2z$  and the **denominator** includes the fraction  $1/3z$ , the common multiplier is  $6z$ .

$$\frac{z - \frac{1}{2z}}{z - \frac{1}{3z}} = \frac{6z \left( z - \frac{1}{2z} \right)}{6z \left( z - \frac{1}{3z} \right)} = \frac{6z^2 - 3}{6z^2 - 2}$$

Click on the green square to return



**Exercise 3(a)** The *least common multiple* of the denominators is  $yz$ .  
Thus

$$\begin{aligned}\frac{2}{y} + \frac{3}{z} &= \frac{2 \times z}{y \times z} + \frac{3 \times y}{z \times y} \\ &= \frac{2z}{yz} + \frac{3y}{yz} \\ &= \frac{3y + 2z}{yz}\end{aligned}$$

Click on the green square to return



**Exercise 3(b)** Here the *least common multiple* of the denominators is  $15y$ , so

$$\begin{aligned}\frac{1}{3y} - \frac{2}{5y} &= \frac{5 \times 1}{5 \times 3y} - \frac{3 \times 2}{3 \times 5y} \\ &= \frac{5}{15y} - \frac{6}{15y} \\ &= \frac{5 - 6}{15y} = -\frac{1}{15y}\end{aligned}$$

Click on the green square to return



**Exercise 3(c)** The *least common multiple* of the denominators of the two fractions in this case is 6. Thus

$$\begin{aligned}\frac{3z+1}{3} - \frac{2z+1}{2} &= \frac{2 \times (3z+1)}{2 \times 3} - \frac{3 \times (2z+1)}{3 \times 2} \\ &= \frac{6z+2}{6} - \frac{6z+3}{6} \\ &= \frac{(6z+2) - (6z+3)}{6} = -\frac{1}{6}\end{aligned}$$

Simplification in this case has shown that the difference of these two fractions is independent of  $z$ .

[Click on the green square to return](#)



**Exercise 3(d)** The *least common multiple* of the denominators of the two fractions in this case is  $2t$ . The sum simplifies as follows.

$$\begin{aligned}\frac{3t+1}{2} + \frac{1}{t} &= \frac{t \times (3t+1)}{t \times 2} + \frac{2 \times 1}{2 \times t} \\ &= \frac{3t^2+t}{2t} + \frac{2}{2t} = \frac{3t^2+t+2}{2t}\end{aligned}$$

Click on the green square to return





**Exercise 3(e)**

Here the required *least common multiple* of the denominators is the factor  $2(x-1)$ . Proceeding as before:

$$\begin{aligned}\frac{x+1}{2} + \frac{1}{x-1} &= \frac{(x-1) \times (x+1)}{(x-1) \times 2} + \frac{2 \times 1}{2 \times (x-1)} \\ &= \frac{(x^2-1)}{2(x-1)} + \frac{2}{2(x-1)} \\ &= \frac{(x^2-1)+2}{2(x-1)} = \frac{x^2+1}{2(x-1)}\end{aligned}$$

Click on the green square to return



**Exercise 3(f)**

Here the required *least common multiple* of the denominators is

$$(w + 3)(w - 1) = w^2 + 2w - 3.$$

With this in mind,

$$\begin{aligned} \frac{2}{w + 3} - \frac{5}{w - 1} &= \frac{(w - 1) \times 2}{(w - 1) \times (w + 3)} - \frac{(w + 3) \times 5}{(w + 3) \times (w - 1)} \\ &= \frac{2w - 2}{(w + 3)(w - 1)} - \frac{5w + 15}{(w + 3)(w - 1)} \\ &= \frac{(2w - 2) - (5w + 15)}{(w + 3)(w - 1)} \\ &= \frac{-3w - 17}{(w + 3)(w - 1)} = - \left( \frac{3w + 17}{(w + 3)(w - 1)} \right) \end{aligned}$$

Click on the green square to return



**Exercise 4(a)** Taking common denominators:

$$\begin{aligned}\frac{a}{x-2} + \frac{b}{x+2} &= \frac{4x}{(x-2)(x+2)} \\ \frac{a(x+2)}{(x-2)(x+2)} + \frac{b(x-2)}{(x-2)(x+2)} &= \frac{4x}{(x-2)(x+2)} \\ \frac{(a+b)x + (2a-2b)}{(x-2)(x+2)} &= \frac{4x}{(x-2)(x+2)}\end{aligned}$$

so that  $(a+b)x + (2a-2b) = 4x$ . Equating coefficients in this case gives

$$\begin{aligned}a+b &= 4 && \text{(coefficients of } x\text{)} \\ 2a-2b &= 0 && \text{(constant terms)}\end{aligned}$$

Solving this set of equations gives  $a = 2$ ,  $b = 2$ . Hence

$$\frac{2}{x-2} + \frac{2}{x+2} = \frac{4x}{(x-2)(x+2)}$$

Click on the green square to return



**Exercise 4(b)** Taking common denominators:

$$\begin{aligned}\frac{a}{z-3} + \frac{b}{z+2} &= \frac{z+7}{(z-3)(z+2)} \\ \frac{a(z+2)}{(z-3)(z+2)} + \frac{b(z-3)}{(z-3)(z+2)} &= \frac{z+7}{(z-3)(z+2)} \\ \frac{(a+b)z + (2a-3b)}{(z-3)(z+2)} &= \frac{z+7}{(z-3)(z+2)}\end{aligned}$$

so that  $(a+b)z + (2a-3b) = z+7$ . Equating coefficients in this case gives

$$\begin{aligned}a+b &= 1 && \text{(coefficients of } z) \\ 2a-3b &= 7 && \text{(constant terms)}\end{aligned}$$

Solving this set of equations gives  $a=2$ ,  $b=-1$ . Hence

$$\frac{2}{z-3} - \frac{1}{z+2} = \frac{z+7}{(z-3)(z+2)}$$

Click on the green square to return



**Exercise 4(c)** Taking common denominators:

$$\begin{aligned}\frac{a}{w-4} + \frac{b}{w+1} &= \frac{3w-2}{(w-4)(w+1)} \\ \frac{a(w+1)}{(w-4)(w+1)} + \frac{b(w-4)}{(w-4)(w+1)} &= \frac{3w-2}{(w-4)(w+1)} \\ \frac{(a+b)w + (a-4b)}{(w-4)(w+1)} &= \frac{3w-2}{(w-4)(w+1)}\end{aligned}$$

so that  $(a+b)w + (a-4b) = 3w-2$ . Equating coefficients in this case gives

$$\begin{aligned}a+b &= 3 && \text{(coefficients of } w\text{)} \\ a-4b &= -2 && \text{(constant terms)}\end{aligned}$$

Solving this set of equations gives  $a = 2$ ,  $b = 1$ . Hence

$$\frac{2}{w-4} + \frac{1}{w+1} = \frac{3w-2}{(w-4)(w+1)}$$

Click on the green square to return



## Solutions to Quizzes

**Solution to Quiz:** The numerator and denominator respectively factorise as

$$t^2 + 3t - 4 = (t - 1)(t + 4) \quad \text{and} \quad t^2 - 3t + 2 = (t - 1)(t - 2)$$

so that

$$\begin{aligned} \frac{t^2 + 3t - 4}{t^2 - 3t + 2} &= \frac{(t - 1)(t + 4)}{(t - 1)(t - 2)} \\ &= \frac{t + 4}{t - 2} \end{aligned}$$

where the factor  $(t - 1)$  has been cancelled from the first equation.

End Quiz

**Solution to Quiz:**

For  $\frac{x - \frac{1}{x+1}}{x - 1}$ , the common multiplier is  $(x + 1)$ . Multiplying the numerator and the denominator by this gives:

$$\begin{aligned}\frac{x - \frac{1}{x+1}}{x - 1} &= \frac{(x + 1) \left(x - \frac{1}{x+1}\right)}{(x + 1)(x - 1)} \\ &= \frac{(x + 1)x - (x + 1) \left(\frac{1}{(x+1)}\right)}{(x^2 - 1)} \\ &= \frac{x^2 + x - 1}{x^2 - 1}\end{aligned}$$

End Quiz

**Solution to Quiz:** Writing all the fractions with a common denominator

$$\begin{aligned}\frac{4x + 5}{(2x + 1)(x + 2)} &= \frac{\mathbf{a}}{2x + 1} + \frac{1}{x + 2} \\ &= \frac{\mathbf{a}(x + 2)}{(2x + 1)(x + 2)} + \frac{(2x + 1)}{(2x + 1)(x + 2)} \\ &= \frac{\mathbf{a}x + 2\mathbf{a} + 2x + 1}{(2x + 1)(x + 2)} \\ &= \frac{(\mathbf{a} + 2)x + (2\mathbf{a} + 1)}{(2x + 1)(x + 2)}\end{aligned}$$

so that  $(\mathbf{a} + 2)x + (2\mathbf{a} + 1) = 4x + 5$ . This gives two equations

$$\begin{aligned}\mathbf{a} + 2 &= 4 && \text{coefficients of } x \\ 2\mathbf{a} + 1 &= 5 && \text{constant coefficients}\end{aligned}$$

The solution is  $\mathbf{a} = 2$ .

End Quiz



**Solution to Quiz:** Writing all the fractions with a common denominator

$$\begin{aligned}\frac{a}{2x-3} + \frac{b}{3x+4} &= \frac{x+7}{(2x-3)(3x+4)} \\ \frac{a(3x+4)}{(2x-3)(3x+4)} + \frac{b(2x-3)}{(2x-3)(3x+4)} &= \frac{x+7}{(2x-3)(3x+4)} \\ \frac{(3a+2b)x + (4a-3b)}{(2x-3)(3x+4)} &= \frac{x+7}{(2x-3)(3x+4)}\end{aligned}$$

so that  $(3a+2b)x + (4a-3b) = x+7$ . This gives two equations

$$3a + 2b = 1 \quad (\text{coefficients of } x)$$

$$4a - 3b = 7 \quad (\text{constant terms})$$

Solving these gives  $a = 1$ ,  $b = -1$ .

End Quiz