Basic Mathematics

Brackets

R Horan & M Lavelle

The aim of this document is to provide a short, self assessment programme for students who wish to acquire a basic competence at simplifying brackets.

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1. Brackets (Introduction)

Quantities are enclosed within brackets to indicate that they are to be treated as a single entity. If we wish to subtract, say, $3a - 2b$ from $4a - 5b$ then we do this as follows.

Example 1

(a)
$$\begin{aligned} (4a - 5b) - (3a - 2b) &= 4a - 5b - 3a - (-2b) \\ &= 4a - 5b - 3a + 2b \\ &= 4a - 3a - 5b + 2b \\ &= a - 3b\,. \end{aligned}$$

and similarly

(b)
$$\begin{aligned} (7x + 5y) - (2x - 3y) &= 7x + 5y - 2x - (-3y) \\ &= 7x + 5y - 2x + 3y \\ &= 7x - 2x + 5y + 3y \\ &= 5x + 8y\,. \end{aligned}$$

When there is more than one bracket it is usually necessary to begin with the inside bracket and work outwards.

Example 2
Simplify the following expressions by removing the brackets.

(a) $3a - c + (5a - 2b - [3a - c + 2b])$,

(b) $-\{3y - (2x - 3y) + (3x - 2y)\} + 2x$.

Solution

(a) We have

$$\begin{aligned} 3a - c + (5a - 2b - [3a - c + 2b]) &= 3a - c + (5a - 2b - 3a + c - 2b) \\ &= 3a - c + (2a - 4b + c) \\ &= 3a - c + 2a - 4b + c \\ &= 3a + 2a - 4b - c + c \\ &= 5a - 4b\,. \end{aligned}$$

(b) Similarly we have

$$\begin{aligned}
-\{3y-(2x-3y)+(3x-2y)\}+2x &= -\{3y-2x+3y+3x-2y\}+2x \\
&= -\{3y+3y-2y+3x-2x\}+2x \\
&= -\{4y+x\}+2x \\
&= -4y-x+2x \\
&= x-4y\,.
\end{aligned}$$

EXERCISE 1. Remove the brackets from each of the following expressions and simplify as far as possible. (Click on green letters for solutions.)

(a) $x-(y-z)+x+(y-z)-(z+x)$,
(b) $2x-(5y+[3z-x])-(5x-[y+z])$,
(c) $(3/a)+b+(7/a)-2b$,
(d) $a-(b+[c-\{a-b\}])$.

2. Distributive Rule

A quantity outside a bracket multiplies *each* of the terms inside the bracket. This is known as the **distributive rule**.

Example 3

(a) $3(x - 2y) = 3x - 6y$.

(b) $2x(x - 2y + z) = 2x^2 - 4xy + 2xz$.

(c) $7y - 4(2x - 3) = 7y - 8x + 12$.

This is a relatively simple rule but, as in all mathematical arguments, a great deal of care must be taken to proceed correctly.

Exercise 2. Remove the brackets and simplify the following expressions. (Click on green letters for solution.)

(a) $5x - 7x^2 - (2x)^2$ (b) $(3y)^2 + x^2 - (2y)^2$
(c) $3a + 2(a + 1)$ (d) $5x - 2x(x - 1)$
(e) $3xy - 2x(y - 2)$ (f) $3a(a - 4) - a(a - 2)$

In the case of *two* brackets being multiplied together, to simplify the expression first choose *one* bracket as a single entity and multiply this into the other bracket.

Example 4 For each of the following expressions, multiply out the brackets and simplify as far as possible.

(a) $(x+5)(x+2)\,,$ (b) $(3x-2)(2y+3)\,.$

Solution

$$
\begin{array}{lrcl}
(a) & (x+5)(x+2) & = & (x+5)x+(x+5)2 \\
 & & = & x(x+5)+2(x+5) \\
 & & = & x^2+5x+2x+10 \\
 & & = & x^2+7x+10\,.
\end{array}
$$

$$
\begin{array}{lrcl}
(b) & (3x-2)(2y+3) & = & (3x-2)2y+(3x-2)3 \\
 & & = & 2y(3x-2)+3(3x-2) \\
 & & = & 6xy-4y+9x-6\,.
\end{array}
$$

Try this short quiz.

Quiz To which of the following does the expression

$$(2x - 1)(x + 4)$$

simplify?

(a) $2x^2 - 2x + 4$
(b) $2x^2 - 7x + 4$
(c) $2x^2 + 7x - 4$
(d) $2x^2 + 2x - 4$

3. FOIL

When it comes to expanding a bracket like $(a + c)(x + y)$ there is a simple way to remember all of the terms. This is the word **FOIL**, and stands for

take products of the

First **O**utside **I**nside **L**ast

This is illustrated in the following.

Example 5

$$(a + c)(x + y) = \overset{\mathbf{F}}{ax} + \overset{\mathbf{O}}{ay} + \overset{\mathbf{I}}{cx} + \overset{\mathbf{L}}{cy} \ .$$

These terms are the products of the pairs highlighted below.

$$\overbrace{(a + c)(x}^{\text{F}} + y),\ \overbrace{(a + c)(x + y)}^{\text{O}},\ (a + \overbrace{c)(x}^{\text{I}} + y),\ (a + \overbrace{c)(x + y)}^{\text{L}}.$$

There are two other brackets that are worth remembering. These are

$(x + y)^2$, which is a *complete square*, and

$(x + y)(x - y)$, which is a *difference of two squares*.

These are included in the following exercises.

EXERCISE 3. Remove the brackets from each of the following expressions using FOIL.

(a) $(x + y)^2$
(b) $(x + y)(x - y)$
(c) $(x + 4)(x + 5)$
(d) $(y + 1)(y + 3)$
(e) $(2y + 1)(y - 3)$
(f) $2(x - 3)^2 - 3(x + 1)^2$

Quiz To which of the following expressions does $9 - (x - 3)^2$ simplify?

(a) $-x^2$
(b) $6x - x^2$
(c) $18 - x^2$
(d) $6x + x^2$

4. Quiz on Brackets

Begin Quiz In each of the following, remove the brackets, simplify the expression and choose the solution from the options given.

1. $(a + 2m)(a - m)$

(a) $a^2 - am - 2m^2$ (b) $a^2 + am - 2m^2$

(c) $a^2 + 2m^2 - am$ (d) $a^2 + 2am + 2m^2$

2. $(3b - a)(2a + 3b)$

(a) $6b^2 + a^2 - 3ab$ (b) $9b^2 + 3ab - 2a^2$

(c) $9b^2 + 9ab - 3b^2$ (d) $6b^2 + 3ab - a^2$

3. $(2x + 1)^2 - (x + 3)^2$

(a) $x^2 - 8$ (b) $x^2 - 2x - 8$

(c) $3x^2 - 8$ (d) $3x^2 - 2x - 8$

4. $3(x + 2)^2 - (x - 2)^2$

(a) $2x^2 + 16x + 8$ (b) $2x^2 + 16$

(c) $4x^2 + 8x + 16$ (d) $4x^2 - 16$

End Quiz Score: Correct

Solutions to Exercises

Exercise 1(a)

$$
\begin{aligned}
& x-(y-z)+x+(y-z)-(z+x) \\
= \;& x-y+z+x+y-z-z-x \\
= \;& x+x-x-y+y+z-z-z \\
= \;& x-z\,.
\end{aligned}
$$

Click on green square to return

□

Exercise 1(b)

$$\begin{aligned} & 2x-(5y+[3z-x])-(5x-[y+z]) \\ = \;& 2x-(5y+3z-x)-(5x-y-z) \\ = \;& 2x-5y-3z+x-5x+y+z \\ = \;& 2x+x-5x-5y+y-3z+z \\ = \;& -2x-4y-2z\,. \end{aligned}$$

Click on green square to return

Exercise 1(c)

$$\begin{aligned} \frac{3}{a} + b + \frac{7}{a} - 2b &= \frac{3}{a} + \frac{7}{a} + b - 2b \\ &= \frac{3+7}{a} - b \\ &= \frac{10}{a} - b\,. \end{aligned}$$

Click on green square to return ☐

Exercise 1(d)

$$
\begin{aligned}
a-(b+[c-\{a-b\}]) &= a-(b+[c-a+b]) \\
&= a-(b+c-a+b) \\
&= a-(2b+c-a) \\
&= a-2b-c+a \\
&= 2a-2b-c\,.
\end{aligned}
$$

Click on green square to return □

Exercise 2(a)

First note that $(2x)^2 = (2x) \times (2x) = 4x^2$.

$$\begin{aligned} 5x - 7x^2 - (2x)^2 &= 5x - 7x^2 - 4x^2 \\ &= 5x - 11x^2 \end{aligned}$$

Click on green square to return

Exercise 2(b)

$$\begin{aligned}(3y)^2 + x^2 - (2y)^2 &= 9y^2 + x^2 - 4y^2 \\ &= 9y^2 - 4y^2 + x^2 \\ &= 5y^2 + x^2\end{aligned}$$

Click on green square to return □

Exercise 2(c)

$$\begin{aligned} 3a + 2(a+1) &= 3a + 2a + 2 \\ &= 5a + 2 \end{aligned}$$

Click on green square to return □

Exercise 2(d)

$$
\begin{aligned}
5x - 2x(x-1) &= 5x - 2x^2 + 2x \\
&= 7x - 2x^2
\end{aligned}
$$

Click on green square to return □

Exercise 2(e)

$$\begin{aligned} 3xy - 2x(y-2) &= 3xy - 2xy + 4x \\ &= xy + 4x \end{aligned}$$

Click on green square to return □

Exercise 2(f)

$$\begin{aligned} 3a(a-4) - a(a-2) &= 3a^2 - 12a - a^2 + 2a \\ &= 3a^2 - a^2 + 2a - 12a \\ &= 2a^2 - 10a \end{aligned}$$

Click on green square to return □

Exercise 3(a)

$$\begin{aligned}(x+y)^2 &= (x+y)(x+y)\\ &= x^2+xy+yx+y^2 \quad \text{using FOIL}\\ &= x^2+2xy+y^2\end{aligned}$$

This is an **IMPORTANT** result and should be committed to memory. Here x is the *first* member of the the bracket and y is the *second*. The rule for the *square* of $(x+y)$, i.e. $(x+y)^2$ is

$$\begin{array}{ccccc} x^2 & + & 2xy & + & y^2 \\ \text{(square the first)} & + & \text{(twice the product)} & + & \text{(square the last)} \end{array}$$

Click on green square to return

Exercise 3(b)

Using FOIL again:

$$\begin{aligned}(x+y)(x-y) &= x^2 - xy + yx - y^2 \\ &= x^2 - y^2\end{aligned}$$

The form of the solution is the reason for the name difference of two squares. This is another important result and is worth committing to memory.

Click on green square to return

□

Exercise 3(c)

Using FOIL:

$$\begin{aligned}(x+4)(x+5) &= x^2+5x+4x+20\\ &= x^2+9x+20\end{aligned}$$

Click on green square to return □

Exercise 3(d)

Using FOIL:

$$\begin{aligned}(y+1)(y+3) &= y^2+3y+y+3 \\ &= y^2+4y+3\end{aligned}$$

Click on green square to return □

Exercise 3(e)

Using FOIL:

$$\begin{aligned}(2y+1)(y-3) &= 2y^2-6y+y-3\\ &= 2y^2-5y-3\end{aligned}$$

Click on green square to return □

Exercise 3(f)

This one is best done in parts. First we have

$$(x-3)^2 = x^2 - 6x + 9$$

and

$$(x+1)^2 = x^2 + 2x + 1$$

Thus

$$\begin{aligned} 2(x-3)^2 - 3(x+1)^2 &= 2(x^2-6x+9) - 3(x^2+2x+1) \\ &= 2x^2 - 12x + 18 - 3x^2 - 6x - 3 \\ &= 2x^2 - 3x^2 - 12x - 6x + 18 - 3 \\ &= -x^2 - 18x + 15 \end{aligned}$$

Click on green square to return □

Solutions to Quizzes

Solution to Quiz:

$$\begin{aligned}(2x-1)(x+4) &= (2x-1)x + (2x-1)4 \\ &= (2x^2 - x) + (8x - 4) \\ &= 2x^2 - x + 8x - 4 \\ &= 2x^2 + 7x - 4\end{aligned}$$

End Quiz

Solution to Quiz:

First note that $(x-3)^2 = x^2 - 6x + 9$, so

$$\begin{aligned} 9 - (x-3)^2 &= 9 - (x^2 - 6x + 9) \\ &= 9 - x^2 + 6x - 9 \\ &= 9 - 9 - x^2 + 6x \\ &= -x^2 + 6x = 6x - x^2 \end{aligned}$$

End Quiz